Math Explorer

MATH & ARCHITECTURE

The Wright design!
The Pantheon speaks volumes?
It’s Hemispheric!!!

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Architecture has always relied on mathematical principles. A Greek structure like the Parthenon is aesthetically pleasing because of its mathematical harmony and balance. The buildings of the famous American architect, Frank Lloyd Wright, who built over 1,000 buildings throughout the world, are noted for their great sense of balance and composition. Wright's architectural education began as a child, when his mother helped him discover principles of design with building blocks. He also studied classical music as a child and later compared the mathematical principles of architecture to those of music. Wright had a unique philosophy of architecture: buildings should express truth and unity. He believed buildings should be simple and suited for their purpose: “form follows function.” Most importantly, he wanted to avoid box-like buildings, and this meant that a composition of many forms was necessary. Though Wright died in 1959, his ideas would continue to influence American architecture to this day.

How do you bring complex shapes together to form a cohesive whole? Often, Wright's geometric designs were interpretations of nature. What shapes do you see in nature that could be re-created in a building? If you were to design a school, for example, how can the many volumes used throughout a school-day be brought together to express a special place. Will you have to calculate how much material you will need when constructing your design? Certainly math is required when designing the structure of the building, the engineering that makes certain that the building sits solidly on the ground. But you will also have to figure out the total area of the school to make sure there is enough room to study, eat, and play sports. You will also need to calculate how many materials you will need to build the school, and how much those materials will cost.

by Laura Chavkin, who is an undergraduate at Yale University. She enjoys American literature and writing.
PROBLEMS OF THE MONTH

1. A 3 x 3 grid of lights has a switch for each row and a switch for each column. In the beginning, all switches are down and “off.” Flipping a switch changes each “on” light to “off” and each “off” light to “on” in that row or column. How many different light patterns can be produced?

2. Susie went to the pet store to buy some pets. She selected her pets from a dog, a rabbit, and a fish. If Susie picked either 0, 1, 2, or 3 pets, how many ways could she make her selection? Suppose she picked 0, 1, 2, 3, or 4 pets from a dog, rabbit, fish, and snake. How many ways could she make her selection? Suppose Susie had 5 pets to choose from. How many ways could she make her selection? Do you see a pattern?

3. A 12-hour digital clock shows times from 1:00 to 12:59. How many times in a 12 hour period does the time read the same forward and backward? Ignore the colon : in each reading. (Examples are 7:17 and 10:01).

4. What is 1000-999+998-997+...-4-3-2-1=

5. If you write out the numbers from 1 to 200, how many zeros do you use?

6. How many zeros are at the end of 100!?
   (Remember 100! = 100 x 99 x 98 x ... x 4 x 3 x 2 x 1.)

7. A fast food store sells chicken sticks in boxes of 5, 9, and 12 pieces. What is the largest number of chicken sticks that you cannot be ordered? For example, you can order 21 sticks (6+6+9) by not 19 sticks.

8. A ball is dropped from a height of 75 feet. Each time it hits the ground, it bounces back to 2/5 of its previous height. How far has the ball traveled when it hits the ground the third time?

9. INGENUITY. Can you find a rectangular array of points such that the number of points on the perimeter equals the number of points in the interior?

   x  x  x  x  x  x
   x  o  o  o  x
   x  o  o  o  x
   x  x  x  x  x

Send us your solutions! Every month, we will publish the best solutions on our website: www.mathexplorer.com. If we print your solutions, we will send you and your teacher free Math Explorers pens!
The Pantheon is one of the best-preserved monuments from ancient Rome. Originally built by Agrippa in 27 B.C. as a Roman temple to honor the Olympic gods, the building was rebuilt by Emperor Hadrian in the 2nd century.

The Pantheon's large bronze doors open into an enormous circular room. Once inside the room there is a sudden feeling of openness and spaciousness. Let's see if we can find out exactly how much space this room really has. To determine this space, or volume, we need to understand the basic shapes from which the room is constructed and how to measure them.

One of the geometric shapes that makes up the Pantheon is the cylinder. A circular cylinder is like a soup can. It consists of two circular disks connected by a tube. (See figure on right.)

The other shape we need to understand is the sphere. A sphere is like a ball. It consists of all the points that are a certain distance from a fixed point called the center. A hemisphere is half a sphere. It has both a flat circular surface and half a sphere's rounded surface. (See figures on left.)

The interior space of the Pantheon is a cylinder with a hemispherical dome on top. An opening at the top of the dome lets in the building's only natural light. The dome has a span, or diameter, of 43.2 m (142 ft.). The repeated use of this dimension in the interior height gives the Pantheon's proportions a beautiful balance. Both the floor diameter and the interior height are 43.3 m, just slightly larger than the diameter of the hemisphere. If we can imagine, an exact replica of the dome, turned upside-down, would touch just the center of the floor. This is how a cross-section looks.
The interior of a three-dimensional figure such as a cylinder or a sphere has a volume, which measures the amount of space that it encloses. Remember that when we measure the area of an enclosed region we use square units such as square meters or square feet.

In measuring the volume of a three dimensional geometric shape we use cubes using units such as cubic meters or cubic feet (see figure right.).

To find the volume of the cylindrical part of the Pantheon we use the geometric formula for the volume of a cylinder given by: \( V = \pi r^2 h \), where \( V \) stands for the volume; \( r \) the radius (half the diameter) of the cylinder; \( h \) is the height of the cylinder; and \( \pi \) is the irrational number approximately equal to 3.14. (\( \pi \) is the constant ratio of the circumference of a circle to its diameter.)

In the case of the Pantheon, we can calculate the volume of the cylindrical part using the measurements \( r = 43.3 / 2 \) m and \( h = 21.7 \) m. Plugging these values into the formula above gives us: \( V = \pi (43.3 / 2 \text{ m})^2 (21.7 \text{ m}) \). You can use your calculator to get an approximation. Did you get about 32,000 cubic meters?

To compute the volume of the hemisphere we use the geometric formula for the volume of a sphere given by: \( V = \frac{4}{3} \pi r^3 \). Because a hemisphere is exactly half a sphere, the geometric formula for the volume of a hemisphere is given by: \( V = \frac{2}{3} \pi r^3 \).

The Pantheon's dome has a span (diameter) of 43.2 m. This means that the radius, which is half of the diameter, is \( r = 43.2 / 2 \text{ m} = 21.6 \text{ m} \).

The volume of the hemisphere must be: \( V = \frac{2}{3} \pi (21.6 \text{ m})^3 \). Using a calculator, we find that the volume of a hemisphere is approximately 21,100 cubic meters. So, we have computed the interior space of the two shapes that make up the Pantheon. The total space, therefore, equals: 32,000 + 21,100 cubic meters, or approximately 53,000 cubic meters. You can use the formula for a cylinder to find the volume of a tin can, or even the amount of liquid a straw will hold. Use the formula for sphere to find the volume of any ball, ice cream scoop, or bubble!
Clever Cuts

After thinking awhile, Julie divided this figure into 4 parts of the same size and shape. Now you have to divide the first figure into 5 parts of the same size and same shape. How can it be done?

Shadows to Shapes

Exercise your spatial imagination! For each set of three shapes at right, create a three-dimensional object that can cast shadows in each of the three shapes.

Well What?

The diagram to the left indicates a land with four wells on it. Divide this land into four regions that have the same size and shape and have one well each.
FIPSE Sponsors Summer Math Camps

Funds for Improvement of Postsecondary Education (FIPSE), is sponsoring 3-year training program for teachers to offer the SWT Junior Summer Math Camp in the Rio Grande Valley. The program is being coordinated by Elaine Hernandez, Director of Continuing Education at South Texas Community College. Call her at (956) 971-3712 for more information.

Thanks Southwestern Bell!

Southwestern Bell Communications (SBC) Foundation sponsored the Rio Grande Valley Summer Math Program, which was coordinated by Adelina Alaniz from Mission. Participating school districts included McAllen, Donna, Progreso, and Mission. Welcome to our new partners! And a special thanks to SBC which made the program possible!

Having Fun at Camp!

Jessie Tijerina gets ready to go off on the integer highway at the McAllen Junior Summer Math Camp, held this summer at Brown Middle School in McAllen, Texas.

Rachelle Meyers, from San Marcos, helps teach students about the “integer highway” at the McAllen Junior Summer Math Camp.

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