TEXAS STATE UNIVERSITY

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Discrete Mathematics Seminar

Time:	Friday, 15 September 2017, 2:15 – 3:15 PM
Location:	237 Derrick Hall
Title:	From Prime Ladders to Goldbach Conjectures
Speaker:	Dr. Nathaniel Dean, Department of Mathematics, Texas State University (retired)

Abstract:

A *prime labeling* of a graph $G = (V, E)$ is a labeling of the vertices with members of $\{1, 2, \ldots, |V|\}$ where the labels are distinct and, for each edge uv, the labels for u an v are relatively prime. A graph is called *prime* if it has a prime labeling. The Prime Ladder Conjecture states that every ladder is prime. First, we consider when a prime labeling may be done in a cyclic manner around the vertices of the ladder. This leads to a conjecture concerning the representation of even numbers as the difference of two primes. Finally, we prove the Prime Ladder Conjecture.

The following result is essential to the proof and relates to the celebrated Goldbach Conjecture. Define a canonical partition of an integer n as a representation of n as the sum of odd primes $p_1, p_2, \ldots, p_m$ where $p_j \geq 2\sum_{i=1}^{j-1} p_i + 3$ for all $j \in \{2, 3, \ldots, m\}$. Every integer $n \geq 50$ has a canonical partition. This leads to the following two conjectures which are stronger than those of Goldbach.

1. Every even integer $n \geq 14$ is the sum of two odd primes, one of which is at most $n/3 - 1$.

2. Every odd integer $n \geq 51$ is the sum of three odd primes, one of which is at most $(n - 12)/9$.