Finite Element Approximations of Singularities

First, we review the finite element formulation in a general mathematical setting. Then, we discuss recent advances in the development of effective finite element algorithms approximating a class of singular solutions, including corner singularities with different boundary conditions and singularities from the non-smooth points on the interface in transmission problems. We establish a-priori estimates (well-posedness, regularity, and the Fredholm property) for the singular solution in weighted Sobolev spaces. Then, based on these theoretical results, we propose a simple and explicit construction of the finite element space to recover the optimal convergence rate of the numerical solution. We also mention applications in physics and engineering.

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