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SECTION 3 – GENERAL TEACHING STRATEGIES Page 8

Strategies that include research-based “what works” teacher actions for English Language Learners as well as for all students who often struggle when learning mathematics concepts. Strategies focus on the learning of mathematics by Hispanic students.

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APPENDIX A – MELL CLASSROOM PRACTICES FRAMEWORK Page 92

Covers the background for the MELL project and provides the conceptual framework for project activities.

Teachers guide created by the SHSU MELL Group, November 2005, in collaboration with the Texas State University System and the Texas Education Agency.
SECTION 1 – PURPOSE AND STRUCTURE OF TEACHER’S GUIDE

This teacher’s guide is designed to help teachers make mathematics more meaningful and understandable for the English language learner (ELL). The initial focus of this guide is for grades 7-11, since these are the grade levels where the mathematics content is taught or reviewed for the Mathematics Grade 11 Exit Level Test. In addition to understanding the common issues that the ELL faces in a mathematics classroom (see Section 2 of this guide), we have included general teaching strategies (see Section 3 of this guide) that help the ELL when learning mathematics content. In Section 4, we provide a more detailed plan for teaching the ten mathematics objectives covered on the Exit TAKS Test.

A standard format was created to focus on four main areas in this teacher's guide and it includes:

1. Mathematics content for grades 7-11
   - Focus on the mathematics concepts covered on the Exit Level TAKS

2. Understanding the English language learner in relation to learning mathematics
   - Understand background and culture of ELLs
   - Diagnose both language needs and current mathematical understanding

3. Instructional strategies for learning mathematics
   - Develop appropriate mathematical skills and applications
   - Develop and improve vocabulary
   - Improve English language acquisition
   - Support diverse learning styles and multiple intelligence

4. Multiple Assessments of Student Learning in Mathematics
   - Use a variety of appropriate assessment tools
   - Use out-of-class projects to connect the mathematical content to contextual experiences of student and family-life of the English Language Learner
   - Ensure assessments are as free of bias as possible.
This teacher’s guide is part of a multi-year project by the Texas State University System, in collaboration with the Texas Education Agency. The project website, www.tsusmell.org, provides more information on the overall project. The Mathematics for English Language Learners (MELL) Classroom Practices Framework was used as a guide in the development of this teacher’s guide. This document is included as Appendix A and is also available on the www.tsusmell.org website.

The ideas and teaching strategies discussed in the teacher’s guide are based on research in the field of teaching mathematics to ELLs, as well as from the experiences of teachers in classrooms with ELLs, and other educators. Sul Ross University educators conducted an exhaustive review in this area in 2005, and the results of this review are available to interested readers at the following website: www.tsusmell.org.

In addition, Sul Ross University educators developed a Quick Start Professional Development Module, which is available on-line for use by teachers and teachers of teachers, beginning in the fall of 2005. This module will include numerous links to applicable ELL documents, which were prepared as a part of this overall project or by external experts in the field. To find out more about the Quick Start Module, look on the following website: www.tsusmell.org.
SECTION 2 -- BACKGROUND

It is recognized that some Spanish-speaking groups prefer to be called Spanish, Hispanic, Latino, Hispanic Americans, or possibly by another name referring to their country of origin (e.g., Cuban or Columbian). Since the U.S. Census Bureau uses the term Hispanic to refer to all Spanish-speaking groups, regardless of race, the term Hispanic will be used in this teacher’s guide. The Hispanic population is the fastest and largest growing minority in the United States. According to the 2000 census, approximately 40 million Hispanics live in the U.S. with more than 40% of Hispanic families reporting to have children under 18 years old. Some Hispanic students live at or below the poverty line, which may lead to issues connected to low socioeconomic status. English language learners (ELLs) among Hispanic students may face challenges related to their learning of mathematics. Many school-related problems that the English language learner (ELL) experiences may be related to cultural differences, difficulties in learning the English language, or minimal prior knowledge of mathematics concepts. This section will help teachers better understand some of the concerns about teaching mathematics to ELLs. Even though the following information specifically targets Hispanic ELLs, these ideas may be applied to many students having difficulties learning mathematics.

Variables that impact English language learners

There are many variables that impact the ELL as a student, other than ethnicity. Some of these include socioeconomic class, geographic region, primary language, religion, family structure, and number of generations in the United States. Many Hispanic children come to school with less developed scholastic skills and little or no early childhood education. In addition, some Hispanic parents exhibit low educational expectations for their children and do not emphasize academic achievement.

- Family connections are important in the Hispanic culture

Hispanic youth feel a moral responsibility and commitment to their families. They are united by customs, language, religion and values and are likely to adopt their parents’ commitment to religious and political beliefs, occupational preferences, and lifestyle. Family commitment involves loyalty, a strong support system, respect of elders, and an obligation to care for family members. Stereotyped sex roles still exist among many Hispanics where the male is perceived as being dominant and the female as nurturing. Cultivating personal relationships and alliances is also important to Hispanics. They would rather submit to peer pressure and choose not to do well in school than face humiliation by being called a “whitie” or “school-boy.” Values like bonding with family and friends are in conflict with that of mainstream adolescents who strive for independence. Therefore, identity formation and individuality are challenging and sometimes problematic for the Hispanic student. The family’s economic situation or
peer rejection may also cause Hispanic youth to have a low self-image. Lack of self-confidence may cause ELLs to have lower educational expectations for themselves, and therefore they may not reach their academic potential.

- Family obligations

Many Hispanic youth will say that school is important to them, but sometimes family and economic issues become a priority. For example, the need for financial support may entice youth at age 13 to work in the family lawn care business rather than go to school. Some youth work long hours at night performing low-skilled jobs and are still expected to go to school during the day, trying to concentrate on classroom instruction. Often there are extended families living in the same household. Older children are expected to take care of (baby-sit) younger children and do household chores. In addition, at home there is seldom any privacy or time to do homework.

- Access to technology

Hispanic households have limited access to updated technology and educational aids. About 40% of Hispanic households have computers, compared to more than 70% of Caucasian households. The television is usually tuned to the Spanish channel with little or no access to educational channels. In addition, since Hispanic families may be of low socioeconomic status, other educational resources such as books, software programs, mathematics games, etc. may not be readily available to use for enrichment or extra help.

Impact of language on mathematics learning

It usually takes an ELL more than one year to develop conversational language and five to seven years to develop sufficient academic language to learn in English. The ELL may have problems with mathematics language because it uses technical terms including homonyms and synonyms. The English language structures such as word order and syntax are sometimes different than the student’s native language. In addition, the teacher may be using idioms, figurative language, and regional dialects that can confuse the ELL. When translating words literally, without regard to language context, the semantics is sometimes lost. Examples include: many different English words that imply “add,” (e.g., plus, combine, and, sum, and increase by); words with multiple meanings like “fix” or “table;” and logical connectors (e.g., therefore, consequently, if, because, and however) used in mathematical problems. Lack of English language knowledge may also result in low self-confidence. Consequently, ELLs sometimes have reservations about participating and interacting in class, asking questions, attempting a task, showing work, and explaining answers, due to limited vocabulary and language proficiency.
• More time is required to interpret new concepts and vocabulary

The ELL needs more time to decipher and understand the language involved with a mathematics concept or word problem. Sometimes mathematics terms, phrases or abstract ideas have no direct translation to the student’s native language. Therefore, it is difficult for the student to stay at the same pace in the classroom as a native English speaker. Without a rich mathematics vocabulary, the ELL will need more time to keep up with native English speakers. Every new term they learn must be embedded in familiar contexts, and this takes time when working in a second language.

• Grade placement and academic background knowledge

When ELLs enter the U.S., they are often placed at a grade level appropriate to their age, not according to their academic background. If the background knowledge needed to be successful at that grade level is not sufficient, the result can be frustrating for the student. For example, as a 14-year old, they may be placed in eighth grade, even though their mathematics knowledge is at the sixth-grade level and their English reading skills are at the fourth-grade level. Remediation of language and prior content knowledge requirements may be necessary for the student to be academically successful.

**Mathematics content related issues**

English language learners may have prior knowledge of mathematic concepts and skills that may conflict with those used in the U.S. classroom. In mathematics, inconsistencies may occur in use of symbols, algorithms, measurement systems and sequencing of content.

• Use of symbols and numbers

The formation of numbers varies from country to country. An example of difference in numbers includes the numbers one and seven where one is a straight vertical line with a base and an acute angled line to the left of the vertical line. This may in some instances be confused with the way the seven is formed. To distinguish between the two numbers, a line is placed through the middle of the vertical line. See figures below:

\[
\begin{align*}
\text{ for 1} \\
\text{ for 7}
\end{align*}
\]

In some cases, the placement of decimal points and commas differ in other countries. For example, the number 1.000,50 in some countries would translate to 1,000.50 in the United States.
• Use of algorithms

The subtraction and division mathematical operations are formatted and sometimes taught differently in other countries. In some countries, subtraction borrows from the bottom numbers rather than the top numbers like in the United States. Also, in division, the dividend and divisor are sometimes reversed and the answer may be placed below the dividend.

• Measurement system

All Hispanic countries use the metric system and do not teach the English measuring system in schools, and so ELLs often have no prior experience in working with the U.S. measurement system. In addition, currency (peso versus dollar), temperature (Celsius versus Fahrenheit), and time (military-style 24-hour clock versus the U. S. 12-hour clock (with AM and PM) are not universal systems among countries.

• Variation in instructional strategies and topics

Mathematics curriculum in other countries is sequenced in a different manner. Calculation may be emphasized more than concept understanding, so they may be reluctant to show work. Mathematics curriculum is not spiral in many countries, and students may understand number operations, but geometry concepts may be new ideas to some students. Many ELLs have not seen or worked with manipulatives, and they may not take the lesson seriously. Estimating skills may not have been previously emphasized.
SECTION 3 – GENERAL TEACHING STRATEGIES

This section provides research-based teaching strategies that mathematics teachers can adapt to reduce some of the barriers often found by English language learners (ELLs). It is based on the MELL Classroom Practices Framework, which is available on the www.tsusmell.org website and in Appendix A of this document. Teachers will find that many of the instructional techniques will also help other students having difficulty learning mathematics (and perhaps other subjects as well). It is important to know the cultural and academic backgrounds of all students, as well as their mathematics background knowledge, which may influence the student’s learning. This section describes strategies for getting to know the English language learner as an individual, focuses on classroom practices, and details some general teaching strategies, giving examples for their use.

The English language learner

There are various strategies for getting to know the English language learner as an individual, and these are described in the following paragraphs.

• Gather assessment data

At the beginning of the school year, assess the needs of ELLs in your classroom. At the time of enrollment, school personnel such as the counselor, registrar, and English as a Second Language (ESL) teacher collect important student documentation, past academic assessment records, and language assessment information (including language(s) spoken in the home), which are maintained in the student’s permanent records. Teachers should review these documents to get a better idea of the current levels of achievement and language understanding of the ELLs in their classrooms. After reviewing the documentation, it may be necessary to administer a diagnostic mathematics test or the teacher can make informal assessments based on in-class individual performance to evaluate the extent of English language and mathematics understanding, as well as the comfort level of each ELL. More information can also be obtained from other teachers having the same students and possibly the student’s parents or other relatives. There may be older children or relatives at home who are more skilled in English and who can also be a resource to help the ELL.

• Value student’s background

Value and respect the language, customs, and culture of the ELL. Be knowledgeable about Spanish customs, traditions, and holidays, and use these as context to make mathematics meaningful for Hispanic students. The teacher’s cultural sensitivity and awareness will help the Hispanic ELL feel comfortable in the classroom.

Teachers guide created by the SHSU MELL Group, November 2005, in collaboration with the Texas State University System and the Texas Education Agency.
• Encourage family involvement

Hispanic family involvement is critical for the student’s academic success. Teachers should respect parents and view them as capable partners in their child’s education. Contact parents at the beginning of the school year. Be the first to contact parents; don’t wait for them to contact you. Make the first contact positive and welcome the family to come to the school at any time. Invite parents to attend a “parent orientation” meeting and go over expectations for the year. If you don’t speak Spanish, ask a Spanish-speaking student, friend, teacher’s aide, or other Spanish-speaking community member to help you contact those parents who only speak Spanish. Some suggestions for involvement in the classroom include tutors, helpers, and field trip volunteers. For continued involvement, have students conduct real-world projects which would include their families, involving such issues as automobile insurance, sales tax on items, installment payments on furniture, etc.

• Use appropriate Spanish phrases to communicate with students

A teacher of Hispanic students does not need to speak or understand the Spanish language. However, using everyday conversational Spanish as “icebreakers” will help the Hispanic learner feel more comfortable in the mathematics classroom. Hispanic students may address you as “teacher” in addition to Mr., Mrs., or Miss as a sign of respect, or may use “maestro or maestra” for highest respect. The following phrases with their Spanish translations and pronunciations might be useful.

<table>
<thead>
<tr>
<th>English phrase</th>
<th>Spanish translation</th>
<th>Pronunciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good morning</td>
<td>Buenos días</td>
<td>Bu-eh-nos dee-as</td>
</tr>
<tr>
<td>Good afternoon</td>
<td>Buenas tardes</td>
<td>Bu-eh-nas tar-des</td>
</tr>
<tr>
<td>Hello</td>
<td>Hola</td>
<td>O-la</td>
</tr>
<tr>
<td>How are you?</td>
<td>¿Cómo está? (one student)</td>
<td>Có-mo es-tá</td>
</tr>
<tr>
<td></td>
<td>¿Cómo están? (class)</td>
<td>Có-mo es-tán</td>
</tr>
<tr>
<td>How is it going?</td>
<td>¿Qué tal?</td>
<td>Kay tal</td>
</tr>
<tr>
<td>What’s wrong?</td>
<td>¿Qué pasa?</td>
<td>Kay pa-sa</td>
</tr>
<tr>
<td>What’s up?</td>
<td>¿Su nombre, por favor?</td>
<td>Su nom-bre, por fa-vor</td>
</tr>
<tr>
<td>Do you understand?</td>
<td>¿Comprende?</td>
<td>Com-pren-de</td>
</tr>
<tr>
<td>Is it clear?</td>
<td>¿Claro?</td>
<td>Cla-ro</td>
</tr>
<tr>
<td>Of course!</td>
<td>¡Claro que sí!</td>
<td>Cla-ro kay see</td>
</tr>
<tr>
<td>It is easy!</td>
<td>¡Es fácil!</td>
<td>Es fah-seal</td>
</tr>
<tr>
<td>Ten minutes left to finish</td>
<td>Faltan diez minutos para terminar</td>
<td>Fahl-tah dee-es mee-nu-tos pah-rah teh-mee-nahr</td>
</tr>
<tr>
<td>Hurry up!</td>
<td>Apúrese</td>
<td>Ah-pu-rey-say</td>
</tr>
<tr>
<td>Listen!</td>
<td>¡Escuche! (one student)</td>
<td>Es-ku-cheh</td>
</tr>
<tr>
<td></td>
<td>¡Escuchen! (many students)</td>
<td>Es-ku-chen</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>English phrase</th>
<th>Spanish translation</th>
<th>Pronunciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Come here</td>
<td>Ven Aquí</td>
<td>Ben ah-key</td>
</tr>
<tr>
<td>Come here now!</td>
<td>¡Ven aquí, ahora!</td>
<td>Ben ah-key ah-oh-rah</td>
</tr>
<tr>
<td>If you make a mistake</td>
<td>¡Válgame dios! ¡Ay Chihuahua!</td>
<td>Bal'-ga-meh dee-ös a-ee chee-wa-wa</td>
</tr>
<tr>
<td>Very good!</td>
<td>¡Muy bien!</td>
<td>Mu-ee bee-ehn</td>
</tr>
<tr>
<td>Fantastic!</td>
<td>¡Fantástico!</td>
<td>Fan-tas'-tee-co</td>
</tr>
<tr>
<td>Magnificent!</td>
<td>¡Magnífico!</td>
<td>Mag-nee'-fee-co</td>
</tr>
<tr>
<td>Thanks for your hard work</td>
<td>Le agrédezco su buen trabajo</td>
<td>Lay ah-greh-des-co su bu-ehn trah-bah-ho</td>
</tr>
<tr>
<td>Attention, please</td>
<td>¡Atención, por favor!</td>
<td>Ah-ten-see-ohn’, por favor</td>
</tr>
<tr>
<td>Like this</td>
<td>Así</td>
<td>A-see’</td>
</tr>
<tr>
<td>Any idea?</td>
<td>¿Tiene idea?</td>
<td>Tee-e-nay ee-dey-ah</td>
</tr>
<tr>
<td>Tell me</td>
<td>Dígame</td>
<td>Dee’-gah-meh</td>
</tr>
<tr>
<td>For example</td>
<td>Por ejemplo</td>
<td>Por eh-hem-plo</td>
</tr>
<tr>
<td>Here is your homework or classwork</td>
<td>Aquí está su tarea</td>
<td>A-key’ es-ta’ su tah-rey-ah</td>
</tr>
<tr>
<td>Book</td>
<td>Libro</td>
<td>Lee-bro</td>
</tr>
<tr>
<td>Pencil</td>
<td>Lápiz</td>
<td>Lah-piece</td>
</tr>
<tr>
<td>Paper</td>
<td>Papel</td>
<td>Pah-pel</td>
</tr>
<tr>
<td>Chair</td>
<td>Silla</td>
<td>See-yah</td>
</tr>
<tr>
<td>Bathroom</td>
<td>Baño</td>
<td>Bah-nyo</td>
</tr>
</tbody>
</table>

- Involve others in bridging the language gap between ELLs and English-speaking students

Enlist the support of English-speaking students to help the ELL. This can be done via role reversal, where they can develop empathy for what the ELL must overcome when learning mathematics in a different language. An example might be for you to develop sample mathematics word problems in a different language (such as German or even a make believe language) and have the students work in groups to try to solve the problems. Using another Latin-based language (such as Portuguese) might be even more effective, since the ELL student would have an advantage over the English-speaking student.

**Classroom Practices**

There are various classroom practices that have been shown to be effective with the English language learner.
• Establish a positive classroom climate

Create an emotionally safe learning environment that helps students feel secure and willing to take risks. Help students set realistic and manageable goals based on the student’s ability. Involve students as active participants. Make the normal classroom sequence of activities structured and predictable, unless teaching a discovery lesson. Display student work, make word walls and colorful posters or pictures for classroom decorations. Add color-coded learning supports when appropriate. Focus on communication, not errors; do not allow other students to correct errors. Give students responsibility for their own learning. Include cooperative grouping activities and other opportunities to work with peers. The room arrangement may be in clusters rather than rows of desks when appropriate to the lesson. Make learning relevant to the students' experiences. Frequently use models, music for motivational purposes or introducing a lesson, gallery-walks (poster-walks), and concept maps for reinforcing mathematic concepts. For beginning or newly arrived ELLs, use conversational native language when possible. Collaborate with other teachers who have the same ELL student in their classes.

• Everyday instructional and teacher practices

Speak slowly and clearly, using a regular rate and tone of voice. When speaking, try to face the student(s) and not speak while your back is turned to the class, since many ELLs read lips and react to facial expressions. Model correct usage and word order, limiting slang words or phrases. Avoid idiomatic expressions such as “pulling your leg” or “back-off.” Use sincere facial expressions, gestures, and body language when speaking. Use longer pauses between phrases. Use shorter sentences with simple syntax. Carefully use logical connectors such as, if, because, as a result, in comparison, however, and consequently, and give examples to support understanding. Stress high frequency vocabulary (e.g., function, operation). Check for comprehension and repeat or restate explanations. Provide a longer wait time for students to answer and process information. Praise students on simple responses that they give and for accomplishments in the classroom.

• Supplement and adapt existing materials

Provide culturally rich learning materials. Have educational resources such as books, software programs, mathematics games, prior knowledge (remedial) learning aids and textbooks, and manipulatives available in the classroom. Highlight essential information and avoid distracters leaving clear, concise content. Simplify the language of instruction, not the concept being taught. Provide and explain (written and oral) lists of instructions for completing assignments. Have students work on each step before moving to the next, scaffolding or building meaningful connections between what they are learning and their personal experience. Simplify the language by incorporating pictures, charts, timelines, diagrams, maps, examples, hands-on activities, and manipulatives. When reading textbooks, have the ELL focus initially on the visuals.
rather than on the text. Assign a supportive partner to talk to the ELL about the pictures and graphs, point to key terms, and read the captions aloud.

• Assignment modifications

Provide chapter outlines or anticipation guides to students at the start of a new topic. Have students write chapter summaries prior to the assessment of their understanding of that chapter’s concepts. For beginning ELLs, have students look at the visuals in the chapter, rather than the words, writing about what they see. Have students write in their native language using English words as they develop the skills. Provide ELLs with a list of key vocabulary words prior to the lesson being taught, or have them develop their own lists as new vocabulary is needed. Allow or provide ELLs bilingual dictionaries. Encourage students to use technology and other sources to reinforce developing knowledge. Divide assignments into smaller segments. Create projects for real events and real audiences.

• Assessment modifications

Advise ELLs of exactly what they are expected to learn. Make assessment an integral part of instruction. Assemble a study guide using semantic mapping, which will help build relationships for key vocabulary, important concepts and connections. Provide multiple types of evaluation including observation, journals, group tasks, oral responses, real-world projects, rubrics, projects, portfolios, and checklists that can record growth over time. Reformat the test as needed (e.g., use more space for recording responses). Develop the test in shorter sections. Allow more time to complete the test, or give shorter tests. Don’t take off points for misspelled words in the mathematics classroom. Design tests using pictures and manipulatives to assess conceptual understanding so that success on the test is not heavily dependent on reading comprehension. Allow students to work in their primary language and then translate their solutions into English when possible.

Teaching Strategies

Various teaching strategies have been shown to be effective for the English language learner. Experienced classroom teachers of ELLS often use the following strategies.

• Identification and use of learning styles

Support multiple learning styles that include visual (seeing), auditory (hearing), kinesthetic (moving), and tactile (hands-on) categories. Learning style preference may depend on the ELL’s culture and customs, mainstream societal influences or expectations, and their previous learning experiences. For visual learners, use multiple representations such as, pictures, graphic organizers, concept maps, flow charts, graphs, diagrams, and charts to augment instruction of a mathematics concept. Many ELLs prefer these instructional aids to help them learn the words and concepts at the
same time. Tactile and kinesthetic learners can benefit by using a variety of mathematics manipulatives to develop concept understanding. For example, Cuisenaire rods help students see the relationship of the part to the whole when learning fraction concepts. And using a metric trundle wheel to measure the length of the hall in meters helps reinforce measurement concepts with a hands-on activity, rather than just solving a problem from a textbook. Allow ELLs to audiotape lessons or their own explanations about how they worked a problem. Auditory learners can review the taped material at a convenient time to reinforce their understanding.

- Grouping strategies

Many ELLs prefer to work with their classmates, rather than by themselves. Assigning (or having the student choose) a supportive learning partner is helpful. A partner can assist the ELL student with mathematics instructions and classroom procedures, as well as with school procedures (e.g., schedules for bus, lunch, fire drills, etc). Partners can also aid English language development, enhance active participation in class, and welcome new students into the established learning community.

Cooperative learning is a strategy where students work together in a positive interdependent manner to accomplish a common goal. It differs from small group work in that each individual is expected to acquire a specific knowledge or skill and be able to demonstrate his or her accomplishment on their own beyond the end of the group task. Cooperative learning techniques particularly useful in teaching mathematics are think-pair-share, jigsaw, and structured problem solving. A list of additional strategies and explanations of their use are available on the website:

http://www.utexas.edu/academic/cte/hewlettcls.html

- Teach for conceptual understanding

Teachers should place emphasis on concept understanding using multiple representations, along with computational accuracy. Understanding why a mathematics procedure works is better than just knowing how to carry out the procedure to get the answer. For example, have students measure the diameter and the circumference of various containers with a circular base, and then ask them to find the ratio of the circumference to the diameter in each case. Discovering that this ratio is always \( \pi \) is more powerful for learning than the teacher giving students a formula and having them plug in numbers to calculate answers to mathematics problems. Some ELLs prefer discovering new mathematics concepts on their own, while others may learn better though a direct teaching approach. Timed drills or repetitive problems are rarely effective with ELLs and seldom help them retain mathematical knowledge for very long. Periodically during the lesson or at the end of the lesson, review important concepts and key vocabulary, and connect these to the objectives for the lesson.
• Enhance English language skills in the mathematics classroom

The ELL can learn mathematics concepts in English on a daily basis through content-based instruction (CBI). This will help them to develop both their linguistic ability and mathematics content. The basic ideas of Sheltered Instruction are often effective for ELLs. This model includes teacher preparation actions, strategies for classroom organization, and delivery of instruction, and is designed to make academic content understandable to ELLs. Emphasize quantitative relationships, such as “more,” “less,” “three times as many,” “smaller,” etc. with models and manipulatives. These key words are important in mathematics and sometimes will help ELLs find solutions to mathematics problems. Avoid using negative statements, such as “all but,” “except,” and “which answer is not”, since these are confusing to the ELL. Also, when learning an algebraic algorithm, have copies of the words and steps for each student (or display words/phrases and steps on an overhead or computer) so that students can read and say the words aloud as they work through the steps. Supplement mathematics textbooks with English and Spanish books, as well as with videos and resources available on the Internet. Become familiar with other second language acquisition theories and English as second language (ESL) teaching strategies.

• Use graphic organizers

Graphic organizers can be helpful for the ELL by showing relationships between mathematics concepts and vocabulary or by breaking down a larger concept into smaller parts. Examples include semantic webs, Venn diagrams, flowcharts, compare and contrast charts, T-charts, timelines, maps, and other diagrams.

• Directly teach math vocabulary

Teach mathematics vocabulary, not just conversational English. Introduce new vocabulary in a variety of ways such as making models, displays, diagrams, demonstrations, charts, and drawings for new words. Review previously-learned mathematics vocabulary when needed for the current lesson. Define vocabulary used in a particular lesson in context. Use brainstorming and have students restate definitions or meanings in their own words.

• Build vocabulary glossaries

Build glossaries using “meanings” of the words in everyday language or applications rather than direct definitions. The “meaning” should describe the word in such a way that anyone who does not know what the word means will understand. In addition, a graphic organizer further reinforces vocabulary words. For example:

<table>
<thead>
<tr>
<th>English term</th>
<th>Spanish term</th>
<th>Description/meaning</th>
<th>Drawing/example</th>
</tr>
</thead>
</table>

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• Create vocabulary games

Create a fun way to explore vocabulary using games. Types of games could include Jeopardy, Charades, Bingo, and Concentration. Other activities include pantomiming, acting out new words, or having students invent their own game, including sets of rules.

• Journal writing

Conclude class discussions with students writing entries in journals or an academic notebook. Journals in general need to be kept confidential (between the student and the teacher) so that students can also communicate important personal thoughts and feelings to the teacher. Journals can also provide insight into a student’s mathematical thinking as part of an alternative assessment of student performance. Newcomers can write partial entries in their native language, but should use the journal as an opportunity to practice their writing in English. They might need assistance from their partner to write entries in English. Examples of topics might include the following:

1) What would you find in your home that would relate to today’s topic?
2) Write the following math problem in words: \(x^2 + 4 = 13\).
3) Tell me the meaning of the word parallelogram in geometry, in everyday English, and draw a picture.
4) Identify definitions and draw examples and non-examples of the vocabulary learned.
5) Create analogies using prior and new vocabulary (for example: square is to cube as rectangle is to prism).
6) List any prefix or suffix that appears regularly and write what it means.
7) List word origins (for example, linear comes from the word “line” and is Latin in origin from the word “linearis”).

• Integrate other disciplines

Thematic units across the curriculum help support the ELLs understanding of mathematics and how it relates to other disciplines. Include literature with mathematics as themes. There is a wealth of materials available that covers mathematics topics using stories. An example would be the *Sir Cumference* series of books by Cindy Neuschwander. Teachers and students can read aloud and follow the words in a book, or copies of an article. Other examples include incorporating musical notes and timing when teaching fractions and using science concepts like density for teaching ratios.
SECTION 4 - TEACHING MATHEMATICS CONCEPTS ON THE EXIT-LEVEL TAKS TEST

This section contains the specific applications of the previous strategies (see Sections 1 and 3) to the mathematics content of the Texas Grade 11 Exit Level TAKS Mathematics Test for English language learners (ELLs). Using each of the mathematics TAKS content objectives for the exit test as focus items, this section includes necessary mathematics vocabulary, specific teaching strategies, examples of performance tasks and projects, and appropriate assessment methods for ELLs. To access this information booklet on the Internet, go to www.tea.state.tx.us/student.assessment/taks/booklets/index.html and click on the exit level test.

The Grade 11 Exit Level TAKS Mathematics Test information booklet contains ten TAKS objectives. These objectives state that the student will be able to:

1. describe functional relationships in a variety of ways;
2. demonstrate an understanding of the properties and attributes of functions;
3. demonstrate an understanding of linear functions;
4. formulate and use linear equations and inequalities;
5. demonstrate an understanding of quadratic and other nonlinear functions;
6. demonstrate an understanding of geometric relationships and spatial reasoning;
7. demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes;
8. demonstrate an understanding of the concepts and uses of measurement and similarity;
9. demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems; and
10. demonstrate an understanding of the mathematical processes and tools used in problem solving.

The following information is provided for each of the above objectives:

Mathematics Content: A mathematical development of the topic and how it relates to previous learning and future learning in mathematics.

Mathematics Vocabulary: A list of appropriate English-Spanish mathematical terms and their meaning, and strategies for developing this vocabulary.

Teaching Strategies: A variety of appropriate teaching strategies for the mathematical content with an emphasis on good practices for English language learners and good strategies for teaching mathematics.

Assessment of Mathematics: Examples of performance tasks and/or projects to connect the mathematics content to contextual experiences and family-life of the

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English language learner. Included are appropriate examples of multiple-choice questions similar to the Grade 11 Exit Level TAKS, including problems to solve, applications of the mathematics topic, problem solving situations, and student projects to integrate the mathematics to student and family-life. Pay particular attention to eliminating assessment bias. In addition, the Charles A. Dana Center developed assessment materials for algebra and geometry. This document is available at the following web site:

http://www.tenet.edu/teks/math/clarifying/algebra1/alg1assess.pdf

In addition to the strategies and ideas presented in this section, Texas State University at San Marcos educators developed a summary of professional development models used in the state of Texas. These models are actively used in school districts throughout the country and provide training programs that will help teachers of English language learners. Several of these professional development models include Sheltered Instruction (SIOP), Cognitively Guided Instruction (CGI), Family Math (EQUALS), and Everyday Math. Information on these programs is available at www.tsusmell.org

Also, the National Council of Teachers of Mathematics (NCTM) published “Principles and Standards for School Mathematics” in 2000. This highly regarded document provides guidelines and examples of how students can learn important mathematical concepts and processes with understanding and is applicable for grades pre-kindergarten through Grade 12. Information on this document and on the NCTM philosophy for teaching mathematics can be found on the NCTM website at www.nctm.org and www.illuminations.nctm.org. These good teaching practices will also benefit English Language Learners.
OBJECTIVE 1

DESCRIBE FUNCTIONAL RELATIONSHIPS IN A VARIETY OF WAYS

Mathematics content:

Prior mathematics knowledge requirements:

1. understand the concept of a variable;
2. write and solve equations with one and two variables, using concrete models and algebraic expressions;
3. write and solve inequalities with one and two variables;
4. represent points as ordered pairs;
5. graph points on an x,y-coordinate plane (complete graphs with axes, positive and negative values, and all four quadrants);
6. draw complete graphs of equations and inequalities on an x,y-coordinate plane;
7. record data for one and two variables in a table form;
8. understand functional relationships as a table of values;
9. identify patterns and proportional relationships between two variables; and
10. determine the nth term of a sequence of numbers or geometric figures.

From pre-algebra activities in middle school, students should have a good understanding of variables, and how to set up and solve equations and inequalities. If students are weak in these areas, remediation may be required. Once students understand multiple ways to show a functional relationship between two variables, they will be prepared to learn Objectives 2-5 of this guide.

Minimum mathematics vocabulary needed for Objective 1:

<table>
<thead>
<tr>
<th>English term</th>
<th>Spanish term</th>
<th>Description/meaning</th>
<th>Drawing/example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cartesian coordinate system</td>
<td>sistema de Cartesiano coordenado</td>
<td>a rectangular system for graphing ordered pairs with x-coordinates and y-coordinates</td>
<td>y</td>
</tr>
<tr>
<td></td>
<td>ses-the’-mah deh</td>
<td></td>
<td>A •</td>
</tr>
<tr>
<td></td>
<td>Cahr-teh-see-ah’-noh coo-or-deh-na’-thoh</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Point A is (-3,2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>English term</strong></td>
<td><strong>Spanish term</strong></td>
<td><strong>Description/meaning</strong></td>
<td><strong>Drawing/example</strong></td>
</tr>
<tr>
<td>-----------------------</td>
<td>----------------------------------</td>
<td>----------------------------------------------------------------------------------------</td>
<td>--------------------------------------</td>
</tr>
<tr>
<td>concrete models</td>
<td>representación física o algebraica</td>
<td>representations of functional relationships with manipulatives or real-world examples</td>
<td>algebra tiles, color tile manipulatives</td>
</tr>
<tr>
<td></td>
<td>reh-preh-sehn-tah-seehohn’</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>fee’-see-cah oh ahl-heh-brah-ee’-ca</td>
<td></td>
<td></td>
</tr>
<tr>
<td>dependent variable</td>
<td>variable dependiente</td>
<td>the output variable whose value is determined by the independent variable (input variable) and the rule applied</td>
<td>income = (number of hours worked) times ($ per hour) Dependent variable is income, which depends on the number of hours worked.</td>
</tr>
<tr>
<td>diagram</td>
<td>diagrama</td>
<td>a picture or drawing that represent the functional relationship between two variables</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>dee-ah-grah’-mah</td>
<td></td>
<td></td>
</tr>
<tr>
<td>equation</td>
<td>ecuación</td>
<td>a mathematical sentence that states that two expressions are equal</td>
<td>y = (-2x - 3)</td>
</tr>
<tr>
<td></td>
<td>eh-koo-ah-seeohn’</td>
<td></td>
<td></td>
</tr>
<tr>
<td>function (functional relationship)</td>
<td>función</td>
<td>a relation where each input is paired with exactly one output and is based on some rule or description</td>
<td>y = (2x - 3) Each value for x gives a different value for y</td>
</tr>
<tr>
<td></td>
<td>foon-seeohn’</td>
<td></td>
<td></td>
</tr>
<tr>
<td>graph</td>
<td>gráfica</td>
<td>the solution set of an equation or inequality on a Cartesian coordinate system</td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td></td>
<td>grah’-fee-cah</td>
<td></td>
<td></td>
</tr>
<tr>
<td>English term</td>
<td>Spanish term</td>
<td>Description/meaning</td>
<td>Drawing/example</td>
</tr>
<tr>
<td>-------------------</td>
<td>--------------------</td>
<td>--------------------------------------------------------------------------------------</td>
<td>-----------------------------------------------------------</td>
</tr>
<tr>
<td>independent variable</td>
<td>variable independiente</td>
<td>the variable whose value is subject to choice (input variable)</td>
<td>income = (number of hours worked) times ($ per hour)</td>
</tr>
<tr>
<td></td>
<td>bah-ree-ah’-bleh</td>
<td></td>
<td>Independent variable is number of hours worked.</td>
</tr>
<tr>
<td></td>
<td>een-deh-pehn-dee-ehn-teh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>inequality</td>
<td>desigualdad</td>
<td>a mathematical sentence that shows a greater than, greater than or equal, less than, or less than or equal relationship between two expressions</td>
<td>$y \geq 3x + 4$</td>
</tr>
<tr>
<td>ordered pair</td>
<td>par ordenado</td>
<td>a pair of numbers in which the order they are written is important.</td>
<td>(3,6) is a different ordered pair than (6,3)</td>
</tr>
<tr>
<td></td>
<td>pahr</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ohr-deh-nah-tho</td>
<td></td>
<td></td>
</tr>
<tr>
<td>relation</td>
<td>relación</td>
<td>a set of ordered pairs, which describe a relationship between two variables. For a relation, it is acceptable for each input value to be paired with more than one output</td>
<td>A = {(1,2), (2, 4), (3,6)}</td>
</tr>
<tr>
<td></td>
<td>reh-elah-seeohn’</td>
<td></td>
<td>B = {(1,2), (1,4), (1,8)}</td>
</tr>
</tbody>
</table>
| table             | tabla              | a display of mathematical data, usually in a vertical or horizontal manner          | $x$  2  4  6  
|                   | tah-blah           |                                        | $y$  5  9  13                                           |
| variable          | variable           | a symbol (usually a letter) used to represent an unknown number in a given set of numbers | in the expression, $4 \ n + 2$, the “$n$” is a variable  |
|                   | bah-ree-ah-bleh    |                                        |                                                            |

**Strategies for learning this vocabulary:**

1. Write definitions in everyday language while still following correct mathematics;
2. Use previously defined or common words in definitions and explanations;
3. Have students develop self-made glossaries of new vocabulary in journals, picture cards, or charts;

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4. as new vocabulary is introduced, add words and definitions with illustrations/explanations to classroom word wall;
5. repeatedly connect the words to mathematical symbols and examples;
6. tape record mathematical words, definitions and verbal examples, for students to play back when needed for extra support; and
7. examine words from Greek and Latin prefixes, roots, and suffixes.

Teaching strategies and examples for Objective 1

1. Use tables to describe relationships of one variable depending on another variable.

Example: A lawn care business charges $25 to cut the grass and trim for a normal sized yard. Build a table to show that the amount of money earned (the dependent variable) depends on the number (#) of normal-sized lawns cut (the independent variable), as shown below: Students should fill in the values in the table.

<table>
<thead>
<tr>
<th># lawns cut</th>
<th>$ earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>75</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
</tr>
<tr>
<td>..</td>
<td>..</td>
</tr>
<tr>
<td>10</td>
<td>?</td>
</tr>
<tr>
<td>..</td>
<td>..</td>
</tr>
<tr>
<td>n</td>
<td>?</td>
</tr>
</tbody>
</table>

Describe in words the connection that the amount of money earned depends on the number of lawns cut.

2. Gather and record data and determine functional relationships among quantities.

Example: have students work with a partner and measure the length of each other’s foot, along with the height of each student. Consolidate data for the entire class, plot the data for each student on an x,y-coordinate system and see if there is a relationship between the length of a person’s foot and the student’s height.

3. Write equations or inequalities for given problem situations.

Example: Jorge joins a club to purchase CD’s of his favorite Tejano groups. There is an initial membership cost of $10, and each CD he buys costs $6. Write an equation to show the relationship between the number of CD’s he buys and his total cost.
4. Represent relationships among quantities in multiple ways.

Example: If the pattern continues to grow at the same rate as shown in the picture below, how many circles will there be in the 8th figure?

**Visual**

```
  O O O O O O O
  O O O O O O O
```

**Verbal** The first figure has one circle, to get the second figure you add a circle horizontally and vertically, then you add two additional circles to get to the third figure, and so on.

**Pattern or rule** If you take the number of the figure, double it, and subtract 1, then you get the number of circles.

**Table**

<table>
<thead>
<tr>
<th>Number of figure</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of O's</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>?</td>
</tr>
</tbody>
</table>

**Ordered Pairs** (1,1), (2,3), (3,5), (4,7), …

**Graph**

![Graph](image)

**Equation** \( y = 2x - 1 \)

**Function notation** \( f(x) = 2x - 1 \)
5. Interpret and make inferences from functional relationships.

Example: The Rodriguez family chooses to send $1 out of every $10 they earn to their parents. If they earned $350 this week, how much would they send to their parents?

Assessment for Objective 1

General strategies for assessment:

1. allow students frequent opportunities to demonstrate mastery in a variety of ways;
2. provide sufficient time for ELL students to complete assessment tasks;
3. use assessment results to design instructional planning for remediation if needed;
4. assign projects for students to work together with their partners;
5. have students write their thoughts and problem-solving actions in a journal;
6. design performance measures with visuals to check concept understanding;
7. design assessments to measure mathematical understanding, not reading comprehension;
8. ensure assignments are as free of bias as possible; and
9. make assignments that require writing explanations in English.

Specific examples for assessment

1. Have students show multiple representations of the relationships between two variables in a writing assignment for a grade. For this task, give a visual pattern and have them show other representations of the problem to include a listing, a table, a graph, a verbal description, and an algebraic equation. For performance assessment, use a rubric that considers concept understanding, approach toward solving a problem, and verbal or picture/diagram explanations. For example:

Example: Given a representation of the relationship between two variables, show other representations.
Scoring Rubric

<table>
<thead>
<tr>
<th>Conceptual Understanding</th>
<th>0</th>
<th>No evidence of understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>Some understanding, but incomplete</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Complete conceptual understanding</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Approach Toward Solving Problem</th>
<th>0</th>
<th>No evidence of trying to solve problem</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>Some evidence of solving problem</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Successful approach to solving problem</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Representation of Relationship</th>
<th>0</th>
<th>No representation attempted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>Incomplete representation</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Complete, accurate representation</td>
</tr>
</tbody>
</table>

Example assessment task:

Marcella wants to build a swimming pool and she wants it to be square. Use color tiles to represent the possible areas of the pool. Assuming that each color tile represents a 1 foot by 1 foot square, use the manipulatives to model the pools, draw pictures of possible pool sizes, and look for a pattern as the pools get larger. Work with your partner and represent your patterns in a table, with ordered pairs, with a graph, and with an algebraic equation. Write an explanation in your journal to describe the area of the fifth pool in increasing size, starting with a 1 by 1 pool as the first possible pool (see drawing below). Try to determine a pattern that would help find the area of the 50th pool, without drawing all of the pools in order.
2. Use traditional assessment methods, including multiple-choice questions to measure mathematics understanding also. Students need to practice solving mathematics problems in the same format of the TAKS test questions. When discussing these problems in class, have students analyze why one answer is correct and the others are incorrect. A sample problem could be:

Diego likes to ride his bike at least 20 miles every weekend, since he is training for a distance riding competition that is 100 miles long. He averages 9 miles per hour on his rides. What is the dependent variable for this functional relationship?

a. the number of miles he rides  
b. his speed  
c. the number of hours he rides  
d. whether he finishes the 100-mile race or not

Additional problems can be found on the Texas Education Agency (TEA) website (www.tea.state.tx.us) from the TAKS information booklets (www.tea.state.tx.us/student.assessment/taks/booklets) and from TAKS released tests (www.tea.state.tx.us/student.assessment/resources/release/taks/index.html). Also on the TEA website, there is a link to the TAKS Study Guide for Grade 11 Exit Level Mathematics and Science: A Student and Family Guide, which explains the key concepts under each objective and gives examples (see www.tea.state.tx.us/student.assessment/resources/guides/study/index.html).

There are additional multiple-choice problems for each objective in this guide. Although it is not designed especially for ELL students, it is a very helpful resource in preparing to take the TAKS test.

3. Design projects that involve the families of students. Have them collect data as homework assignments. For example:

Have students extend the measurement activity that was done in class. Students can measure the foot size (in centimeters or millimeters) and compare it to the height of every family member (aunts and uncles and cousins count too). Then take other measurements also, such as the length of a person’s foot compared to the distance between their wrist and their elbow. In addition, compare the circumference of a family member’s neck to their height. For each set of measurements, have the students verbally describe what they did, what they found, and what patterns they discovered. Patterns should be represented in a variety of ways, including by a table, a list, a graph, and an equation.

4. Let students work with a partner in class. Provide extra time for students to talk together about an assigned mathematics problem, decide how to approach it, and make a summary paper on the problem. Provide a format sheet for their mini-reports, such as a) what is the problem, b) what do we need to find out, c) how do we get started, c) how do we solve the problem, d) what is our solution, and e) how can I
describe what we did? Allow time for both partners to discuss their report together, before presenting it to the class.

5. Make sure that all students have the resources available to accomplish every assignment. For example, do not assign projects that involve working on the Internet as a homework assignment, since not every student has access to a computer at home. If you want students to measure something, provide them with rulers or the tools to do the measuring. If you assign a project that requires the use of graphing calculators, provide the calculators and make it an in-class assignment, since many students will not have access to graphing calculators outside the classroom.
OBJECTIVE 2

DEMONSTRATE AN UNDERSTANDING OF THE PROPERTIES AND ATTRIBUTES OF FUNCTIONS

Mathematics content:

Prior mathematics knowledge requirements:

1. write and solve equations with one and two variables;
2. graph points, equations and inequalities on an x,y-coordinate plane;
3. interpret graphical representations of data and functional relationships;
4. plot the relationship between two variables in a scatter plot;
5. identify patterns and relationships between two variables;
6. multiply variable terms (e.g., $x^2$ times $x^3 = x^5$);
7. use alphabetical symbols to represent unknowns and variables; and
8. apply the associative, commutative, and distributive properties of numbers and variables.

From pre-algebra activities in middle school, students should have a good understanding of variables, and how to set up and solve equations and inequalities. They also need to understand patterns and relationships for number or visual representations, as well as the properties of numbers and variables. If students are weak in these areas, remediation may be required. Once students understand the properties and attributes of linear and quadratic functions, they will be prepared to learn Objectives 3-5 of this guide.

Minimum mathematics vocabulary needed for Objective:2

<table>
<thead>
<tr>
<th>English term</th>
<th>Spanish term</th>
<th>Description/meaning</th>
<th>Drawing/example</th>
</tr>
</thead>
<tbody>
<tr>
<td>dependent variable</td>
<td>variable dependiente</td>
<td>the output variable whose value is determined by the independent variable (input variable) and the rule applied</td>
<td>income = (number of hours worked) times ($ per hour) Dependent variable is income, which depends on the number of hours worked.</td>
</tr>
<tr>
<td>bah-ree-ah'-ble</td>
<td>deh-pehn-dee-ehn'-te</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th><strong>English term</strong></th>
<th><strong>Spanish term</strong></th>
<th><strong>Description/meaning</strong></th>
<th><strong>Drawing/example</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>domain</td>
<td>dominio de una función</td>
<td>the input values of a function (independent variable)</td>
<td>$y = 2x$ For every x value input into the function (the domain), the y value is twice as much</td>
</tr>
<tr>
<td>function (functional relationship)</td>
<td>función</td>
<td>a relation where each input is paired with exactly one output and is based on some rule or description</td>
<td>$y = 2x - 3$ Each different value for x gives a different value for y</td>
</tr>
<tr>
<td>independent variable</td>
<td>variable independiente</td>
<td>the variable whose value is subject to choice (input variable)</td>
<td>income = (# hours worked) times ($ per hour) Independent variable is # hours worked.</td>
</tr>
<tr>
<td>inequality</td>
<td>desigualdad</td>
<td>a mathematical sentence that shows a greater than, greater than or equal, less than, or less than or equal relationship between two expressions</td>
<td>$y \geq 3x - 4$</td>
</tr>
<tr>
<td>linear function</td>
<td>función lineal en dos variables</td>
<td>a function that can be represented by a straight line on a graph. The function would have a constant rate of change</td>
<td>$y = 0.5x + 7$</td>
</tr>
<tr>
<td>monomial</td>
<td>monomio</td>
<td>a term that is a number, a variable, or the product of a number and one or more variables</td>
<td>$3x \quad 3y^2 \quad 4m^2n$</td>
</tr>
<tr>
<td>polynomial expressions</td>
<td>polinomino</td>
<td>a monomial or the sum or difference of monomials</td>
<td>$2x \quad 4x^2 - 3x + 2$</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>English term</th>
<th>Spanish term</th>
<th>Description/meaning</th>
<th>Drawing/example</th>
</tr>
</thead>
<tbody>
<tr>
<td>quadratic function</td>
<td>función cuadrática</td>
<td>a function of the form $f(x) = ax^2 + bx + c$, where $a$, $b$, and $c$ are real numbers and $a \neq 0$. Quadratic functions must have an $x^2$ term.</td>
<td>$f(x) = 2x^2 - 4x + 3$</td>
</tr>
<tr>
<td>range</td>
<td>alcance de una función</td>
<td>the output values (y-coordinates on a graph) of a function</td>
<td>if the domain (input values of x) are 1 and 2 for the function $y = 3x$, then the range (values of y) are 3 and 6.</td>
</tr>
<tr>
<td>scatter plot</td>
<td>gráfica de dispersión</td>
<td>a graph in which the data are shown as points in a coordinate plane. Scatter plots show the relationship between two variables.</td>
<td>Ht</td>
</tr>
<tr>
<td></td>
<td>grah'-fee-cah deh</td>
<td></td>
<td>Wt</td>
</tr>
<tr>
<td></td>
<td>dees-pehr-seoohn'</td>
<td></td>
<td>. . .</td>
</tr>
</tbody>
</table>

**Strategies for learning this vocabulary:**

1. write definitions in everyday language while still following correct mathematics;
2. use previously defined or common words in definitions and explanations;
3. have students develop self-made glossaries of new vocabulary in journals, picture cards, or charts;
4. as new vocabulary is introduced, add words and definitions with illustrations/explanations to classroom word wall;
5. repeatedly connect the words to mathematical symbols and examples;
6. tape record mathematical words, definitions and verbal examples, for students to play back when needed for extra support; and
7. examine words from Greek and Latin prefixes, roots, and suffixes

**Teaching strategies and examples for Objective 2**

1. Identify and sketch the general forms of linear ($y = x$) and quadratic ($y = x^2$) functions.
Example: The perimeter and the area of a square are functions of the length of the side of the square. Esmarelda uses color tiles to make up squares on her desk as shown below.

Next, she decides to make a table for both of the perimeter and area patterns of growth. She decides to make the length of a side of a color tile equal to 1 and finds the area by counting the number of color tiles. Her tables are:

<table>
<thead>
<tr>
<th>Length of side</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>perimeter</td>
<td>4</td>
<td>8</td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Length of side</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>area</td>
<td>1</td>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

Which of these functions is linear? Which is not linear? How can you tell? Sketch each of these functions on graph paper.

2. Identify the domain and range of various functions.

Example: Maria sees an advertisement for a sale at the mall for music CD’s, and every CD costs $9.95, including tax. She would like to buy 5 to 10 CD’s. What could her total cost be? Write the function described in this problem as an equation, with one variable as the number of CD’s bought and the other variable as the total cost. What would the range be for this function? What would the domain be? Is this a linear function?

3. Make and interpret scatterplots.

Example: Selma went shopping at the grocery store and noticed that the prices for potato chips varied. The following table shows the different prices she found for different sizes of bags of chips.

<table>
<thead>
<tr>
<th>Cost</th>
<th>$1.50</th>
<th>$0.95</th>
<th>$2.99</th>
<th>$1.99</th>
<th>$1.75</th>
<th>$1.45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of ounces</td>
<td>9</td>
<td>6</td>
<td>24</td>
<td>16</td>
<td>11</td>
<td>10</td>
</tr>
</tbody>
</table>

Draw a scatterplot with cost on the vertical axis and number of ounces on the horizontal axis. Is there a trend among the points that you plotted? Do the points appear to be on the same line? Draw a line that appears to go through the middle of the points, with approximately the same number of points above and below the line. Does your line appear to “fit” the points very well? If it does, what does this mean? According to your line, what would you expect a 20-ounce bag of chips to cost?
4. Identify patterns and represent these patterns algebraically.

Example: Jose notices that his pickup truck always gets the same amount of miles per gallon. The following table shows the amount of miles that he drove for various numbers of gallons of gas purchased.

<table>
<thead>
<tr>
<th>Number of miles</th>
<th>108</th>
<th>252</th>
<th>162</th>
<th>372</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gallons</td>
<td>9</td>
<td>21</td>
<td>13.5</td>
<td>31</td>
</tr>
</tbody>
</table>

Write an equation that shows the relationship between number of gallons of gas (g) and the number of miles driven (d) for Jose’s truck.

5. Simplify polynomial expressions.

Example: When you divide 20 meters squared per second by 8 meters per second, what do you get?

6. Use the associative, commutative, and distributive properties to simplify algebraic expressions.

Example: Simplify the expression $8(3x - 2) + 7(2x + 3)$.

Assessment for Objective 2

**General strategies for assessment**

1. allow students frequent opportunities to demonstrate mastery in a variety of ways;
2. provide sufficient time for ELL students to complete assessment tasks;
3. use assessment results to design instructional planning for remediation if needed;
4. assign projects for students to work together with their partners;
5. have students write their thoughts and problem-solving actions in a journal;
6. design performance measures with visuals to check concept understanding;
7. design assessments to measure mathematical understanding, not reading comprehension;
8. ensure assignments are as free of bias as possible; and
9. make assignments that require writing explanations in English.
Specific examples for assessment

1. Give students the assignment to sketch the graphs of \( y = 2x \) and \( y = x^2 \). Discuss the domain and range of each function, as well as maximum and minimum values for input and output variables. Also discuss which are the dependent and independent variables for each function. List possible \( x \) and \( y \) values for each function in a table. In a report, discuss properties that these two functions have in common and properties that are different about them.

2. Use traditional assessment methods, including multiple-choice questions, to measure mathematics understanding also. Students need to practice solving mathematics problems in the same format of the TAKS test questions. When discussing these problems in class, have students analyze why one answer is correct and the others are incorrect. A sample problem could be:

Eddie is stacking oranges on his job at the local grocery store. He arranges them so that every level is a perfect square. He started with a 6 by 6 square on the bottom level. He wants to know how many oranges total there are in his square pyramid arrangement. Because it is difficult to see all of the oranges on each level, he looks for a pattern.

Which of the following will give the total amount of oranges in the arrangement?

a. \( 1 + 2 + 3 + 4 + 5 + 6 \)
b. \( 1^2 + 6^2 \)
c. \( 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 \)
d. \( 6^2 \)

Additional problems can be found on the Texas Education Agency (TEA) website (www.tea.state.tx.us) from the TAKS information booklets (www.tea.state.tx.us/student.assessment/taks/booklets) and from TAKS released tests (www.tea.state.tx.us/student.assessment/resources/release/taks/index.html). Also on the TEA website, there is a link to the TAKS Study Guide for Grade 11 Exit Level Mathematics and Science: A Student and Family Guide, which explains the key concepts under each objective and gives examples (see www.tea.state.tx.us/student.assessment/resources/guides/study/index.html). There are additional multiple-choice problems for each objective in this guide. Although it is not designed especially for ELL students, it is a very helpful resource in preparing to take the TAKS test.

3. Design projects that involve the families of students. For example, students can collect data from their family members. For each person, measure his or her height and the length of an average step. Record the data for each person in a table, with each column labeled. Then plot the data for each family member on a coordinate plane, where each point represents a family member. If you don’t have at least six points on the graph, then obtain additional measurements from friends or neighbors.
Next, look at the points on the graph. What do you observe from the pattern of the points? Does there appear to be a trend? As the height of a person increases, what happens to that person’s step length? Is this true for every family member? Do the points appear to form a line? Why or why not? Write your findings in a report.

4. Let students work with a partner in class. Provide extra time for students to talk together about an assigned mathematics problem, decide how to approach it, and make a summary paper on the problem. Provide a format sheet for their mini-reports, such as a) what is the problem, b) what do we need to find out, c) how do we get started, c) how do we solve the problem, d) what is our solution, and e) how can I describe what we did? Allow time for both partners to discuss their report together, before presenting it to the class.

5. Make sure that all students have the resources available to accomplish every assignment. For example, do not assign projects that involve working on the Internet as a homework assignment, since not every student has access to a computer at home. If you want students to measure something, provide them with rulers or the tools to do the measuring. If you assign a project that requires the use of graphing calculators, provide the calculators and make it an in-class assignment, since many students will not have access to graphing calculators outside the classroom.
OBJECTIVE 3

DEMONSTRATE AN UNDERSTANDING OF LINEAR FUNCTIONS

Mathematics content:

Prior mathematics knowledge requirements:

1. use symbols and letters to represent unknowns and variables;
2. write and solve equations with one and two variables;
3. show multiple representations of functions using graphs, tables, equations, and verbal descriptions;
4. graph equations on an $x,y$-coordinate plane;
5. write ratios to represent the relationship between two quantities;
6. set up and solve proportions;
7. identify patterns and relationships between two variables using graphical, numeric, algebraic, and verbal representations;
8. identify domain and range of linear functions;
9. identify and understanding dependent and independent variables; and
10. understand functions with a constant rate of change.

From pre-algebra activities in middle school, students should have a good understanding of variables, and how to set up and solve equations and inequalities. They also need to understand patterns and relationships for number or visual representations, as well as the properties of numbers and variables. If students are weak in these areas, remediation may be required. Once students understand multiple representations of functions (Objective 1) and the properties and attributes of linear and quadratic functions (Objective 2), they will be prepared to learn Objectives 3-5 of this guide.

Minimum mathematics vocabulary needed for Objective 3:

<table>
<thead>
<tr>
<th>English term</th>
<th>Spanish term</th>
<th>Description/meaning</th>
<th>Drawing/example</th>
</tr>
</thead>
<tbody>
<tr>
<td>direct variation</td>
<td>variación directa</td>
<td>when the $y$ value of a function varies at the same proportional rate as the $x$ value of the function</td>
<td>$y = 4x$</td>
</tr>
<tr>
<td></td>
<td>bah-ree-ah-seeohn’</td>
<td></td>
<td>$y$ will always be 4 times whatever $x$ is</td>
</tr>
<tr>
<td></td>
<td>dee-rehk-tah</td>
<td></td>
<td></td>
</tr>
<tr>
<td>English term</td>
<td>Spanish term</td>
<td>Description/meaning</td>
<td>Drawing/example</td>
</tr>
<tr>
<td>------------------------</td>
<td>---------------------------------------</td>
<td>--------------------------------------------------------------------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>linear function</td>
<td>función lineal en dos variables</td>
<td>a function that can be represented by a straight line on a graph. The function would have a constant rate of change</td>
<td>[ y = 0.5x + 7 ]</td>
</tr>
<tr>
<td>constant rate of change</td>
<td>taza de cambio consante</td>
<td>where one variable changes at the same proportion to another variable</td>
<td>an example is the slope of a line.</td>
</tr>
<tr>
<td>slope of a line</td>
<td>pendiente de una linea</td>
<td>the change in vertical distance divided by the change in horizontal distance</td>
<td>for the equation [ y = 3x + 5 ], the slope of the line is 3, which shows a vertical change of 3 units for every 1 unit of horizontal change</td>
</tr>
<tr>
<td>slope-intercept form of a linear function</td>
<td>formula ( y = mx + b )</td>
<td>( y = mx + b ), where ( m ) is the slope and ( b ) is the y-intercept</td>
<td>for the equation [ y = -2x - 3 ], the slope is -2 and the y-intercept is -3</td>
</tr>
<tr>
<td>y-intercept</td>
<td>y-intercepta</td>
<td>The ( y ) coordinate of the point where the graph of the function crosses the ( y )-axis</td>
<td>for the equation [ y = 5x - 2 ], the ( y )-intercept is -2</td>
</tr>
</tbody>
</table>

Strategies for learning this vocabulary:

1. write definitions in everyday language while still following correct mathematics;
2. use previously defined or common words in definitions and explanations;
3. have students develop self-made glossaries of new vocabulary in journals, picture cards, or charts;
4. as new vocabulary is introduced, add words and definitions with illustrations/explanations to classroom word wall;
5. repeatedly connect the words to mathematical symbols and examples;
6. tape record mathematical words, definitions and verbal examples, for students to play back when needed for extra support; and
7. examine words from Greek and Latin prefixes, roots, and suffixes.

Teachers guide created by the SHSU MELL Group, November 2005, in collaboration with the Texas State University System and the Texas Education Agency.
Teaching strategies and examples for Objective 3

1. Convert among various representations of linear functions, to include equations, tables, graphs, and verbal descriptions.

Example: Benji wants to buy his first car. He finds a used Mustang for $4000 cash, but it will cost $5000 if he makes monthly payments. Unfortunately, he doesn’t have any cash. He does have a job at Whataburger and makes about $500 a month and he can afford to pay $200 per month in payments. Show how much he will pay for each of the first six months in a table. Make a graph of amount paid versus number of monthly payments made. Write an equation that represents his car purchase and use it to determine how many months it will take Benji to completely pay off his car loan.

2. Develop the concept of slope as a constant rate of change and determine slopes from multiple representations of linear functions.

Example: Cynthia is playing a game of “guess my rule” with her friend Liza. Liza knows that they are working on linear functions, and she is trying to guess the linear function that Cynthia has chosen. She guesses several numbers and Cynthia gives her back the value for each of Liza’s guesses, as shown in the following table:

<table>
<thead>
<tr>
<th>Liza’s guess</th>
<th>3</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cynthia’s answer</td>
<td>9</td>
<td>13</td>
<td>19</td>
</tr>
</tbody>
</table>

What would Cynthia’s answer be if Liza guessed 6? What is the constant rate of change for Cynthia’s answers? What linear function represents the rule that Cynthia is using to give her answers? Plot these values on a coordinate plane. Does the same line connect all three points? If so, what is the slope of the line?

3. Investigate, describe and predict effects of changes in \( m \) and \( b \) on the graph of \( y = mx + b \), the slope-intercept form of a linear equation.

Example: Use a graphing calculator for this example, and have students work in pairs, each with a calculator if possible. Plot the equation \( y = x \) (enter this equation in \( y_1 \)) and note where the graph crosses the \( y \)-axis. Moving from left to right on the curve, by how much is the graph going up for every one unit on the \( x \)-axis? Is the rate of change always the same? Next, enter the equation \( y = x + 2 \) in \( y_2 \) and leave the equation \( y = x \) in \( y_1 \). The calculator will graph \( y = x \) first and then \( y = x + 2 \) second. How does the second equation differ from the first? What is the \( y \)-intercept of the equation \( y = x + 2 \)? What do you think would happen to your original equation \( (y = x) \) if we changed it to \( y = x - 3 \)? What would the new \( y \)-intercept be? Without graphing it, what do you think the \( y \)-intercept be for the function \( y = 3x - 8 \)? Graph it on your graphing calculator and check your guess. Clear this function from \( y_2 \), and leave the function \( y = x \) in \( y_1 \). Enter \( y = 2x \) in \( y_2 \). Graph both equations. How did your graph change for \( y = 2x \)? Why do
you think it changed? Replace your equation in $y_2$ with $y = 4x$. What happened? Without using your graphing calculator, what do you think will happen if you graph $y = 8x$? Graph it and check your guess. What do you think the number in front of the $x$ term (called the coefficient) has to do with the resulting graph? As a challenge, see what happens when you put a negative sign in front of the $x$ in several equations. Write a summary of your ideas from this exercise in your journal.

4. Graph and write equations for lines given characteristics such as two points, a point and a slope, or a slope and the y-intercept.

Example: Give several sheets of graph paper to each student. For each of the following examples, draw the line on graph paper that has the following characteristics. Then for each example, write the equation that represents the line in $y = mx + b$ form.

   a. two points with the coordinates of (1, 4) and (3,8).
   b. two points with the coordinates of (2,5) and (-3,4)
   c. a point with the coordinates of (3,2) and a slope of 2
   d. a point with the coordinates of (-4, -2) and a slope of 1
   e. a y-intercept of 2 and a slope of -3
   f. a y-intercept of -4 and a slope of 0.5

Were any of the examples above difficult for you? What problems did you have? Can your partner explain these examples to you?

5. Solve problems involving direct variation and proportional change.

Example: Marco was looking at a map of Texas, and he noticed 1 cm on the map represented 40 kilometers. He wanted to know how many kilometers the distance was from Houston to San Antonio. He measured the distance on the map and it totaled 4.8 cm. What was the distance between Houston and San Antonio?

Assessment for Objective 3

General strategies for assessment:

   1. allow students frequent opportunities to demonstrate mastery in a variety of ways;
   2. provide sufficient time for ELL students to complete assessment tasks;
   3. use assessment results to design instructional planning for remediation if needed;
   4. assign projects for students to work together with their partners;
   5. have students write their thoughts and problem-solving actions in a journal;
   6. design performance measures with visuals to check concept understanding;

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7. design assessments to measure mathematical understanding, not reading comprehension;
8. ensure assignments are as free of bias as possible; and
9. make assignments that require writing explanations in English.

Specific examples for assessment

1. Mario and his band have been practicing in the garage of his home, but his parents decide that this is too noisy for the family. He finds a studio to rent for band practices, and the cost is $30 plus $10 per hour. Parts of an hour would cost a proportional amount. Mario makes up a problem for his friend Chad to solve in algebra class, based on the rental costs for the studio. Mario gives Chad the following tasks:
   a. Make a table of values showing the total costs for various time periods, including 2, 3, 5, and 10 hours.
   b. Write an equation in slope-intercept form to represent this functional relationship.
   c. What is the slope of the equation? What does this number represent? How are slope and the idea of constant rate of change related?
   d. Graph the function on an x,y coordinate plane. What is the y-intercept? What does this mean?
   e. If they wanted to practice three times before playing at the school dance and they only had $200, how many total hours could they practice?

2. Use traditional assessment methods, including multiple-choice questions, to measure mathematics understanding also. Students need to practice solving mathematics problems in the same format of the TAKS test questions. When discussing these problems in class, have students analyze why one answer is correct and the others are incorrect. A sample problem could be:

Antonio visits his grandmother in El Paso and he takes a taxi from the bus station to her house. The taxi costs $3.00 plus $1.00 for every quarter mile. Antonio calculates the costs for trips of various distances, and records this in the table below.

<table>
<thead>
<tr>
<th>Number of miles driven</th>
<th>0.5</th>
<th>1</th>
<th>3</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost of taxi</td>
<td>$5.00</td>
<td>$7.00</td>
<td>$15.00</td>
<td>$27.00</td>
</tr>
</tbody>
</table>

If x represents the number of miles driven and y equals the total cost of the taxi, which of the following would be the equation for a line passing through these points?

a. \( y = 0.25x + 3 \)
b. \( y = 3x + 0.25 \)
c. \( y = 3x + 4 \)
d. \( y = 4x + 3 \)
Additional problems can be found on the Texas Education Agency (TEA) website (www.tea.state.tx.us) from the TAKS information booklets (www.tea.state.tx.us/student.assessment/taks/booklets) and from TAKS released tests (www.tea.state.tx.us/student.assessment/resources/release/taks/index.html). Also on the TEA website, there is a link to the TAKS Study Guide for Grade 11 Exit Level Mathematics and Science: A Student and Family Guide, which explains the key concepts under each objective and gives examples (see www.tea.state.tx.us/student.assessment/resources/guides/study/index.html). There are additional multiple-choice problems for each objective in this guide. Although it is not designed especially for ELL students, it is a very helpful resource in preparing to take the TAKS test.

3. Design projects that involve the families of students. For example, Enrique’s father is complaining about the increasing cost of gasoline for the truck that he uses in his tree cutting business. He knows that more and more of the business income is used for purchasing gasoline this summer, when the price is about $2.35 per gallon. He is even more worried lately, since he heard that the price of gasoline might rise to $3 a gallon by next summer. He knows that he usually drives his truck for an average of 1850 miles every month. Enrique uses his algebraic skills to determine the effects on monthly gasoline costs, when the price of gasoline gradually rises from $2.35 to $3.00 a gallon. Set up a table to show monthly costs, based on 1850 miles driven per month, for various prices of gasoline.

4. Let students work with a partner in class. Provide extra time for students to talk together about an assigned mathematics problem, decide how to approach it, and make a summary paper on the problem. Provide a format sheet for their mini-reports, such as a) what is the problem, b) what do we need to find out, c) how do we get started, c) how do we solve the problem, d) what is our solution, and e) how can I describe what we did? Allow time for both partners to discuss their report together, before presenting it to the class.

5. Make sure that all students have the resources available to accomplish every assignment. For example, do not assign projects that involve working on the Internet as a homework assignment, since not every student has access to a computer at home. If you want students to measure something, provide them with rulers or the tools to do the measuring. If you assign a project that requires the use of graphing calculators, provide the calculators and make it an in-class assignment, since many students will not have access to graphing calculators outside the classroom.
OBJECTIVE 4

FORMULATE AND USE LINEAR EQUATIONS AND INEQUALITIES

Mathematics content:

Prior mathematics knowledge requirements:

1. understand the property of equality;
2. write and solve equations with one and two variables, using concrete models and algebraic expressions;
3. write and solve inequalities with one and two variables;
4. draw complete graphs of equations and inequalities on an x,y-coordinate plane;
5. record data for one and two variables in a table form;
6. understand functional relationships as a table of values, an equation, and a graph; and
7. understand the problem-solving process, including analyzing the problem, choosing a solution strategy, carrying out the strategy, and checking the reasonableness of the solution.

From pre-algebra activities in middle school and Objectives 1-3, students should have a good understanding of variables, how to set up and solve equations and inequalities, and how to graph sets of points on a coordinate plane. They should also be familiar with the problem-solving process in general. If students are weak in these areas, remediation may be required. Once students understand how to use linear equations and inequalities to solve problems, they will be prepared to learn Objective 5 of this guide.

Minimum mathematics vocabulary needed for Objective 4:

<table>
<thead>
<tr>
<th>English term</th>
<th>Spanish term</th>
<th>Description/meaning</th>
<th>Drawing/example</th>
</tr>
</thead>
<tbody>
<tr>
<td>concrete models</td>
<td>representación física o algebraica</td>
<td>representations of functional relationships with manipulatives or real-world examples</td>
<td>algebra tiles, color tile manipulatives</td>
</tr>
<tr>
<td>reh-preh-sehn-tah-seehn fee-see-ca oh ahl-heh-brah-ee-cah</td>
<td>show how to translate algebraic expressions into concrete models and vice versa</td>
<td>show how to translate algebraic expressions into concrete models and vice versa</td>
<td></td>
</tr>
<tr>
<td>English term</td>
<td>Spanish term</td>
<td>Description/meaning</td>
<td>Drawing/example</td>
</tr>
<tr>
<td>------------------------------</td>
<td>--------------------------------------------------</td>
<td>---------------------------------------------------------------------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>linear function</td>
<td>función lineal en dos variables</td>
<td>a function that can be represented by a straight line on a graph. The function would have a constant rate of change</td>
<td>y = 4x + 7</td>
</tr>
<tr>
<td>properties of equality</td>
<td>propiedad de igualdad</td>
<td>the reflective, symmetric, transitive, and substitution properties</td>
<td>see each definition for specific examples of each equality property</td>
</tr>
<tr>
<td>reflective property</td>
<td>propiedad reflexiva</td>
<td>for all real numbers a, a = a</td>
<td>4 = 4</td>
</tr>
<tr>
<td>substitution property</td>
<td>propiedad de sustitución</td>
<td>for all real numbers a and b, if a = b, then a may replace b or b may replace a in any statement</td>
<td>if y = 6, then 6 may be substituted for y in the expression 2y + 3, giving a value of 15 for the expression</td>
</tr>
<tr>
<td>symmetric property</td>
<td>propiedad simétrica</td>
<td>for all real numbers a and b, if a = b, then b = a</td>
<td>if 2x = 6, then 6 = 2x</td>
</tr>
<tr>
<td>system of linear equations</td>
<td>sistema de ecuaciones lineales</td>
<td>two or more linear equations that are considered or plotted together</td>
<td>y = 2x + 3</td>
</tr>
<tr>
<td></td>
<td>sees-the'-ma deh eh-koo-ah-seehohn'-ehs lee-neh-ah'-lehs</td>
<td></td>
<td>2y = −2x + 12</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>transitive property</td>
<td>propiedad transitiva</td>
<td>for all real numbers a, b, and c, if a = b and b = c, then a = c</td>
<td>if a = 2 and 2 = c, then a = c</td>
</tr>
</tbody>
</table>

Strategies for learning this vocabulary:

1. write definitions in everyday language while still following correct mathematics;
2. use previously defined or common words in definitions and explanations;

Teachers guide created by the SHSU MELL Group, November 2005, in collaboration with the Texas State University System and the Texas Education Agency.
3. have students develop self-made glossaries of new vocabulary in journals, picture cards, or charts;
4. as new vocabulary is introduced, add words and definitions with illustrations/explanations to classroom word wall;
5. repeatedly connect the words to mathematical symbols and examples;
6. tape record mathematical words, definitions and verbal examples, for students to play back when needed for extra support; and
7. examine words from Greek and Latin prefixes, roots, and suffixes.

Teaching strategies and examples for Objective 4

1. Analyze situations involving linear equations and formulate linear equations or inequalities to solve problems.

Example: Rafael loves to wear colorful T-shirts to school. The Old Navy Store recently received a shipment of T-shirts, ranging in cost from $8 to $13. He received $135 for his 15th birthday, and wants to buy as many T-shirts as possible with this money. His favorite two shirts cost the maximum price. Write an inequality that could be used to solve this problem. What is the maximum number of T-shirts that he can buy with his remaining money at Old Navy?

2. Solve equations and inequalities using concrete models, graphs, and the properties of equality.

Example: Raquel has some quarters and nickels. She realizes that the number of quarters that she has is two less than three times the number of nickels that she has. She also counts her money and she has a total value of $2.70 in her coins. How many quarters and nickels does she have?

3. Determine the reasonableness of solutions to linear equations and inequalities.

Example: Consuela needs to earn $400 for a trip to go visit her grandmother in Honduras. She earns money babysitting at $3.50 per hour. She is able to work babysitting no more than 48 hours per month. She only has two months remaining before she leaves on her trip. She decides that she will have enough money at the end of the two months. Do you agree?

4. Solve systems of linear equations using concrete models, graphs, tables, and algebraic methods.

Example: Maria has finally convinced her parents that she needs a cell phone. She visited all of the cell phone booths in the mall and narrowed her choice to one of two plans. Plan A costs $29.95 a month for 300 minutes of use, with a charge of $0.10 for every minute over 300 in a month. Plan B costs $39.95 a month for 500 minutes, and costs $0.05 a minute for every minute above 500 in a month. Write the linear equations
for both plan A and B, and graph them on a coordinate plane. The two lines should intersect on the graph. What does the point of intersection mean? Solve the two equations algebraically. Describe in words what the solution means. Which plan is cheaper if she uses 450 minutes a month?

Assessment for Objective 4

General strategies for assessment:

1. allow students frequent opportunities to demonstrate mastery in a variety of ways;
2. provide sufficient time for ELL students to complete assessment tasks;
3. use assessment results to design instructional planning for remediation if needed;
4. assign projects for students to work together with their partners;
5. have students write their thoughts and problem-solving actions in a journal;
6. design performance measures with visuals to check concept understanding;
7. design assessments to measure mathematical understanding, not reading comprehension;
8. ensure assignments are as free of bias as possible; and
9. make assignments that require writing explanations in English.

Specific examples for assessment

1. Have students show their understanding of the application of linear functions in real-world applications. For this project, have students investigate the costs for Internet provider service. For performance assessment, use a rubric that considers concept understanding, approach toward solving a problem, and written or picture/diagram explanations.

Example assessment task:

The Speedy Internet Company charges a monthly fee of $8 plus additional costs of $0.15 per hour for its Internet provider service. Perform the following tasks and answer each question.

a. Choose five different numbers of hours used on the Internet and make a table showing the relationship of the number of hours used to the total cost in that month. Label the input and output values in the table.

b. Plot the input and output values on a total cost versus number of hours used graph, and label the axes of the graph. What input values make sense for this problem?

c. Are the points on a line? If so, draw the line, and extend it to the y-axis.
d. Write a sentence and an equation describing the function rule for the total cost in terms of the number of hours used.

e. What would the total cost be if you used 200 hours? Is a time of 1000 hours reasonable?

f. Write an inequality to describe this problem if you wanted to keep your monthly costs below $18. What is the maximum amount of hours you could use the Internet and stay below $18?

g. What if the company raised their rates to $8 a month plus $0.20 an hour? Could you use the Internet for more or less hours than in the original problem for the same total cost? What is the maximum number of hours you could use it under the new rates to keep your total bill below $18?

2. Use traditional assessment methods, including multiple-choice questions, to measure mathematics understanding also. Students need to practice solving mathematics problems in the same format of the TAKS test questions. When discussing these problems in class, have students analyze why one answer is correct and the others are incorrect. A sample problem could be:

Sarah runs around her block several times every evening, while training for the cross-country team. She always keeps her speed at or above 6 miles per hour. Which of the following represents how many miles she will run, where \( h \) is the number of hours, and \( m \) is the number of miles?

a. \( m = 6h \)

b. \( h = 6m \)

c. \( m \geq 6h \)

d. \( h \geq 6m \)

Additional problems can be found on the Texas Education Agency (TEA) website (www.tea.state.tx.us) from the TAKS information booklets (www.tea.state.tx.us/student.assessment/taks/booklets) and from TAKS released tests (www.tea.state.tx.us/student.assessment/resources/release/taks/index.html). Also on the TEA website, there is a link to the TAKS Study Guide for Grade 11 Exit Level Mathematics and Science: A Student and Family Guide, which explains the key concepts under each objective and gives examples (see www.tea.state.tx.us/student.assessment/resources(guides/study/index.html).

There are additional multiple-choice problems for each objective in this guide. Although it is not designed especially for ELL students, it is a very helpful resource in preparing to take the TAKS test.

3. Design projects that involve the families of students. For example, Pedro decides to help his father determine how many tables he needs to rent for his sister’s wedding reception. Since they expect 200 guests at the reception, he does not want to make a drawing of all of those tables, but decides to use his algebraic skills instead. In this manner, he plans to show his father how much he is learning in his algebra class. They plan to put tables end-to-end, to form two long rows of tables. On each table, two
people can sit on one side, and, of course, an additional person can sit at the ends of the long rows of tables. He shows the following partial drawing to his father, to make sure that this is what he wants.

His father approves this design. What equation can Pedro use to find out how many tables he needs to seat all 200 people, if he decides to let $t$ equal the number of tables?

Pedro's little brother, Mark, wants to help, so Pedro asks him to make a drawing of the long tables and chairs, and to count up the number of tables when he has 200 chairs drawn. Mark worked hard on his drawing and counts the same number of tables that Pedro calculated. What is the minimum number of tables needed to seat the 200 people?

4. Let students work with a partner in class. Provide extra time for students to talk together about an assigned mathematics problem, decide how to approach it, and make a summary paper on the problem. Provide a format sheet for their mini-reports, such as a) what is the problem, b) what do we need to find out, c) how do we get started, c) how do we solve the problem, d) what is our solution, and e) how can I describe what we did? Allow time for both partners to discuss their report together, before presenting it to the class.

5. Make sure that all students have the resources available to accomplish every assignment. For example, do not assign projects that involve working on the Internet as a homework assignment, since not every student has access to a computer at home. If you want students to measure something, provide them with rulers or the tools to do the measuring. If you assign a project that requires the use of graphing calculators, provide the calculators and make it an in-class assignment, since many students will not have access to graphing calculators outside the classroom.
OBJECTIVE 5

DEMONSTRATE AN UNDERSTANDING OF QUADRATIC AND OTHER NONLINEAR FUNCTIONS

Mathematics Content:

Prior mathematics knowledge requirements:

1. understand and apply functional relationships as tables of values,
2. understand and apply functional relationships as graphs, and
3. understand and apply functional relationships algebraically

Minimum Mathematics Vocabulary Needed for Objective 5:

<table>
<thead>
<tr>
<th>English term</th>
<th>Spanish term</th>
<th>Description/meaning</th>
<th>Drawing/example</th>
</tr>
</thead>
<tbody>
<tr>
<td>axis of symmetry</td>
<td>eje de simetría</td>
<td>a line over which a graph is the mirror image of itself. Also called a line of symmetry.</td>
<td>Axis of Symmetry</td>
</tr>
</tbody>
</table>
| completing the square | completando el cuadrado | adding a term to an expression of the form $ax^2 + bx$ to produce a binomial square. | $x^2 + 3x = x^2 + 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2$
<p>|                     | cohm-pleh-than-do ehl coo-ah-drah-tho | $= \left(x + \frac{3}{2}\right)^2 - \frac{9}{4}$ |                 |
| parabola           | parábola           | the set of points in a plane equidistant from a point called the focus and a line called the directrix. | focus directrix |
| quadratic function | función cuadrática | a function of the form $f(x) = ax^2 + bx + c$ where $a \neq 0$. | $f(x) = x^2 - 3x + 5$ |</p>
<table>
<thead>
<tr>
<th>English term</th>
<th>Spanish term</th>
<th>Description/meaning</th>
<th>Drawing/example</th>
</tr>
</thead>
<tbody>
<tr>
<td>vertex</td>
<td>vértice o cima</td>
<td>the point on the axis of symmetry of a parabola equidistant from the focus and the directrix.</td>
<td>![Vertex Diagram]</td>
</tr>
<tr>
<td>x-intercept</td>
<td>x-intercepción</td>
<td>the value(s) of x where a graph intersects the x-axis.</td>
<td>![X-Intercepts Diagram]</td>
</tr>
<tr>
<td>y-intercept</td>
<td>y-intercepción</td>
<td>the value of y where a graph intersects the y-axis.</td>
<td>![Y-Intercept Diagram]</td>
</tr>
</tbody>
</table>

**Strategies for learning this vocabulary:**

1. write definitions in everyday language while still following correct mathematics;
2. use previously defined or common words in definitions and explanations;
3. have students develop self-made glossaries of new vocabulary in journals, picture cards, or charts;
4. as new vocabulary is introduced, add words and definitions with illustrations/explanations to classroom word wall;
5. repeatedly connect the words to mathematical symbols and examples;
6. tape record mathematical words, definitions and verbal examples, for students to play back when needed for extra support; and
7. examine words from Greek and Latin prefixes, roots, and suffixes.
Teaching strategies and examples for Objective 5

1. Use geometric construction to develop concept of parabola.

Example: Consider the graph paper below made up of concentric circles and parallel lines.

Label the center of the circles $F$ and the line 2 units below $F$ as $d$.

Begin constructing points equidistant from $F$ and $d$. Each of the points below are equidistant from $F$ and $d$. 

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Connecting these points with a smooth curve will produce a parabola with focus $F$ and directrix $d$.

![Diagram of a parabola with focus F and directrix d]

2. Use tables and graphs to introduce the quadratic function as a parabola.

Example: Consider the graph of $y = x^2 - 5x + 3$.

Make a table of values of $y$ for each integer value of $x$ from -2 through 7. Then plot the points. The points should appear as a smooth curve called a parabola. The low point of the parabola is called the vertex. Write the coordinates of the vertex as an ordered pair. The graph is symmetrical to a vertical line through the vertex. Draw a line on your graph representing the axis of symmetry. What does the $y$-intercept equal? Where does this number appear in the equation of the parabola? There are two $x$-intercepts. What are their approximate values? Why is this function called a quadratic function?

3. Horizontal and Vertical Shifts of the Parabola

Example: Sketch a graph of $y = x^2$.

Sketch a graph of $y = (x - 2)^2$. How are the graphs of $y = x^2$ and $y = (x - 2)^2$ related?

Sketch the graph of $y = (x - 3)^2$. How is the graph of $y = (x - 3)^2$ related to the graph of $y = x^2$? If $h > 0$, how is the graph of $y = (x - h)^2$ related to the graph of $y = x^2$?

Sketch the graph of $y = (x + 2)^2 = (x - (-2))^2$. How is this related to the graph of $y = x^2$?

How is the graph of $y = (x - h)^2$ for $h < 0$ related to the graph of $y = x^2$? Sketch a graph of $y = x^2$ and $y = x^2 + 2$? How are these graphs related? How are the graphs of $y = x^2$ and $y = x^2 + k$ related?
4. Completing the Square

Example: Graph the parabola \( y = (x - 2)^2 + 3 \)

To graph the parabola \( y = (x - 2)^2 + 3 \) we simply observe that \( y = (x - 2)^2 + 3 \) is the parabola \( y = x^2 \) shifted 2 units to the right horizontally and 3 units vertically. Thus, the vertex of \( y = (x - 2)^2 + 3 \) is \( V(2,3) \).

Suppose we are given the parabola \( y = x^2 - 6x + 1 \) and we want to find the coordinates of the vertex. If we would write \( y = x^2 - 6x + 1 \) in the form \( y = (x - h)^2 + k \) then we would know the vertex is \( V(h,k) \). Thus, our problem is to transform \( y = x^2 - 6x + 1 \) into the form \( y = (x - h)^2 + k \). Observing that \( x^2 - 6x \) would be a perfect square if we added 9, i.e., \( y = x^2 - 6x + 9 = (x - 3)^2 \), so we will add 9 and subtract 9 from \( y = x^2 - 6x + 1 \).

\[
\begin{align*}
y &= x^2 - 6x + 1 \\
   &= x^2 - 6x + 9 - 9 + 1 \\
   &= (x - 3)^2 - 9 + 1 \\
   &= (x - 3)^2 - 8.
\end{align*}
\]

Thus, the vertex is \( V(3, -8) \). This process is called completing the square and allows us to transform any quadratic function \( y = x^2 + bx + c \) into the form \( y = (x - h)^2 + k \). If the coefficient of the quadratic term is one, as in \( x^2 + bx \), then the number that completes the square is found by talking half of the linear coefficient, \( b \), and squaring it. The result is \( x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2 \).

Assessment for Objective 5

General strategies for assessment

1. allow students frequent opportunities to demonstrate mastery in a variety of ways;
2. provide sufficient time for ELL students to complete assessment tasks;
3. use assessment results to design instructional planning for remediation if needed;
4. assign projects for students to work together with their partners;
5. have students write their thoughts and problem-solving actions in a journal;
6. design performance measures with visuals to check concept understanding;
7. design assessments to measure mathematical understanding, not reading comprehension;
8. ensure assignments are as free of bias as possible; and
9. make assignments that require writing explanations in English.

Specific examples for assessment

1. Performance Assessment Tasks

   See the Charles A. Dana Center assessments for algebra 1, pages 329-389. This document is available at the following web site:

   http://www.tenet.edu/teks/math/clarifying/algebra1/alg1assess.pdf

2. Use traditional assessment methods, including multiple-choice questions, to measure mathematics understanding.

   See Grade 11 Mathematics TAKS Information Booklet, page 29 for three examples of multiple-choice questions. This booklet is available at the following web site:


Additional problems can be found on the Texas Education Agency (TEA) website (www.tea.state.tx.us) from TAKS released tests (www.tea.state.tx.us/student.assessment/resources/release/taks/index.html). Also on the TEA website, there is a link to the TAKS Study Guide for Grade 11 Exit Level Mathematics and Science: A Student and Family Guide, which explains the key concepts under each objective and gives examples (see www.tea.state.tx.us/student.assessment/resources/guides/study/index.html). There are additional multiple-choice problems for each objective in this guide. Although it is not designed especially for ELL students, it is a very helpful resource in preparing to take the TAKS test.

3. Design projects that involve the families of students.

   Example: Suppose your family has a store with a cold drink machine. You find that sales average 2,600 cans per month when you charge $0.50 per can. For each nickel increase in price, the sales per month drops by 100 cans.
Complete the following table:

<table>
<thead>
<tr>
<th>Number of $0.05 increases</th>
<th>Number of cans sold</th>
<th>Price per can</th>
<th>Revenue generated</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2600</td>
<td>$0.50</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2500</td>
<td>$0.55</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Describe a function $R(x)$ that models the total revenue realized by your cold drink machine, where $x$ is the number of $0.05$ increases in price of a can.

Construct a graph of $y = R(x)$ that clearly shows a maximum for $R(x)$.

How much should you charge for each can to maximize your revenue? What will the maximum revenue be?

4. Let students work with a partner in class. Provide extra time for students to talk together about an assigned mathematics problem, decide how to approach it, and make a summary paper on the problem.

Example: A little league team uses a baseball-throwing machine to help 10-year-old players to catch high fly balls. It throws the baseball straight up with an initial velocity of 48 ft/sec. The height $s$ of an object in free fall is given by

$$s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$$

where $g = -32$ ft/sec$^2$ is the acceleration due to gravity, $v_0$ is the initial velocity, $s_0$ is the object's initial height, and $t$ is the time in seconds. Find an equation that models the height of the ball $t$ seconds after it is thrown. What is the maximum height of the baseball? How many seconds will it take the baseball to reach its maximum height?

5. Make sure that all students have the resources available to accomplish every assignment. For example, do not assign projects that involve working on the Internet as a homework assignment, since not every student has access to a computer at home. If you want students to measure something, provide them with rulers or the tools to do the measuring. If you assign a project that requires the use of graphing calculators, provide the calculators and make it an in-class assignment, since many students will not have access to graphing calculators outside the classroom.
OBJECTIVE 6

DEMONSTRATE AN UNDERSTANDING OF GEOMETRIC RELATIONSHIPS AND SPATIAL REASONING

Mathematics Content:

Prior mathematics knowledge requirements:

1. describe, model, draw and classify shapes
2. understand the properties of two-dimensional and three-dimensional shapes
3. relate geometric ideas to number and measurement ideas
4. know the components of the coordinate plane, including axes and quadrants
5. understand how to use translations, rotations, reflections and dilations to illustrate similarities, congruencies and symmetries of figures
6. understand how to use formulas to find length, perimeter, area, and volume of geometric figures
7. classify three-dimensional shapes such as: cubes, prisms, spheres, cones, and pyramids, based on their properties, such as edges and faces

From construction of concepts in the early grades and continued interaction with their local environment, students should have a good understanding of spatial sense and geometric reasoning. Spatial reasoning plays a key role in understanding geometric relationships. Connecting these components helps students understand geometric reasoning and its relationship with real world mathematics. Once students understand spatial reasoning and geometric relationships, they will be prepared to learn Objectives 7-8 of this guide.

Minimum Mathematics Vocabulary Needed for Objective 6:

<table>
<thead>
<tr>
<th>English term</th>
<th>Spanish term</th>
<th>Description/meaning</th>
<th>Drawing/example</th>
</tr>
</thead>
<tbody>
<tr>
<td>angle</td>
<td>ángulo</td>
<td>A figure formed by two non-collinear rays that have the same endpoint. The two rays are called the sides of the angle. Their common endpoint is the vertex.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>English term</td>
<td>Spanish term</td>
<td>Description/meaning</td>
<td>Drawing/example</td>
</tr>
<tr>
<td>-------------------</td>
<td>-----------------------</td>
<td>-------------------------------------------------------------------------------------</td>
<td>--------------------------</td>
</tr>
<tr>
<td><strong>area</strong></td>
<td>área</td>
<td>the number of square units that cover a figure</td>
<td><img src="image" alt="Area" /></td>
</tr>
<tr>
<td></td>
<td>ah'-reh-ah</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>circumference(c)</strong></td>
<td>circunferencia</td>
<td>length of the curve that forms the circle.</td>
<td><img src="image" alt="Circumference" /></td>
</tr>
<tr>
<td></td>
<td>seer-coon-feh- rehn-seea</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>congruent triangles</strong></td>
<td>triángulos congruentes</td>
<td>triangles whose corresponding sides and angles have the same measures</td>
<td><img src="image" alt="Congruent Triangles" /></td>
</tr>
<tr>
<td></td>
<td>tree-ahn'-goo-lohs</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>cohn-groo-ehn-tehs</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>cube</strong></td>
<td>cubo</td>
<td>a solid figure with six faces, each of which is a square</td>
<td><img src="image" alt="Cube" /></td>
</tr>
<tr>
<td></td>
<td>koo-boh</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>cylinder</strong></td>
<td>cilindro</td>
<td>a solid figure whose top and bottom are circular and parallel to each other</td>
<td><img src="image" alt="Cylinder" /></td>
</tr>
<tr>
<td></td>
<td>see-leen'-droh</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>diameter (d)</strong></td>
<td>diámetro</td>
<td>a line segment drawn through the center of a circle that intersects the circumference (connects two points on the circle)</td>
<td><img src="image" alt="Diameter" /></td>
</tr>
<tr>
<td></td>
<td>dee-ah'-meh-troh</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>dimension</strong></td>
<td>dimensión</td>
<td>in flat (plane) figures, the length and width of the figure; in solid figures, the length, width, and height of the figure</td>
<td><img src="image" alt="Dimension" /></td>
</tr>
<tr>
<td></td>
<td>dee-mehn-seeohn’</td>
<td></td>
<td></td>
</tr>
<tr>
<td>English term</td>
<td>Spanish term</td>
<td>Description/meaning</td>
<td>Drawing/example</td>
</tr>
<tr>
<td>-----------------</td>
<td>------------------</td>
<td>-------------------------------------------------------------------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>hypotenuse</td>
<td>hipotenusa</td>
<td>the side of a right triangle opposite the right angle</td>
<td><img src="hypotenuse.png" alt="Diagram" /></td>
</tr>
<tr>
<td>pi</td>
<td>pi</td>
<td>a number that expresses the ratio of a circle’s circumference and its diameter.</td>
<td><img src="pi.png" alt="Formula" /></td>
</tr>
<tr>
<td>perimeter</td>
<td>perímetro</td>
<td>the sum of the lengths of the sides of polygon</td>
<td><img src="perimeter.png" alt="Diagram" /></td>
</tr>
</tbody>
</table>
| Pythagorean theorem | teorema de Pitágoras | a formula that shows the relationship among the lengths of the three sides of a right triangle:  
  \[ a^2 + b^2 = c^2 \], where c is the hypotenuse, and a and b are the legs of the triangle. | ![Diagram](Pythagorean.png) |
| radius (r)      | radio            | a line segment from the center of a circle to a point on the circumference (half of the diameter) | ![Diagram](radius.png) |
| rectangle       | rectángulo       | a quadrilateral with four right angles                                              | ![Diagram](rectangle.png) |
| right angle     | ángulo recto     | an angle with a measure of 90 degrees                                               | ![Diagram](right_angle.png) |

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<table>
<thead>
<tr>
<th>English term</th>
<th>Spanish term</th>
<th>Description/meaning</th>
<th>Drawing/example</th>
</tr>
</thead>
<tbody>
<tr>
<td>right triangle</td>
<td>triángulo recto</td>
<td>a triangle that contains one 90-degree (right) angle</td>
<td><img src="image" alt="right_triangle" /></td>
</tr>
<tr>
<td>tree-an’-goo-loh rehk-toh</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sphere</td>
<td>esfera</td>
<td>the set of points in space that are an equal distance from the center of the figure.</td>
<td><img src="image" alt="sphere" /></td>
</tr>
<tr>
<td>ehs-feh’-rah</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>square</td>
<td>cuadro</td>
<td>a rectangle whose four sides are congruent</td>
<td><img src="image" alt="square" /></td>
</tr>
<tr>
<td>koo-ah’-droh</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tessellation</td>
<td>enlosado</td>
<td>an arrangement of closed figures that completely cover a surface, without any gaps or overlaps</td>
<td><img src="image" alt="tessellation" /></td>
</tr>
<tr>
<td>ehn-loh-sah-thoh</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Strategies for learning this vocabulary:**

1. write definitions in everyday language while still following correct mathematics;
2. use previously defined or common words in definitions and explanations;
3. have students develop self-made glossaries of new vocabulary in journals, picture cards, or charts;
4. as new vocabulary is introduced, add words and definitions with illustrations/explanations to classroom word wall;
5. repeatedly connect the words to mathematical symbols and examples;
6. tape record mathematical words, definitions and verbal examples, for students to play back when needed for extra support; and
7. examine words from Greek and Latin prefixes, roots, and suffixes.

**Teaching strategies and examples for Objective 6:**

1. Choose from a variety of different methods to solve problems.

Example: Ana Osorio was replacing her kitchen and dining room flooring with ceramic tile. The area of these rooms was 220 ft². Ms. Osorio chose a 16” by 16” tile to cover
her floors. Daniel Rodriguez told her she needed 13 boxes of tiles. Each box contained nine tiles and each tile cost $1.79. Ana picked up her tiles and took them home with her. Later that afternoon, Ana called Mr. Rodriguez and told him she would need another box of tiles, because thirteen boxes would not be enough. Daniel disagreed with her and said she really didn’t need another box of tiles. Who was right? Be sure you justify your answer.

2. Make generalizations about geometric properties using numeric and geometric relationships.

Example: Emelio has two differently sized cubes. The smaller cube has a length of an edge of 4 cm, while the length of an edge for the larger cube is 8 cm. Even though the length of an edge of the larger cube was doubled, he noticed that the volume was more than doubled. By what factor did the volume increase?

3. Apply mathematics to practical situations, utilizing the properties of transformations and compositions.

Example: Have students work in groups and model this problem. Julio draws a copy of a polygon figure found in his classroom. His partner Suzette tries to make a tessellation using the figure chosen. Then they switch roles. Students also look for examples of tessellations in their classroom, around the school, in their home, and in their community and bring examples (pictures or drawings) to class to share with all students.

4. Apply patterns of right triangles to problem situations.

Example: Juan has been walking along the sidewalks of his neighborhood, which was built with all blocks as rectangles, as shown in the drawing below. He wants to walk from point A to point C by cutting across the block diagonally. How much distance will he save by walking diagonally, rather than walking down the sidewalks from A to B and then B to C?

---

Teachers guide created by the SHSU MELL Group, November 2005, in collaboration with the Texas State University System and the Texas Education Agency.
5. Make conjectures from congruence transformations.

**Example:** A video game designer is creating a new computer game. Before writing the program, he maps out one of the figures on grid paper. The vertices of the figure are (-3, 1), (4, 2), (4, -2), and (-5, -3). During the game, the figure shifted to (0, 3), (7, 4), (7, 0), and (-2, -1). What type of transformation is this? Justify your answer.

Assessment for Objective 6

**General strategies for assessment:**

1. allow students frequent opportunities to demonstrate mastery in a variety of ways;
2. provide sufficient time for ELL students to complete assessment tasks;
3. use assessment results to design instructional planning for remediation if needed;
4. assign projects for students to work together with their partners;
5. have students write their thoughts and problem-solving actions in a journal;
6. design performance measures with visuals to check concept understanding;
7. design assessments to measure mathematical understanding, not reading comprehension;
8. ensure assignments are as free of bias as possible; and
9. make assignments that require writing explanations in English.

**Specific examples for assessment**

1. For performance assessment, use a rubric that considers concept understanding, approach toward solving a problem, and verbal or picture/diagram explanations.

   Example assessment task: Juan and Maria want to find the most cost effective way to package three-dimensional objects. These objects include shapes of cones, cubes, cylinders, rectangular prisms and spheres, all of which have volumes that are approximately equal. They go to the post office to obtain the sizes and costs of rectangular cardboard boxes. The students were told that the boxes cost $0.70 per 100 cubic inches. Juan and Maria must decide which of their objects would be the most cost effective in packaging. Explain in your journals the steps used to determine which object was the most cost effective, along with diagrams of the objects in the boxes.

2. Use traditional assessment methods, including multiple-choice questions, to measure mathematics understanding. Students need to practice solving mathematics problems in the same format of the TAKS test questions. When
discussing these problems in class, have students analyze why one answer is correct and the others are incorrect. A sample problem could be:

Felipe is flying a kite. Alma is directly underneath the kite. If Felipe has let out 200 feet of string and Alma is 170 feet away from Felipe, which of the following is the approximate height of the kite?

- a. 30 feet
- b. 105 feet
- c. 170 feet
- d. 370 feet

Additional problems can be found on the Texas Education Agency (TEA) website (www.tea.state.tx.us) from the TAKS information booklets (www.tea.state.tx.us/student.assessment/taks/booklets) and from TAKS released tests (www.tea.state.tx.us/student.assessment/resources/release/taks/index.html). Also on the TEA website, there is a link to the TAKS Study Guide for Grade 11 Exit Level Mathematics and Science: A Student and Family Guide, which explains the key concepts under each objective and gives examples (see www.tea.state.tx.us/student.assessment/resources/guides/study/index.html).

There are additional multiple-choice problems for each objective in this guide. Although it is not designed especially for ELL students, it is a very helpful resource in preparing to take the TAKS test.

3. Design projects that involve the families of students. For example: have students work with their family and create a tessellation design to tile their floor or decorate a wall in their home. The design should incorporate at least two of the following transformations – reflections, rotations and translations. Have students work along with their family, describing the steps and the pattern of transformations used to complete their tessellation design. To complete this project, a drawing of the tessellation will be included, along with how transformations were used to create the tessellation.
4. Let students work with a partner in class. Provide extra time for students to talk together about an assigned mathematics problem, decide how to approach it, and make a summary paper on the problem. Provide a format sheet for their mini-reports, such as a) what is the problem, b) what do we need to find out, c) how do we get started, c) how do we solve the problem, d) what is our solution, and e) how can I describe what we did? Allow time for both partners to discuss their report together, before presenting it to the class.

5. Make sure that all students have the resources available to accomplish every assignment. For example, do not assign projects that involve working on the Internet as a homework assignment, since not every student has access to a computer at home. If you want students to measure something, provide them with rulers or the tools to do the measuring. If you assign a project that requires the use of graphing calculators, provide the calculators and make it an in-class assignment, since many students will not have access to graphing calculators outside the classroom.
OBJECTIVE 7

DEMONSTRATE AN UNDERSTANDING OF TWO- AND THREE-DIMENSIONAL REPRESENTATIONS OF GEOMETRIC RELATIONSHIPS AND SHAPES

Mathematics Content:

Prior mathematics knowledge requirements:

1. describe, model, draw and classify shapes
2. know the components of the coordinate plane, to include axes and quadrants
3. understand two-dimensional and three-dimensional shapes and their properties
4. represent problem situations with geometric models and apply properties of figures
5. visualize and draw two- and three-dimensional geometric figures with special attention to analyzing and reasoning informally about their properties
6. demonstrate an understanding of how to apply geometric properties and relationships such as congruence, similarity, angle measure, parallelism and perpendicularity to real-world situations
7. understand incidence relationships among points, lines, and planes, such as intersection, parallelism, and perpendicularity
8. understand the concept of slope in linear equations

From construction of concepts in the early grades and continued interaction with their local environment, students should have a good understanding of geometric reasoning. Studying geometric shapes and figures helps students develop techniques for representing mathematical situations and making generalizations about spatial relationships. Connecting these components helps students understand geometric reasoning and its relationship with real world mathematics. Once students understand spatial reasoning and geometric relationships, they will be prepared to learn measurement and similarity concepts in objective 8 of this guide.

Minimum Mathematics Vocabulary Needed for Objective 7:

<table>
<thead>
<tr>
<th>English term</th>
<th>Spanish term</th>
<th>Description/meaning</th>
<th>Drawing/example</th>
</tr>
</thead>
<tbody>
<tr>
<td>edge</td>
<td>orilla</td>
<td>a line segment on a solid figure where two faces intersect.</td>
<td><img src="image" alt="Edge" /></td>
</tr>
<tr>
<td></td>
<td>oh-ree-ya</td>
<td></td>
<td></td>
</tr>
<tr>
<td>English term</td>
<td>Spanish term</td>
<td>Description/meaning</td>
<td>Drawing/example</td>
</tr>
<tr>
<td>-------------------</td>
<td>--------------------</td>
<td>--------------------------------------------------------------------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>face</td>
<td>cara</td>
<td>a flat surface of a solid figure</td>
<td><img src="image" alt="Face" /></td>
</tr>
<tr>
<td>midpoint of a segment</td>
<td>punto medio de un segmento</td>
<td>the point on a line segment that is equal distance from the two endpoints</td>
<td><img src="image" alt="8 cm" /></td>
</tr>
<tr>
<td>net</td>
<td>patrón</td>
<td>a diagram of a two-dimensional shape that can be folded to form a three dimensional shape</td>
<td><img src="image" alt="Net" /></td>
</tr>
<tr>
<td>parallel lines</td>
<td>líneas paralelas</td>
<td>two lines in a plane that do not intersect</td>
<td><img src="image" alt="Parallel Lines" /></td>
</tr>
<tr>
<td>perpendicular lines</td>
<td>líneas perpendiculares</td>
<td>two lines in a plane that form right angles at their point of intersection</td>
<td><img src="image" alt="Perpendicular Lines" /></td>
</tr>
<tr>
<td>reflection</td>
<td>reflexión</td>
<td>a congruence transformation that reflects (flips) a figure over a line called the line of reflection</td>
<td><img src="image" alt="Reflection" /></td>
</tr>
<tr>
<td><strong>English term</strong></td>
<td><strong>Spanish term</strong></td>
<td><strong>Description/meaning</strong></td>
<td><strong>Drawing/example</strong></td>
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<tr>
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</tr>
<tr>
<td>rotation</td>
<td>rotación</td>
<td>a congruence transformation that rotates (turns) a figure about a fixed point</td>
<td><img src="image" alt="Rotation" /></td>
</tr>
<tr>
<td>slope of a line</td>
<td>pendiente de una línea</td>
<td>the change in vertical distance divided by the change in horizontal distance</td>
<td>for the equation $y = 3x + 5$, the slope of the line is 3, which shows a vertical change of 3 units for every 1 unit of horizontal change</td>
</tr>
<tr>
<td>translation</td>
<td>translación</td>
<td>a congruence transformation that slides a figure in the plane without changing its orientation or size</td>
<td><img src="image" alt="Translation" /></td>
</tr>
<tr>
<td>transversal</td>
<td>transversal</td>
<td>a line that intersects two or more different lines in a plane. When the lines are parallel, certain angle measurements are equal</td>
<td><img src="image" alt="Transversal" /></td>
</tr>
</tbody>
</table>

**Strategies for learning this vocabulary:**

1. write definitions in everyday language while still following correct mathematics;
2. use previously defined or common words in definitions and explanations;
3. have students develop self-made glossaries of new vocabulary in journals, picture cards, or charts;
4. as new vocabulary is introduced, add words and definitions with illustrations/explanations to classroom word wall;
5. repeatedly connect the words to mathematical symbols and examples;
6. tape record mathematical words, definitions and verbal examples, for students to play back when needed for extra support; and
7. examine words from Greek and Latin prefixes, roots, and suffixes.
Teaching strategies and examples for this objective

1. Construct three dimensional objects using two dimensional drawings or models.

Example: Pablo and his friend Manual are making cans for their business. They want to maximize the volume of the cans using an 8-inch by 11-inch piece of metal. If they roll the two long sides together and close the top and bottom with other circular pieces of metal, they get a long thin cylinder. If they roll the two shorter sides together they get a short fat cylinder. Which has the greater volume? What is the area of the circular top and bottom of the can that gives the greatest volume? Justify your answer.

2. Use different views for three-dimensional objects to solve problems.

Example: When Maria looks out her bedroom window, she can see a back corner view of the store shown below. Maria decides to draw a picture of the store. Draw a picture that represents the front view (which has two tall towers on it) of the store that Maria saw out her window.

3. Use the coordinate system to represent different geometric figures.

Example 1: Four boys have decided to make a tent for a party in the backyard with the shape of a quadrilateral. To be accurate they plot the vertex points of the figure at A (-5,2), B (3,8), C (6,4), and D (-2, -2). Their sisters want to hang party lights from the midpoints of each side, stringing them tightly from midpoint to midpoint. The boys and girls have a discussion concerning the shape of the figure that will be formed by the lights. The boys believe it will be a square, and the girls feel it will be a rhombus. Who is correct? Justify your answer.

Example 2: Miguel draws a triangle with the following vertices: A (-3, 2), B (7, 1) and C (5, -3). Determine if this triangle is isosceles, equilateral or scalene.
4. Investigate geometric relationships using properties of lines

Example: graph the equation \(2x + 3y = 30\) on a coordinate plane. Draw another line on the same graph that is parallel to the original graph. Calculate the slope of both lines. What do you notice? Next draw the graph of the equation \(2x + 3y = 30\) again on another page of graph paper. Graph a line that is perpendicular to this line. Calculate the slope of the new (perpendicular) line and compare it to the slope of the original line. What do you notice? Discuss your findings with your partner and write them in your journal.

5. Investigate with formulas to solve various geometric problems

Example: Plot the points (6,8) and (2,2) on a coordinate plane. Measure the line with a ruler and mark the point that is exactly in the middle of the line segment formed by these two points. What are the coordinates of that midpoint? What is the relationship between the coordinates of the midpoint and the coordinates of the two endpoints? Do you think this will always be true? Try several more examples on your graph paper. Can you develop a formula for finding the coordinates of the midpoint, when given the coordinates of the two endpoints of a line segment?

6. Investigate the attributes of three-dimensional objects.

Example: For the figure below, find the number of vertices, faces and edges. Can you find a relationship among these three attributes? Try another figure to see if your conjecture works.

![Three-dimensional object](image)

Assessment for Objective 7

General strategies for assessment

1. allow students frequent opportunities to demonstrate mastery in a variety of ways;
2. provide sufficient time for ELL students to complete assessment tasks;
3. use assessment results to design instructional planning for remediation if needed;
4. assign projects for students to work together with their partners;
5. have students write their thoughts and problem-solving actions in a journal;

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6. design performance measures with visuals to check concept understanding;
7. design assessments to measure mathematical understanding, not reading comprehension;
8. ensure assignments are as free of bias as possible; and
9. make assignments that require writing explanations in English.

Specific examples for assessment

1. Have students work together designing nets that can form three-dimensional shapes. For performance assessment, use a rubric that considers concept understanding, approach toward solving a problem, and verbal or picture/diagram explanations.

Example assessment task: Students will work in groups making models of an icosahedron (20-sides) and octahedron (8-sides) using equilateral triangles only. These nets will be joined together with tape (or include flaps on the net to glue the edges) to form a three-dimensional figure. Describe the process and time it took to design these nets using only equilateral triangles.

2. Use traditional assessment methods, including multiple-choice questions, to measure mathematics understanding. Students need to practice solving mathematics problems in the same format of the TAKS test questions. When discussing these problems in class, have students analyze why one answer is correct and the others are incorrect. A sample problem could be:

The midpoint of segment xy is (3, 4). If the coordinates of y are (6, 6), which of the following are the coordinates of x?

a. (0,2)
b. (0,8)
c. (9,2)
d. (9,8)

Additional problems can be found on the Texas Education Agency (TEA) website (www.tea.state.tx.us) from the TAKS information booklets (www.tea.state.tx.us/student.assessment/taks/booklets) and from TAKS released tests (www.tea.state.tx.us/student.assessment/resources/release/taks/index.html).

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3. Design projects that involve the families of students.

For example: Extend the group activity of the net designs for three-dimensional figures. Students will work with their family designing decorative three-dimensional objects for their families. The net design will consist of at least four polygons. These figures can include pictures, drawings or any other decorative objects found around the home. Write about your experience with your family regarding the process for designing these figures and bring your figures to class for display.

4. Let students work with a partner in class. Provide extra time for students to talk together about an assigned mathematics problem, decide how to approach it, and make a summary paper on the problem. Provide a format sheet for their mini-reports, such as a) what is the problem, b) what do we need to find out, c) how do we get started, c) how do we solve the problem, d) what is our solution, and e) how can I describe what we did? Allow time for both partners to discuss their report together, before presenting it to the class.

5. Make sure that all students have the resources available to accomplish every assignment. For example, do not assign projects that involve working on the Internet as a homework assignment, since not every student has access to a computer at home. If you want students to measure something, provide them with rulers or the tools to do the measuring. If you assign a project that requires the use of graphing calculators, provide the calculators and make it an in-class assignment, since many students will not have access to graphing calculators outside the classroom.
OBJECTIVE 8

DEMONSTRATE AN UNDERSTANDING OF THE CONCEPTS AND USES OF MEASUREMENT AND SIMILARITY

Mathematics Content:

Prior mathematics knowledge requirements:

1. understand how to set up and solve problems involving proportions
2. know how to use characteristics of two and three-dimensional objects to solve problems
3. understand how to use formulas to find length, perimeter, area, surface area and volume of geometric figures
4. solve problems using various dimensional objects
5. relate geometric ideas to number and measurement ideas
6. understand how to use translations, rotations, reflections, dilations, and contractions to illustrate similarities, congruencies and symmetries of figures

Understanding the concepts of measurement and similarity has many real-world applications and provides a basis for developing skills in geometry and in other academic disciplines. With the knowledge about geometric relationships and shapes gained from Objectives 6 and 7, students are now ready to explore and learn about measurement and similarity. Objective 8 is related to Objective 9, where students solve application problems with percents and proportional relationships, as well as determining probabilities.

Minimum Mathematics Vocabulary Needed for Objective 8:

<table>
<thead>
<tr>
<th>English term</th>
<th>Spanish term</th>
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<th>Drawing/example</th>
</tr>
</thead>
<tbody>
<tr>
<td>arc length</td>
<td>longitud de arco</td>
<td>the length of a portion of the circumference of a circle</td>
<td>arc AB The portion of the circle between A and B is the arc AB</td>
</tr>
<tr>
<td>lohn’-hee-tood deh ahr-koh</td>
<td>lohn’-hee-tood deh ahr-koh</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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</thead>
<tbody>
<tr>
<td>area</td>
<td>área</td>
<td>the number of square units that cover a figure</td>
<td><img src="image" alt="area" /> Area = 2 X 4 = 8 sq. units</td>
</tr>
<tr>
<td>perimeter</td>
<td>perímetro</td>
<td>the sum of the lengths of the sides of a polygon</td>
<td><img src="image" alt="perimeter" /> 9cm + 5cm + 11cm = 25cm</td>
</tr>
<tr>
<td>Pythagorean theorem</td>
<td>teorema de Pitágoras</td>
<td>a formula that shows the relationship among the lengths of the three sides of a right triangle: $a^2 + b^2 = c^2$, where $c$ is the hypotenuse, and $a$ and $b$ are the legs of the triangle.</td>
<td><img src="image" alt="Pythagorean theorem" /> $a^2 + b^2 = c^2$ $5^2 + 12^2 = 13^2$ $25 + 144 = 169$ $169 = 169$</td>
</tr>
<tr>
<td>regular polygon</td>
<td>polígono regular</td>
<td>a polygon that is both equilateral and equiangular</td>
<td><img src="image" alt="regular polygon" /> $n = 6$</td>
</tr>
<tr>
<td>sector</td>
<td>sector</td>
<td>a region of a circle bounded by two radii and their intercepted arc</td>
<td><img src="image" alt="sector" /> Major sector Minor sector</td>
</tr>
<tr>
<td>similar figures</td>
<td>figuras similares</td>
<td>figures that have the same shape but may have different sizes. The lengths of corresponding sides of similar figures are proportional</td>
<td><img src="image" alt="similar figures" /></td>
</tr>
</tbody>
</table>

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<tr>
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<th>Description/meaning</th>
<th>Drawing/example</th>
</tr>
</thead>
<tbody>
<tr>
<td>surface area</td>
<td>area lateral (superficial)</td>
<td>for a solid figure, the sum of the areas of all the faces or surfaces than enclose the solid</td>
<td><img src="image" alt="Diagram of a cube with surface area calculation" /> The surface area of this cube is $6 \times 37^2 = 8214 \text{ mm}^2$</td>
</tr>
<tr>
<td>trigonometric ratios</td>
<td>radio trigonométrico función trigonométrica</td>
<td>relationships between the sides of right triangles. Most commonly used are the sine, cosine, and tangent</td>
<td><img src="image" alt="Diagram of a right triangle with sine calculation" /> The sine of angle A is $12/13$</td>
</tr>
<tr>
<td>volume</td>
<td>volúmen</td>
<td>the measured size (how much it will hold) of a solid figure; measurement given in “cubic” units</td>
<td><img src="image" alt="Diagram of a cube with volume calculation" /> volume = $2 \times 2 \times 6 = 24$</td>
</tr>
</tbody>
</table>

**Strategies for learning this vocabulary:**

1. write definitions in everyday language while still following correct mathematics;
2. use previously defined or common words in definitions and explanations;
3. have students develop self-made glossaries of new vocabulary in journals, picture cards, or charts;
4. as new vocabulary is introduced, add words and definitions with illustrations/explanations to classroom word wall;

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5. repeatedly connect the words to mathematical symbols and examples;
6. tape record mathematical words, definitions and verbal examples, for students to play back when needed for extra support; and
7. examine words from Greek and Latin prefixes, roots, and suffixes.

Teaching strategies and examples for Objective 8

1. Find the area of composite figures.

Example: The garage at Juan’s house is outlined by polygon BCDEFG. There is also a concrete patio with corners at A, B, F, and G. Juan needs to paint the floor of his garage and the patio with different colors, and needs to determine how much paint to order for each. If a gallon of floor paint covers 250 square feet, how many gallons of paint should he buy to cover the floor of the garage and the patio?
2. Find the arc lengths of circles.

Example: A hotel has a revolving door with 3 equal sections for people to enter. Its diameter is 6 ft. In order to minimize the effects of bad weather, the hotel wants only one section at a time open to the outside. In order to show the boundary for the revolving door, the hotel is adding a bright green tile to mark its outer edge. If tiles are 4 inches long, how many are needed to mark the boundary for the revolving door outside the hotel, as indicated in the drawing below? Justify your answer.

3. Use the Pythagorean Theorem to solve problems.

Example: Sarah, Maria, and Shanelle are best friends. Sarah lives 4 mi. west and 4 mi. south of the school, and Maria lives 5 mi. north and 5 mi. east of the school. Sarah and Maria are riding their dirt bikes to Shanelle’s house, which is located 6 miles south and 8 miles east of the school. Assuming that the girls can all ride across open fields to Shanelle’s house, who has the shortest distance to travel, Sarah or Maria? Make a drawing on graph paper to help solve this problem. Justify your answer.
4. Find surface area and volume in problem situations.

Example: Mayra wants to create a piñata made of thick cardboard that will hold birthday surprises and candy. Her design consists of a cone, cylinder and hemisphere as shown below. Each of three solid sections has a height of 20 centimeters and a radius of 15 centimeters. How many cubic centimeters of birthday surprises and candy will Mayra’s piñata hold? Justify your answer.

5. Use similarity properties and transformations to justify conjectures about geometric figures.

Example: Julio and Sheanna want to find the distance between two points $A$ and $B$. The difficulty is that there is a lake between the two points so a direct measurement cannot be taken. The drawing they were given is shown at right where $\angle A = \angle B = 60^\circ$; $\overline{DE} \parallel \overline{GF}$; $AG = 15'; BE = 18'; EF = 16'$.

Point $C$ is on $\overline{GF}$. If they can only measure one more distance to complete the problem, what would it be? Julio and Sheanna gave the following directions for finding the distance from $A$ to $B$:

**Julio:** Measure $CF$ and use similar triangles $BED$ and $BFC$ to find the distance from $C$ to $B$. Add this to the distance from $C$ to $A$ (determined by using proportional sides of similar triangles $BFC$ and $AGC$) to get the distance from $A$ to $B$.

**Sheanna:** Measure the distance from $A$ to $C$. Use similar triangles $AGC$ and $BFC$ to get the distance from $B$ to $C$. Add this to $AC$ to get the entire distance.

Critique these two plans and determine whether either can be used to find the distance from $A$ to $B$. 

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6. Use ratios to solve problems involving similar figures.

Example: Sammy uses a mirror to project an image of a baseball card. The card measures 9 centimeters by 12 centimeters. In the image, the shorter side of the card is now 16 centimeters. How long is the larger side of the baseball card in the image?

7. Apply right triangle ratios to solve problems.

Example:

Julie has been given the task of determining the distance between two buildings without measuring. She and her friend, Ricky, climb to the roof of one of the buildings and find that the buildings are the same height. Ricky holds her father’s triangle square so that the right angle is at the top and directly over the edge of their building. Julie adjusts the triangle square so that when she sights along one edge she lines up with the roof of the other building. Then, without moving the device, she sites down the other edge and places a rock at the spot on the roof that she sees. The rock is 2 feet from the right corner of the building and the top of the triangle square is 4.5 feet from the rock. Julie next measures the angle that the triangle square makes to the roof of the building where she is standing and finds that it is 52 degrees. Find the distance between the two buildings using trigonometric ratios.

8. Describe the effect on perimeter, area, and volume when length, width, or height of a three-dimensional solid is changed, and use this idea in solving problems.

Example:

An oil tank is constructed so that the middle part is a cylinder with length 25' and radius 5' and two ends that are each hemispheres. If the length of the cylindrical part of the tank is doubled, what effect does this have on the surface area and volume of the tank? Explain your answer.
Assessment for Objective 8

General strategies for assessment

1. allow students frequent opportunities to demonstrate mastery in a variety of ways;
2. provide sufficient time for ELL students to complete assessment tasks;
3. use assessment results to design instructional planning for remediation if needed;
4. assign projects for students to work together with their partners;
5. have students write their thoughts and problem-solving actions in a journal;
6. design performance measures with visuals to check concept understanding;
7. design assessments to measure mathematical understanding, not reading comprehension;
8. ensure assignments are as free of bias as possible; and
9. make assignments that require writing explanations in English.

Specific examples for assessment

1. Task students to work with their partners to develop at least two geometric proofs of the Pythagorean Theorem, using graph paper, manipulatives, or drawings. Each proof should be explained in a written report, along with diagrams, examples, and student reflections on their work and experiences. If students cannot develop two proofs on their own, they can use the supplemental mathematics textbooks in the classroom or library, as well as the Internet on the classroom computers.

2. Use traditional assessment methods, including multiple-choice questions, to measure mathematics understanding also. Students need the practice solving mathematics problems in the same format of the TAKS test questions. When discussing these problems in class, have students analyze why one answer is correct and the others are incorrect. A sample problem could be:

Two rectangular prisms are similar. The smaller rectangular prism has a height of 24 centimeters. The larger prism has a height of 48 centimeters. By what factor is the volume of the bigger prism increased compared to the smaller prism?

   a. 2
   b. 4
   c. 6
   d. 8
Additional problems can be found on the Texas Education Agency (TEA) website (www.tea.state.tx.us) from the TAKS information booklets (www.tea.state.tx.us/student.assessment/taks/booklets) and from TAKS released tests (www.tea.state.tx.us/student.assessment/resources/release/taks/index.html). Also on the TEA website, there is a link to the TAKS Study Guide for Grade 11 Exit Level Mathematics and Science: A Student and Family Guide, which explains the key concepts under each objective and gives examples (see www.tea.state.tx.us/student.assessment/resources/guides/study/index.html). There are additional multiple-choice problems for each objective in this guide. Although it is not designed especially for ELL students, it is a very helpful resource in preparing to take the TAKS test.

3. Design projects that involve the families of students. Have them collect data as homework assignments. For example:

David wants to show his father what he is learning about surface area in his geometry class. When looking around his house, he notices that the garage needs painting and plans to help his father with this project on a weekend. David decides to calculate how much paint they would need to paint both sides. He measures the length and height of each side of the garage, and records 25 feet by 8 feet on each side. But then he notices that there are a total of three windows, which will not be painted. So he measures the windows, which were each 3 feet by 5 feet. How many square feet will need to be painted on the garage?

4. Let students work with a partner in class. Provide extra time for students to talk together about an assigned mathematics problem, decide how to approach it, and make a summary paper on the problem. Provide a format sheet for their mini-reports, such as a) what is the problem, b) what do we need to find out, c) how do we get started, c) how do we solve the problem, d) what is our solution, and e) how can I describe what we did? Allow time for both partners to discuss their report together, before presenting it to the class.

5. Make sure that all students have the resources available to accomplish every assignment. For example, do not assign projects that involve working on the Internet as a homework assignment, since not every student has access to a computer at home. If you want students to measure something, provide them with rulers or the tools to do the measuring. If you assign a project that requires the use of graphing calculators, provide the calculators and make it an in-class assignment, since many students will not have access to graphing calculators outside the classroom.
OBJECTIVE 9

DEMONSTRATE AN UNDERSTANDING OF PERCENTS, PROPORTIONAL RELATIONSHIPS, PROBABILITY, AND STATISTICS IN APPLICATION PROBLEMS

Mathematics content:

Prior mathematics knowledge requirements:

1. represent relationships between numbers as fractions, percents, and decimals
2. convert numbers from one form to another (fractions, ratios, percents, and decimals)
3. identify, set up, and solve proportional relationships
4. list possible outcomes for probability problems
5. find measures of central tendency (mean, median, mode)
6. use appropriate terms to describe probabilities of events
7. represent and interpret data using circle graphs, bar graphs, and histograms

Teachers need to prepare students to become informed consumers who can describe data and interpret statistical information. Students need to determine percents, predict results of probability experiments, account for all possible outcomes of a given situation, and to apply proportional reasoning in everyday problems. Objective 9 logically follows Objective 8, where proportional reasoning was necessary to solve similarity problems. Further applications of mathematical thinking and problem solving will be covered in Objective 10.

Minimum mathematics vocabulary needed for Objective 9:

<table>
<thead>
<tr>
<th>English term</th>
<th>Spanish term</th>
<th>Description/meaning</th>
<th>Drawing/example</th>
</tr>
</thead>
<tbody>
<tr>
<td>dependent event</td>
<td>evento dependiente</td>
<td>in probability, an event that depends on some previous outcome</td>
<td>pick a card from a deck of playing cards, do not replace it, and then pick another card. The pick of the second card is dependent on which card was picked on the first draw</td>
</tr>
<tr>
<td>event</td>
<td>evento</td>
<td>a subset of a sample space</td>
<td>rolling a die and getting a 4 is an event</td>
</tr>
</tbody>
</table>

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<td>evento</td>
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<td>rolling a die and getting a 4 is an event</td>
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<td><strong>Spanish term</strong></td>
<td><strong>Description/meaning</strong></td>
<td><strong>Drawing/example</strong></td>
</tr>
<tr>
<td>-------------------------------</td>
<td>---------------------------</td>
<td>--------------------------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>experimental probability</td>
<td>probabilidad experimental</td>
<td>the probability of an event determined by observation or measurement</td>
<td>toss a coin 100 times and you had 60 heads and 40 tails. The experimental probability for getting a head is 60/100 or 3/5</td>
</tr>
<tr>
<td>independent event</td>
<td>evento independiente</td>
<td>in probability, an event that does not depend on some previous outcome</td>
<td>pick a card from a deck of playing cards, replace it, and then pick another card. The pick of the second card does not depend on which card was picked on the first draw</td>
</tr>
<tr>
<td>mean</td>
<td>media</td>
<td>the calculated average of a set of numbers, found by adding all of the numbers in the set and dividing by the number of numbers in the set</td>
<td>for the data set {80, 76, 89, 91} the mean is ( \frac{80 + 76 + 89 + 91}{4} = \frac{336}{4} = 84 )</td>
</tr>
<tr>
<td>median</td>
<td>mediana</td>
<td>the middle value (or the average of the middle two values) of a set of data arranged in numerical order</td>
<td>for the data set {2, 3, 4, 6, 8, 10, 15}, the median is 6</td>
</tr>
<tr>
<td>mode</td>
<td>moda</td>
<td>in a data set, the number or element that occurs most often.</td>
<td>for the data set {5, 8, 6, 7, 8, 1, 4, 8}, the mode is 8</td>
</tr>
<tr>
<td>possible outcome</td>
<td>resultado posible</td>
<td>one of the elements or events in the sample space.</td>
<td>if you pull a card from a deck of 52 cards, an ace of spades is a possible outcome.</td>
</tr>
<tr>
<td>probability</td>
<td>probabilidad</td>
<td>a measure of the chance or likelihood of an event to occur.</td>
<td>the probability of drawing a heart out of a standard deck of playing cards is 13/52 or 1/4</td>
</tr>
</tbody>
</table>
| proportion                    | proporción                | an equation that states that two ratios are equal                                         | \[ \frac{8}{11} = \frac{14}{x} \]
<table>
<thead>
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<th>Description/meaning</th>
<th>Drawing/example</th>
</tr>
</thead>
<tbody>
<tr>
<td>ratio</td>
<td>relación ó razón</td>
<td>a comparison of two or more quantities.</td>
<td>200 miles</td>
</tr>
<tr>
<td></td>
<td>reh-lah-seeohn' oh</td>
<td></td>
<td>9 gallons</td>
</tr>
<tr>
<td></td>
<td>rah-sohn'</td>
<td></td>
<td></td>
</tr>
<tr>
<td>sales tax</td>
<td>impuesto</td>
<td>an amount added to the cost of items, calculated</td>
<td>a sales tax of 8.5% on a $50 purchase would be $4.25</td>
</tr>
<tr>
<td></td>
<td>eem-poo-ehs'-toh</td>
<td>as a percentage of the cost</td>
<td>(50 x 0.085)</td>
</tr>
<tr>
<td>sample space</td>
<td>espacio muestral</td>
<td>the set of all possible outcomes for an experiment.</td>
<td>the sample space for tossing a 6-face die is S = {1, 2, 3, 4, 5, 6}</td>
</tr>
<tr>
<td></td>
<td>ehs-pah'-seeoh moo-ehs-trahl'</td>
<td></td>
<td></td>
</tr>
<tr>
<td>similar figures</td>
<td>figuras similares</td>
<td>figures that have the same shape but may have</td>
<td></td>
</tr>
<tr>
<td></td>
<td>fee-goo'-rahs see-mee-lah'-rehs</td>
<td>different sizes. The lengths of corresponding</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>sides of similar figures are proportional</td>
<td></td>
</tr>
<tr>
<td>theoretical</td>
<td>probabilidad</td>
<td>the probability of an event determined by the</td>
<td>when a coin is tossed, the theoretical probability of</td>
</tr>
<tr>
<td>probability</td>
<td>teórica</td>
<td>ratio of the number of favorable outcomes to</td>
<td>getting a head is 1/2</td>
</tr>
<tr>
<td></td>
<td>proh-bah-bee-lee-dah'</td>
<td>the number of possible outcomes in the sample space</td>
<td></td>
</tr>
<tr>
<td></td>
<td>teh-oh'-ree-cah</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Strategies for learning this vocabulary:**

1. write definitions in everyday language while still following correct mathematics;
2. use previously defined or common words in definitions and explanations;
3. have students develop self-made glossaries of new vocabulary in journals, picture cards, or charts;
4. as new vocabulary is introduced, add words and definitions with illustrations/explanations to classroom word wall;
5. repeatedly connect the words to mathematical symbols and examples;
6. tape record mathematical words, definitions and verbal examples, for students to play back when needed for extra support; and
7. examine words from Greek and Latin prefixes, roots, and suffixes.
Teaching strategies and examples for Objective 9

1. identify proportional relationships and use them to solve problems.

Example: An antique rectangular table has a width of 60 inches and a length of 72 inches as shown below. Raul wanted to make an exact scale model of the table and he wants the scale model to fit into his hand. What would be a good size (width and length) for the table to be for the scale model? Justify your answer.

2. find probabilities of compound events.

Example: Aarón wants to go out with his friends on Friday night, and he estimates that he has a 50% chance that his parents will let him. If he does get to go out, there is a probability of 1/3 that his friends will go to the mall. What is the overall probability that Aarón will end up at the mall?

3. have students use theoretical probabilities and experimental results to make predictions and decisions.

Example: Ana has a six-sided cube that has sides that are painted red, green or yellow. She tosses the cube 50 times and records 28 reds, 15 greens and 7 yellows. She gives these results to her teacher (who can’t see the cube) and asks her to predict how many sides on her colored cube are red, green, and yellow. What should her teacher say? Why?

4. select an appropriate measure of central tendency to describe a set of data.

Example: Create a data set with at least 10 homework grades so that the mean is 85 and the median is 80. Explain how you completed this problem. Do you think that your teacher will use the mean or the median to determine your homework grade? Why?

5. construct circle graphs, bar graphs, and histograms, with and without technology.

Example: Carlos makes $120 per week working at McDonalds. Even though his parents provide him with a room and food at home, he must pay for all of his other expenses out of his salary. Develop a reasonable monthly budget for Carlos, including gasoline for his pickup truck (his parents pay for repairs and other car costs), clothing, food (lunches, hamburgers, snacks, sodas, etc.), music CD's, movie theater tickets, girlfriend expenses for dates and gifts, other entertainment, and other miscellaneous expenses. Represent each category of expenses on both a circle graph and a bar
graph. The highest grades on this assignment will be awarded to students who create their graphs on a computer or graphing calculator (both will be available in our classroom before and after school, if needed).

6. recognize misuses of graphical or numerical information and evaluate predictions and conclusions based on data analysis.

Example: Diego and Geraldo are working on a project for their mathematics class, which requires them to find statistical errors in newspapers, magazines, or on Internet reports. For each of the following, decide whether there is a statistical error or not, and justify your response.

   a. On a line graph, the horizontal axis has increments of 10, while the vertical axis has increments of 100.
   b. A circle graph shows percentages of different types of restaurants in town, with four sections for Mexican (22%), Italian (18%), Seafood 11%, and Steakhouses 24%.
   c. A line graph starts at 0 on the horizontal axis, and starts at 1000 on the vertical axis.
   d. A histogram showing ages of students in the high school has intervals (such as 15 to 16.9 years old) on the horizontal axis.
   e. A picture graph shows the average number of cans of soda bought by students aged 14 to 18 in 1995 and 2005. The average doubled, so the picture of the can of soda representing 2005 is twice as tall and twice as wide as the picture of the can representing 1995.

Assessment for Objective 9

General strategies for assessment:

1. allow students frequent opportunities to demonstrate mastery in a variety of ways;
2. provide sufficient time for ELL students to complete assessment tasks;
3. use assessment results to design instructional planning for remediation if needed;
4. assign projects for students to work together with their partners;
5. have students write their thoughts and problem-solving actions in a journal;
6. design performance measures with visuals to check concept understanding;
7. design assessments to measure mathematical understanding, not reading comprehension;
8. ensure assignments are as free of bias as possible; and
9. make assignments that require writing explanations in English.

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Specific examples for assessment

1. Task students to design a probability experiment, using playing cards, dice, coins or chips of different colors. The experiment must involve 50 trials. Compute the theoretical probability for the experiment, and then have your partner or family member perform the experiment. Compare and discuss the experimental results with the theoretical probability. Were the experimental results surprising? How likely were the results from the experiment? How do you account for any differences between the theoretical and the experimental probabilities from the experiment? Write a report of your findings and conclusions, and discuss any changes you would make to the design of the experiment before you would perform it again.

2. Use traditional assessment methods, including multiple-choice questions, to measure mathematics understanding. Students need to practice solving mathematics problems in the same format of the TAKS test questions. When discussing these problems in class, have students analyze why one answer is correct and the others are incorrect. A sample problem could be:

A data set gives the blood pressure readings for 12 people, where the mean is 140. One of the 12 people had a reading of 180. If the person with this reading is eliminated from the group, which of the following will decrease?

a. The mode
b. The median
c. The mean
d. It cannot be determined from the information given

Additional problems can be found on the Texas Education Agency (TEA) website (www.tea.state.tx.us) from the TAKS information booklets (www.tea.state.tx.us/student.assessment/taks/booklets) and from TAKS released tests (www.tea.state.tx.us/student.assessment/resources/release/taks/index.html). Also on the TEA website, there is a link to the TAKS Study Guide for Grade 11 Exit Level Mathematics and Science: A Student and Family Guide, which explains the key concepts under each objective and gives examples (see www.tea.state.tx.us/student.assessment/resources/guides/study/index.html). There are additional multiple-choice problems for each objective in this guide. Although it is not designed especially for ELL students, it is a very helpful resource in preparing to take the TAKS test.

3. Design projects that involve the families of students. For example: have students ask their family members to predict how many minutes of a half-hour television program will be advertisements. After recording predictions for all family members, the family will need to watch a half-hour program (on a channel such as NBC or Telemundo if available) and record the number of seconds for every commercial or advertisement. Next, the family will need to watch a half-hour local news program, and again record the

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number of seconds for each commercial or advertisement. Students and their families will then calculate the percentage of time for commercials and the actual program, and compare the results. A short paper will be submitted describing this experiment and the results, as well as comparing the results with the original predictions and contrasting the two types of programs.

4. Let students work with a partner in class. Provide extra time for students to talk together about an assigned mathematics problem, decide how to approach it, and make a summary paper on the problem. Provide a format sheet for their mini-reports, such as a) what is the problem, b) what do we need to find out, c) how do we get started, c) how do we solve the problem, d) what is our solution, and e) how can I describe what we did? Allow time for both partners to discuss their report together, before presenting it to the class.

5. Make sure that all students have the resources available to accomplish every assignment. For example, do not assign projects that involve working on the Internet as a homework assignment, since not every student has access to a computer at home. If you want students to measure something, provide them with rulers or the tools to do the measuring. If you assign a project that requires the use of graphing calculators, provide the calculators and make it an in-class assignment, since many students will not have access to graphing calculators outside the classroom.
OBJECTIVE 10

DEMONSTRATE AN UNDERSTANDING OF THE MATHEMATICAL PROCESSES AND TOOLS USED IN PROBLEM SOLVING

Mathematics Content:

Prior mathematics knowledge requirements:

1. understand and apply geometry and spatial reasoning concepts
2. understand and apply proportional reasoning skills
3. understand and apply measurement and similarity concepts
4. understand and apply probability and statistics concepts
5. identify, describe, and extend patterns in visual and numerical representations
6. apply numerical reasoning to solve problems
7. use mathematical tools to set up and solve problems

Students need to apply 8th grade mathematics to solve problems related to everyday experiences, investigations in other disciplines, and activities in and outside of school. They should be able to use various strategies for solving problems, such as drawing a picture, looking for patterns, and working on a simpler problem first. Students are also expected to communicate mathematics using formal and informal language, representations, and models. Students are also expected to use logical reasoning to make conjectures and verify conclusions. Mathematics vocabulary described in Objectives 6-9 will also be required to solve assorted problems in Objective 10.

Minimum Mathematics Vocabulary Needed for Objective 10:

<table>
<thead>
<tr>
<th>English term</th>
<th>Spanish term</th>
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</tr>
</thead>
<tbody>
<tr>
<td>conjecture</td>
<td>conjetura</td>
<td>a generalization</td>
<td>after measuring and finding the sum of the interior angles of several different triangles, a conjecture would be that the sum is always 180 degrees.</td>
</tr>
<tr>
<td></td>
<td>cohn-heh-too'-rah</td>
<td>based on inductive reasoning</td>
<td></td>
</tr>
</tbody>
</table>

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</thead>
<tbody>
<tr>
<td>logical reasoning</td>
<td>razonamiento lógico</td>
<td>reasoning based on inferences of logic</td>
<td>a geometric proof uses logical reasoning to take what information is given and to use mathematical definitions, axioms, and theorems to determine a result.</td>
</tr>
<tr>
<td>mathematical properties</td>
<td>propiedades matemáticas</td>
<td>rules or relationships that are valid for a set of numbers</td>
<td>the commutative property states that, for any real numbers $a$ and $b$, $a + b = b + a$</td>
</tr>
<tr>
<td>model</td>
<td>modelo</td>
<td>a representation of a mathematical concept (such as physical, algebraic, or geometric representations)</td>
<td>algebra tiles are used to represent algebraic relationships.</td>
</tr>
</tbody>
</table>
| pattern                      | regularidad o patrón             | a sequence of observations that follows a trend                                       | $3 + 5 = 8$  
$7 + 13 = 20$  
$21 + 3 = 24$  

an odd whole number plus another odd whole number equals an even number. |
| problem solving strategies   | estrategias para soluciones de problemas matemáticos | techniques or approaches for solving a problem                                       | strategies may include make a list, look for a pattern, make a drawing, solve an easier problem, etc. |
Strategies for learning this vocabulary:

1. write definitions in everyday language while still following correct mathematics;
2. use previously defined or common words in definitions and explanations;
3. have students develop self-made glossaries of new vocabulary in journals, picture cards, or charts;
4. as new vocabulary is introduced, add words and definitions with illustrations/explanations to classroom word wall;
5. repeatedly connect the words to mathematical symbols and examples;
6. tape record mathematical words, definitions and verbal examples, for students to play back when needed for extra support; and
7. examine words from Greek and Latin prefixes, roots, and suffixes.

Teaching strategies and examples for Objective 10

1. identify and apply mathematics to everyday experiences.

Example: The capacity of an elevator at the mall is either 16 children or 12 adults. There are 12 children already on the elevator. How many adults can get on the elevator with them?

2. use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness.

Example: Manuel’s family has a small farm and he wants to fence in an area for his sister’s chickens. He has a barn that is 200 feet long, and he wants to use one side of the barn as part of the rectangular enclosed area to save on the cost of fencing. He has a total of 360 feet of fencing to use (he wants to use all of the fencing), and he wants the side of the rectangle opposite the barn to be 80 feet long. Use the four-step problem-solving model and write explanations of each step in a written report to answer the following questions:

a. how far out from the barn (the distance from the side of the barn to the opposite side of the rectangular area) would his sister’s chicken pen be?

b. if the last 20 feet of fencing on the inside of the roll were damaged and could not be used, how far out from the barn would the fence now extend?

c. write an algebraic expression for the distance of the fencing from the barn for $n$ feet of available fencing, if the rest of the problem remained the same.
3. select or develop an appropriate problem-solving strategy from a variety of techniques.

Example: Hieu is planning for her classroom end-of-semester party. She made 10 gallons of grape punch, which was 30% pure grape juice and the rest water. However, her friend Amy tastes the punch and says it is too strong. How much water would they need to add to make a 20% solution? Choose an appropriate problem-solving strategy, justify why you chose that particular strategy, solve the problem, and explain each step of the process. What would you have to do to make the original 30% solution stronger?

4. communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models.

Example: Algebra tiles can be used to enhance algebraic skills. For this manipulative, 

\[ \square \] represents the variable \( x \), \( \Box \) represents the number 1, and 

\[ \square \] represents \( x^2 \).

Mrs. Garcia shows the following diagram of algebra tiles to her students, and asks them to tell her what algebraic problem is shown by the algebra tiles. (Note: the problem displayed below is \((x + 2)(x + 4)\)).

Mrs. Garcia next asks students to get with their partner and try to find how the algebra tiles model is related to the FOIL method for multiplying binomial expressions (which students learned last month in their algebra class).

5. make conjectures from patterns or sets of examples and non-examples.

Example: Paulino’s mathematics teacher assigned a magic squares project. As an example, the following 3 by 3 magic square was provided to students.
Paulino and his group were asked to find as many patterns as they could with the numbers in the magic square. The teacher then asked each group to tell the class about the patterns they found. Most groups discovered that the sum of all rows, columns, and diagonals was 18 (the magic sum). Paulino’s teacher then extended the activity by asking students to:

a. look for a relationship between the middle number and the magic sum for this 3 by 3 magic square.

b. look for any patterns for the numbers in any one row, column, or diagonal (hoping that students would notice the “balancing” of the numbers on each side of the center number).

c. write algebraic expressions to represent the relationships for every number in the magic square to the middle number.

d. using these algebraic expressions, create additional magic squares with different numbers than the one provided.

e. write an entry in their journals describing what they discovered about the patterns found in magic squares.

6. validate conclusions using mathematical properties and relationships.

Example: Draw a trapezoid on a rectangular coordinate grid. Reflect the trapezoid across the vertical axis and then across the horizontal axis. Draw the new trapezoid in another color on the grid. Using the original trapezoid, reflect it across the horizontal axis first, and then across the vertical axis. Note the position of this second image of the trapezoid. What can you conclude? What property is demonstrated by this activity? To extend this activity, draw another trapezoid on another rectangular coordinate grid. Rotate the trapezoid 90 degrees clockwise about the origin, and draw the new trapezoid on the grid in another color. Using the original trapezoid, rotate it 270 degrees counterclockwise about the origin. Note the position of the second trapezoid image. What can you conclude from these two rotations?

Assessment for Objective 10

1. allow students frequent opportunities to demonstrate mastery in a variety of ways;
2. provide sufficient time for ELL students to complete assessment tasks;
3. use assessment results to design instructional planning for remediation if needed;
4. assign projects for students to work together with their partners;
5. have students write their thoughts and problem-solving actions in a journal;
6. design performance measures with visuals to check concept understanding;
7. design assessments to measure mathematical understanding, not reading comprehension;
8. ensure assignments are as free of bias as possible; and
9. make assignments that require writing explanations in English.

Specific examples of assessment

1. For performance assessment, use a rubric that considers concept understanding, approach toward solving a problem, and verbal or picture/diagram explanations. A sample assessment task could be:

During the fall season, geese normally migrate from the north to the south. Cristina studied the pattern in which geese fly as a group. She noticed that they form a V-shape with the leader on the front (see the sequence of pictures below). Cristina started recording data to help her find how many geese there would be if this V pattern continued for larger groups of geese. Help Cristina draw pictures of the next two groups of geese in this pattern, complete the table and answer the following questions related to the geese migration. Turn in your response in a written report.

   a. What pattern did you notice for the total number of geese in the group, in relation to the number of rows of geese?

   b. How did you figure out how many birds are in the group with N birds on one line?

   c. Is the total number of geese in a group a linear or a quadratic function? How can you tell?

   d. What if three groups of five geese each decided to merge into one V formation? Make a drawing of how you think they would be arranged.
2. Use traditional assessment methods, including multiple-choice questions, to measure mathematics understanding also. Students need to practice solving mathematics problems of the same format as the TAKS test questions. Students should use the four-step problem-solving process and analyze different strategies to approach problems. When discussing these problems in class, have students analyze why one answer is correct and the others are incorrect. The sample problem below is from the 2004 released TAKS test. More than half of the students missed this question.

Jake’s square backyard covers an area of 104 square meters. How can Jake best determine the length of each side of his backyard?

A. divide the area by the number of sides  
B. square the area  
C. find the square root of the area  
D. divide the area in half

<table>
<thead>
<tr>
<th>Number of rows of geese</th>
<th>Total number of geese in the group</th>
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Additional problems can be found on the Texas Education Agency (TEA) website (www.tea.state.tx.us) from the TAKS information booklets (www.tea.state.tx.us/student.assessment/taks/booklets) and from TAKS released tests (www.tea.state.tx.us/student.assessment/resources/release/taks/index.html). Also on the TEA website, there is a link to the TAKS Study Guide for Grade 11 Exit Level Mathematics and Science: A Student and Family Guide, which explains the key concepts under each objective and gives examples (see www.tea.state.tx.us/student.assessment/resources/guides/study/index.html). There are additional multiple-choice problems for each objective in this guide. Although it is not designed especially for ELL students, it is a very helpful resource in preparing to take the TAKS test.

3. Design projects that involve the families of students. For example:

Julio wants to build a patio at the back of his mother's house that measures 12 feet by 10 feet. The patio will also have a square barbecue pit in the center that measures 3 feet on each side. He wants to tile the entire patio, excluding the barbecue pit. After shopping around for tile squares (that measure 12 inches by 12 inches), he found one his mother liked that cost $1.25 per square foot. If the sales tax on this purchase was 8.25%, how much would the total cost of the tile squares for the project be? Write a report that discusses in detail each step of the problem-solving process you used for this problem, showing all calculations.

4. Let students work with a partner in class. Provide extra time for students to talk together about an assigned mathematics problem, decide how to approach it, and make a summary paper on the problem. Provide a format sheet for their mini-reports, such as a) what is the problem, b) what do we need to find out, c) how do we get started, c) how do we solve the problem, d) what is our solution, and e) how can I describe what we did? Allow time for both partners to discuss their report together, before presenting it to the class.

5. Make sure that all students have the resources available to accomplish every assignment. For example, do not assign projects that involve working on the Internet as a homework assignment, since not every student has access to a computer at home. If you want students to measure something, provide them with rulers or the tools to do the measuring. If you assign a project that requires the use of graphing calculators, provide the calculators and make it an in-class assignment, since many students will not have access to graphing calculators outside the classroom.
APPENDIX A – MELL CLASSROOM PRACTICES FRAMEWORK CPF

The MELL Classroom Practices Framework is a synthesis document compiled by the Texas State University System (TSUS) Math for English Language Learners (MELL) Initiative funded by a grant from the Texas Education Agency. In the summer of 2004 TEA, in response to the lingering achievement gap in mathematics between Limited English Proficient (LEP) students and other students, worked with TSUS and its five partner institutions to establish the MELL Initiative. The primary purpose of the MELL Initiative is to develop resources for professional development targeted at improving mathematics instruction for English Language Learners, especially those at the secondary level.

In Phase I of the Year 1 scope of work, several avenues were simultaneously pursued in order to identify specific needs related to math instruction for English Language Learners and to identify existing resources. An extensive review of research and literature was coordinated by Sul Ross State University, while mathematicians at Texas State University explored 12 different professional development models designed to support the math instruction for struggling students. Lamar University gathered information directly from teachers through focus groups and survey instruments to identify their views about professional development needed to support math instruction for English Language Learners. Statisticians at Sam Houston State University worked with TAKS results to further analyze achievement trends by groups of students and by geographic and ESC regions. Angelo State University analyzed the preparation program for math teachers at each of the TSUS institutions to identify current practices in preparing math teachers to deal with the specific needs of English Language Learners. Periodic meetings involving TEA staff, faculty from the participating TSUS institutions, and experts in the field, were conducted throughout the year to guide and shape the work and to keep all parties updated on the progress of others. Each of these Phase I avenues of investigation has resulted in a specific MELL product and information about these products is available through the MELL website at www.tsusmell.org.

Phase II of the Year 1 scope of work is devoted to developing various professional development resources designed to address the specific needs identified in Phase I. MELL and TEA staff both identified the need for a concise document that could not only capture the essence of the Phase I work, but could also provide a roadmap for use in Phase II products. The MELL Classroom Practices Framework (CPF) was developed in response to this need.

The MELL CPF was generated collaboratively by MELL and TEA staff and was guided by the question of “What do the findings of our Phase I investigations suggest in regard to classroom practices that contribute to successful math instruction for English Language Learners?” This framework represents the current collective thinking of MELL partners about what Phase I investigations revealed, and it is our intention that MELL professional development products support teachers in implementing these
classroom practices. Over time, as additional insights are gleaned from ongoing work, it is likely that this evolving framework will be revised.

Reaching consensus on this framework was a lengthy and labor-intensive process and our group understandably has reservations about suggesting specific classroom practices in the absence of a definitive body of research. Admittedly, there is limited “hard data” to document that specific achievement gains are the result of specific classroom practices. Additional research in this area is critically needed and contributing to this body of research is a major goal of MELL. In the meantime, there is a pressing need to serve ELL students better, especially in mathematics. There is also a growing understanding among educational practitioners of the instructional needs of such students and how to address these needs. The MELL initiative is an attempt to connect this emerging understanding to the pressing need in a format that can be readily communicated.

Much, perhaps most, of this framework is comprised of elements of effective instruction appropriate for all students, and clearly students would be well-served by these suggested practices, regardless of their language proficiency. It appears, however, from our investigations, that the success of ELL students is more highly dependent on receiving instruction geared to their specific needs. In other words, while many students who are not experiencing a language barrier might be able to experience success with less than optimal instructional practices, few ELL students can thrive in such an environment. For this reason, creating a rich classroom experience for ELL students is not simply desirable, but rather is necessary if they are to have a chance to succeed. The MELL Framework is targeted at achieving this goal.

Learning Atmosphere & Physical Environment

A caring classroom atmosphere of mutual respect and support is facilitated by the teacher who:

- Knows each child as an individual,
- Embraces languages, customs, and cultures of ELL students,
- Provides culturally rich learning materials,
- Encourages self-expression and provides positive recognition,
- Builds student confidence and esteem, and
- Fosters an emotionally safe environment that allows students to feel secure and to take risks.
**The classroom is visually rich to support student learning.**
Incorporates displays of student produced work, whenever possible,
Is colorful and thought stimulating,
Contains pertinent, real-world information and applications,
Reinforces math-specific vocabulary and concepts, and
Provides color-coded learning supports when appropriate.

**Room arrangement facilitates student interaction and group work.**

**Instructional Practices**

*Instructional practices foster cooperation and collaboration.*

*Concepts are presented accurately, logically, and in engaging ways.*

*Multiple representations incorporate mathematics learning levels: concrete, semi-concrete, and abstract.*

*The teacher employs student-centered instructional practices.*
Approaches content from a concept-oriented constructivist method,
Surrounds students with different modalities (e.g., aural, visual, kinesthetic),
Connects new concepts to prior learning,
Encourages students to refine and reflect about their own work and verbalize concept understanding “in their own words,”
Chooses homework to optimize individual content development, and
Provides extra help and resources on an individual basis.

*Students are frequently partnered with peer learners to enhance learning opportunities.*
To develop math content,
To aid English language development,
To insure sustained active participation in the class, and
To welcome new students into an established learning community.

*Instructional activities are varied and support diverse learning styles and multiple intelligences, including for instance:*:
Frequent use of models,
Music as a motivator and anchor,
Mind maps, poster-walks, and word walls,
Key vocabulary and cognates presented in different forms, and
Vivid adjectives.

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Teachers guide created by the SHSU MELL Group, November 2005, in collaboration with the Texas State University System and the Texas Education Agency.
Mathematics Content & Curriculum

Glossary of mathematical terms is always available for reference.

Content is aligned to appropriate grade-level, mathematics TEKS and professional standards.

Content is based on diagnosed student needs.

*Content is systematically designed to incorporate sound learning principles.*

To incorporate increased complexity,
To present a cohesive big-picture through chunking,
To connect concepts through bridging and scaffolding,
To emphasize multidisciplinary understandings, and
To reflect on inherent patterns by comparing and contrasting concepts.

*Curriculum is challenging, relevant, age-appropriate, and well-paced*

To include contextually-based problems,
To incorporate student realities, and
To involve interactive problem solving.

Language Practices

Language support is offered without supplanting English instruction.

Support is aligned with student’s diagnosed language needs.

Language used is appropriate to age and grade level and presented in a socially meaningful context.

Mathematics-specific vocabulary is explicitly and implicitly taught and reinforced through repetition.

Teachers are knowledgeable about the second language acquisition theories and best practices embodied in Texas Administrative Code, Title 19, Part II, Chapter 128.

Ideally, dual language instructional support should be offered.

When dual language teachers are not available, sheltered instruction should be utilized to provide strong language support by addressing content through ESL.
Family & Community Involvement

Schools connect to student’s family-life by embedding contextual experiences and skills in teaching and curriculum.

Projects are relevant and promote family interaction.

Opportunities are available for English-speaking higher grade-level students to mentor ELL lower grade-level students either in an in-school or after-school program, as appropriate.

Teacher engages in frequent communication with families
About activities and events in which parents can participate, and
About student progress.

Teacher utilizes services provided by a community liaison and is knowledgeable about community resources.

Parents are informed about the benefits of using their most cognitively advanced language at home.

Assessment of Student Learning

Classroom assessment is designed to foster student success.

Assessment methods allow students frequent opportunities to demonstrate mastery in a variety of ways.

Various assessment techniques are used to measure student understandings.

Grades are oriented to promote and emphasize valid step-by-step logical reasoning processes.

Assessment data and results shape instructional planning.

Flexible time allotments are given to demonstrate concept mastery.