The theory of cluster algebras is one of the most mathematically well-studied areas in mathematical physics. Since its discovery in 2001, it has been shown that cluster algebras are related to diverse areas of mathematics such as algebraic geometry, total positivity, quiver representations, string theory, statistical physics models, non-commutative geometry, Teichmüller theory, hyperbolic geometry, tropical geometry, Kadomtsev-Petviashvili (KP) solitons, discrete integrable systems, quantum mechanics, Lie theory, algebraic combinatorics, Wentzel-Kramers-Brillouin (WKB) analysis, and Poisson geometry.

We will start with a problem from the most recent Putnam exam, promote it to a research level, and make a connection to cluster algebras. No background is required, as we use algebra at the pre-calculus level only.