# FACTORING

**Summary of steps** for factoring a polynomial:

1. Factor out any common factors.
2. If the polynomial is a binomial, check whether it can be rewritten as the difference of two squares, the difference of two cubes, or the sum of two cubes.
3. If the polynomial is a trinomial, check whether it is a perfect square trinomial. If not, factor by trial and error (guessing), or apply the AC Method.
4. If the polynomial has more than three terms, try factoring by grouping.

Factoring out a common factor:

EXAMPLE 1: $12xy^4 - 27xy^2 + 15xy^3 = 3xy^2(4y^2 - 9 + 5y)$

EXAMPLE 2: $44A^{2n+1}B^4 - 33A^{2n}B^6 = 44A^{2n}AB^4 - 33A^{2n}B^4B^2 = 11A^{2n}B^4(4A - 3B^2)$

## The Difference of Two Squares: (The **sum** of **two squares** is **not** factorable!)

Factoring formula: $x^2 - y^2 = (x + y)(x - y)$

Example 1: $x^2 - 9 = x^2 - 3^2$ break down to squares
$= (x + 3)(x - 3)$ factor using formula

Example 2: $25x^2 - 81 = (5x)^2 - 9^2$ break down to squares
$= (5x + 9)(5x - 9)$ factor using formula

Example 3: $x^4 - 81 = x^4 - 9^2$ break down to exponents
$= (x^2)^2 - 9^2$ break down to squares
$= (x^2 - 3^2)(x^2 + 9)$ factor using formula
$= (x + 3)(x + 3)(x^2 + 9)$ factor using formula

## The Difference and Sum of Two Cubes:

Factoring formulas: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

Example 1: $x^3 - 27 = x^3 - 3^3$ break down to cubes
$= (x - 3)(x^2 + 3x + 3^2)$ factor using formula

Example 2: $27x^3 - 8y^3 = (3x)^3 - (2y)^3$ break down to cubes
$= (3x - 2y)((3x)^2 + 3x \cdot 2y + (2y)^2)$ factor using formula
$= (3x - 2y)(9x^2 + 6xy + 4y^2)$ simplify

Example 3: $(A+B)^3 - 8 = (A+B)^3 - (2)^3$ break down to cubes

$= ((A+B)-2)((A+B)^2 + (A+B)(2) + (2)^2)$ factor using formula

$= (A+B-2)(A^2 + 2AB + B^2 + 2A + 2B + 4)$ simplify

Example 4: $216x^{3m} - 27b^{9m+3} = (6x^m)^3 - (3b^{3m+1})^3$ break down to cubes

$= (6x^m - 3b^{3m+1})((6x^m)^2 + (6x^m)(3b^{3m+1}) + (3b^{3m+1})^2)$ factor using formula

$= (6x^m - 3b^{3m+1})(36x^{2m} + 18x^m b^{3m+1} + 9b^{6m+2})$ simplify

Example 5: $x^6 - 64 = x^6 - 2^6$

$= (x^3)^2 - (2^3)^2$ break down to squares

$= (x^3 + 2^3)(x^3 - 2^3)$ factor using formula

$= (x+2)(x^2 - 2x + 4)(x-2)(x^2 + 2x + 4)$ factor using formula

Factoring as the difference of two cubes yields $x^6 - 64 = (x^2)^3 - (2^2)^3 = (x+2)(x-2)(x^4 + 4x^2 + 16)$. To factor completely, continue and complete the square in the trinomial

Example 6: $(x+y)^3 + 27 = (x+y)^3 + 3^3$

$= ((x+y)+3)((x+y)^2 - (x+y)(3) + 3^2)$

$= (x+y+3)(x^2 + 2xy + y^2 - 3x - 3y + 9)$

Example 7: $8x^3 + 343c^3 = (2x)^3 + (7c)^3 = (2x+7c)((2x)^2 - (2x)(7c) + (7c)^2)$

$= (2x+7c)(4x^2 - 14xc + 49c^2)$

Example 8: $27A^{9n} + 216B^{3n+6} = (3A^{3n})^3 + (6B^{n+2})^3$

$= (3A^{3n} + 6B^{n+2})((3A^{3n})^2 - (3A^{3n})(6B^{n+2}) + (6B^{n+2})^2)$

$= (3A^{3n} + 6B^{n+2})(9A^{6n} - 18A^{3n}B^{n+2} + 36B^{2n+4})$

$x^4 + 4x^2 + 16$. Given both options, **a l w a y s** break down to the difference of two squares first!

## Perfect Square Trinomial:

Factoring formulas: $x^2 + 2xy + y^2 = (x+y)^2$

$x^2 - 2xy + y^2 = (x-y)^2$

Example 1: $4x^2 + 4xy^2 + y^4 = (2x)^2 + 2(2x)(y^2) + (y^2)^2 = (2x + y^2)^2$

Example 2: $25a^4b^8 - 60a^2b^4c^3 + 36c^6 = (5a^2b^4)^2 - 2(5a^2b^4)(6c^3) + (6c^3)^2 = (5a^2b^4 - 6c^3)^2$

## Grouping:

Example 1: $x^2 - 9y^2 - 3x - 6y = (x^2 - 9y^2) - (3x + 6y)$
$= (x^2 - (3y)^2) - 3(x + 2y)$
$= (x - 3y)(x + 3y) - 3(x + 2y)$

Example 2: $a^2x^2 + b^2y^2 - a^2y^2 - b^2x^2 = (a^2x^2 - a^2y^2) - (b^2x^2 - b^2y^2)$
$= a^2(x^2 - y^2) - b^2(x^2 - y^2)$
$= (x^2 - y^2)(a^2 - b^2)$
$= (x + y)(x - y)(a + b)(a - b)$

## AC Method: (For an in-depth description of this procedure see SLAC handout: *AC Method*)

Example 1:

$20x^2 - 31xy + 12y^2 = 20x^2 - 16xy - 15xy + 12y^2$ $(20)(12) = 2 \cdot 2 \cdot 5 \cdot 2 \cdot 2 \cdot 3 = (16)(15) = -31$
$= 4x(5x - 4y) - 3y(5x - 4y)$ factor out any common factors
$= (5x - 4y)(4x - 3y)$ apply Factoring by Grouping

Example 2:

$28a^2 - 51ac + 20c^2 = 28a^2 - 35ac - 16ac + 20c^2$ $(28)(20) = 2 \cdot 2 \cdot 7 \cdot 2 \cdot 2 \cdot 5 = (35)(16) = -51$
$= 7a(4a - 5c) - 4c(4a - 5c)$ factor out any common factors
$= (4a - 5c)(7a - 4c)$ apply Factoring by Grouping

**Exercise:** Factor the following polynomials.

1. $15x^2 + 41x + 14$
2. $3x^2 - 16x + 5$
3. $5x^4y^4 - 135xy$
4. $b^3 + a^2 + b^2 - a^3 - 2ab$
5. $(a - b)^6 + 125$
6. $6zx^2 + 2yx - 3zyx - y^2$

**Answers:**

1. $(3x + 7)(5x + 2)$
2. $(x - 5)(3x - 1)$
3. $5xy(xy - 3)(x^2y^2 + 3xy + 9)$
4. $(b - a)(b^2 + ab + a^2 + b - a)$
5. $((a - b)^2 + 5)((a - b)^4 - 5(a - b)^2 + 25)$
6. $(2x - y)(3zx + y)$

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