On the Classification of Graphs Based on Their Rank Numbers

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Abstract

A $k$-ranking of a graph $G$ is a function $f : V(G) \to \{1, 2, \ldots, k\}$ such that if $f(u) = f(v)$ then every $uv$ simple path contains a vertex $w$ such that $f(w) > f(u)$. The rank number of $G$, denoted $\chi_r(G)$, is the minimum $k$ such that a $k$-ranking exists for $G$. Rank number is a variant of graph colorings. It is known that given a graph $G$ and a positive integer $t$ the question of whether $\chi_r(G) \leq t$ is NP-complete. In this paper we completely characterize $n$-vertex graphs whose rank number is equal to $n - 1$ or $n - 2$. Also, we establish rank numbers of some dense subgraphs of complete graphs, some dense subgraphs of complete bipartite graphs, and complements of trees. In addition, we completely characterize the rank number of subdivided star graphs and establish the rank number of all trees that contain a complete binary tree of the same height.