Texas Mathworks
Texas State University – San Marcos
Primary Mathematics World Contest
Qualifying Test
November 12, 2009

COVER SHEET

Name:________________________________________________________

Current Grade in School: __________

Home Address:________________________________________________

City:______________________  State: ________  Zip: ________________

Home Phone: (______) ___________________________

School Name: _______________________________

School Address: ______________________________________

Teacher First Name: _____________ Teacher Last Name: ______________

Check Math Courses Taken:
□ Pre-Algebra    □ Algebra 1   □ Algebra 2   □ Geometry

Birth date (Including year): _______ – _______ – _______

Gender: □ Male        □ Female

Are you a U.S. Citizen? □ Yes    □ No

Return Completed Test to:
Attn: Dr. Max Warshauer
Texas State-Mathworks
601 University Drive
San Marcos, TX 78666

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Directions: This test has 15 problems, with a time limit of 120 minutes. Do not use a calculator. Show all your work and how you obtained each answer. Use additional paper as needed. Partial credit will be given even if you do not obtain an answer. Do not worry if you cannot do all the problems.

1. A square board with 10 unit sides is divided into a 100 unit squares each painted red, white or blue. A bug lands on the board and hops repeatedly from square to square but never choosing the same color twice in a row. When on a red square, each of the non-red squares have equal probability of being chosen next. When on a white square, each of the non-white squares have equal probability of being chosen next. When on a blue square, each of the non-blue squares have equal probability of being chosen next. From a blue square the probability of hopping to a red a square is 3/4 and from a red square the probability of hopping to a white square is 1/7. How many white squares does the board contain?

2. The first three terms of a sequence are 1, 2, 4 respectively and for \( n \geq 4 \), \( t_n = t_{n-3} + t_{n-2} - t_{n-1} \). What is the 100th term of the sequence?

3. If you write out all the integers from 1 to 1000, how many times do you use the digit 0?
4. In triangle ABC, AB = 10, AC = 9, BC = 7, and point D lies on AB. DC is perpendicular to AB. Find the length DC.

5. Triangle ABC has AB = 4 units with $\angle A = 30^\circ$ and $\angle B = 135^\circ$. What is the area of the triangle?

6. Let $x, y$ be non-zero real numbers. Suppose three of the following four numbers are equal: $x+y, x-y, xy, x/y$. Find the absolute value of $xy$.

7. Find the value of $(x^3 - x^2 - 502x - 3)^5$ where $x = \frac{1+\sqrt{2009}}{2}$.
8. What is the first time after 2 pm at which the angle between the hour hand and minute hand of a clock equals $39^\circ$?

9. How many words of length 5 using the letters \{A, B, C, D, E\} have at least one A and one B?

10. Circles X, Y and Z have radii of 1, 2 and 4 respectively and are externally tangent as shown below. Points X, Y, Z and E are collinear and points A, B, C and E are points of tangency. What is the area of the region XADE?
11. In how many ways can 4 squares be selected from an 8x8 chessboard so that no two of the four squares are in the same row or column?

12. Clark made up a computer password consisting of exactly 8 uppercase English letters from the set \{C, L, A, R, K\}. Exactly 4 of the letters in the password were “A”. How many possibilities are there for Clark’s password?

13. An isosceles triangle has a vertex angle of 36° and a base of 2 units. What is the length of one of its legs?
14. Triangle ABC is shown below. Ratios of selected side lengths are given by:

\[
\frac{AE}{EB} = \frac{1}{1}, \quad \frac{BF}{FC} = \frac{4}{1}, \quad \frac{CD}{DA} = \frac{5}{2}.
\]

Find \( \frac{DG}{GE} \)

![Triangle ABC with points labeled A, B, C, D, E, and F.](image)

15. How many odd numbers are there in the 131st row of the modified Pascal’s Triangle below?

Note that each new row of the Triangle is constructed as follows:

I. The first and last entry of each row is a “1”

II. Each other entry of a row is the sum of the two entries diagonally above it.

For example, the “5” in the 5th row is obtained by adding the “1” and “4” diagonally above it. Similarly, the “10” in the 5th row is obtained by adding the “4” and “6” diagonally above it.

1. 1
2. 1 2 1
3. 1 3 3 1
4. 1 4 6 4 1
5. 1 5 10 10 5 1

1st row
2nd row
3rd row
4th row
5th row