Discrete Mathematics Seminar

Time: Friday, 11 March 2016, 2:15 – 3:15 PM
Location: 237 Derrick Hall
Title: Solution to a Combinatorial Problem arising in Group Theory
Speaker: Dr. Eugene Curtin, Department of Mathematics

Abstract:

In a 2014 paper Thomas Keller conjectured that given any \( n \times \infty \) matrix of \( n \) element sets \( (S_{i,j}) \), it is possible to construct an \( n \times \infty \) matrix \( (x_{i,j}) \) satisfying the following conditions: (i) For all \( i \) and \( j \), \( x_{i,j} \in S_{i,j} \). (ii) The first \( n-2 \) elements in each row are distinct and never repeated later in the row. (iii) For all \( t \) the \( n \) sets \( \{x_{i,1}, x_{i,2}, \ldots, x_{i,t}\} \) are distinct.

He proved the \( n = 4 \) case in his paper, and we will outline a proof for the general case. We will also show the following:

Let \( X \) be a subset of the Boolean lattice on \( [n] \) satisfying the following conditions: (i) \( \{i\} \in X \) for all \( i \in [n] \). (ii) For all \( A \in X \) with \( |A| \leq n-2 \) there exist elements \( i \neq j \) in \( [n]-A \) such that \( A \cup \{i\} \in X \) and \( A \cup \{j\} \in X \). Then \( X \) contains \( n \) disjoint chains of length \( n-1 \).

We conjecture that if \( \{X_i\}_{i=1}^n \) is a collection of \( n \) subsets of the Boolean lattice on \( [n] \) each satisfying (i) and (ii) above then there exist \( n \) disjoint chains \( C_i \) of length \( n-1 \) with \( C_i \subset X_i \).

This Boolean lattice conjecture implies a stronger version of the infinite matrix result. There is a combinatorial-game version of the conjecture which is stronger still.

No specialized background is needed to follow the arguments. The techniques are elementary, with the Max-Flow Min-Cut Theorem and the Konig Infinity Lemma making guest appearances.

This is joint work with Suho Oh.