Texas State Topology Seminar

Thursday, 2018, September 27, 2:00-3:00 p.m., in DERR 227
Speaker: David Snyder

Topic: Generalized Dunce Hats Are Not Splittable

ABSTRACT

In 2008, David Gabai proved an astonishing result: the Whitehead manifold (a contractible non-compact 3-manifold that is not homeomorphic to $\mathbb{R}^3$) is splittable, a union of two open 3-manifolds whose intersection is a 3-manifold, all three of which are homeomorphic to $\mathbb{R}^3$. A generalized dunce hat is a 2-dimensional polyhedron created by attaching the boundary of a disk $\Delta$ to a circle $J$ via a map $f: \partial \Delta \to J$ with the property that there is a point $v \in J$ such that $f^{-1}([v])$ is a finite set containing at least 3 points and $f$ maps each component of $\partial \Delta - f^{-1}([v])$ homeomorphically onto $J - \{v\}$. Theorem: No generalized dunce hat is the union of two proper subpolyhedra that each have finite first homology groups. This result undermines a strategy for proving that the interior of the Mazur compact contractible 4-manifold $M$ is splittable in the sense of Gabai (i.e., $M = U \cup V$ where $U$, $V$ and $U \cap V$ are each homeomorphic to Euclidean 4-space).