Texas Mathworks  
Texas State University – San Marcos  
Primary Mathematics World Contest  
**Qualifying Test**  
January 13, 2011  

**COVER SHEET**

Student First Name:____________________ Last Name:____________________

Current Grade in School: ______

Home Address:________________________________________________

City:____________ State: _____ Zip: __________

Home Phone: (____) ______________________

School Name:________________________________________

School Address:______________________________________

City: ______________ State: _____ Zip: __________

Teacher:__________________________________________

Check Math Courses Taken:

□ Pre-Algebra   □ Algebra 1   □ Algebra 2   □ Geometry

Birth date (MM/DD/YYYY): _____ / _____/ _______

Gender: □ Male    □ Female

Are you a U.S. Citizen or Permanent Resident? □ Yes    □ No

**Return Completed Test to:**  
Texas Mathworks  
Texas State University  
601 University Drive  
San Marcos, TX 78666
Directions: This test has 15 problems, with a time limit of 120 minutes. Do not use a calculator. Show all your work and how you obtained each answer. Use additional paper as needed. Partial credit will be given even if you do not obtain an answer. Do not worry if you cannot do all the problems.

1. Find the number of integers between 10,000 and 99,999 that have no two adjacent digits equal in their usual base 10 representation.

2. Find the smallest whole number $x$ such that $x$ divided by the sum of its digits results in a prime number quotient.

3. Hypotenuse AB of right triangle ABC is 18 units in length. The bisectors of the two acute angles of the triangle intersect at X. If the perimeter of the triangle is 42 units, what is the length of segment CX?
4. An arithmetic progression, (A.P.), is a sequence of terms with a common difference between consecutive terms and might be represented as $a, a + d, a + 2d$, etc. A geometric progression, (G.P.), is a sequence of terms with a common ratio between consecutive terms and might be represented as $b, br, br^2$, etc. The sequences $\{4, 8, 12, 18\}$ and $\{6, 9, 12, 16\}$ have three common characteristics:
   a) All four terms are integers
   b) The first three terms form an A.P. and the last three terms form a G.P
   c) The sum of the first and last term is 22

Find another such sequence that also has these three characteristics and whose second number equals 8.

5. In the diagram below, ABCD is a square with sides of length 4. Square A'B'C'D' is obtained when ABCD is rotated 45 degrees about its center. Find the area of the shaded octagon.
6. A sequence of integers $x_1, x_2, x_3, \ldots$ is formed by the following rules:
   - The first two terms are $x_1 = 1$ and $x_2 = 2$.
   - To get any other term $x_n$ we first add the two previous terms $x_{n-1}$ and $x_{n-2}$, but then subtract 19 if this sum is 19 or greater, as explained below.
     - If $(x_{n-1} + x_{n-2}) < 19$ then $x_n = x_{n-1} + x_{n-2}$
     - If $(x_{n-1} + x_{n-2}) \geq 19$ then $x_n = x_{n-1} + x_{n-2} - 19$

Find $x_{100}$, the 100th term in the sequence.

7. Triangle A has sides of length 10-$x$, 3, and $x$. Triangle B has sides of length 10-$y$, 3 and $y$. What is the least possible value of $z$ such that the inequality $z > (x-y)$ must hold?

8. Four friends - Alice, Bobby, Cathy and Doug - come to a bridge. They have one flashlight. It’s dark, so nobody can walk without the flashlight. Anyone can walk either alone, or together with someone else, but the bridge can’t hold more than two people at a time. It takes Alice 1 minute to cross the bridge (walking either way), 2 minutes for Bobby, 5 minutes for Cathy, and 10 minutes for Doug. Any two of them together must walk at the speed of the slower one. What is the smallest amount of time needed for all four friends to end up on the other side of the bridge?
9. Find the remainder when the number 1234567891011121314…200820092010 is divided by 9.

10. In the diagram below, AB = 6, DC = 4, D is the midpoint of AB, angles ADC and AEB are right angles. Find the length of EF.
11. On triangle ABC below, point E lies on side AB and point D lies on side BC. The segments AD and CE intersect at F. Now AE:EB = 1:3 and CD:DB = 1:2. The area of triangle ABC is 30. Find the area of the quadrilateral BDFE.

12. How many possibilities are there for \((w, x, y, z)\) if \(w, x, y,\) and \(z\) are positive integers such that \((w + 3) < (x + 2) < (y + 1) < z < 12\)?
13. Two adjacent jars each have 90 balls. Each jar has \( b \) black balls, \( w \) white balls and \( r \) red balls. A ball is drawn at random from each jar. Let \( x \) be the probability that both are black, \( y \) be the probability that both are white and \( z \) be the probability that both are red. Suppose that \( x = y + z \). What is the number \( b \) of black balls in each jar?

14. Find the exact value of the summation below

\[
\frac{1}{1 \times 2 \times 3} + \frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \ldots + \frac{1}{98 \times 99 \times 100}
\]

15. Triangle ABC has side lengths of \( AB = x \), \( AC = x + 1 \), and \( BC = x + 2 \). Suppose the measure of angle A is twice that of angle B. Find \( x \).