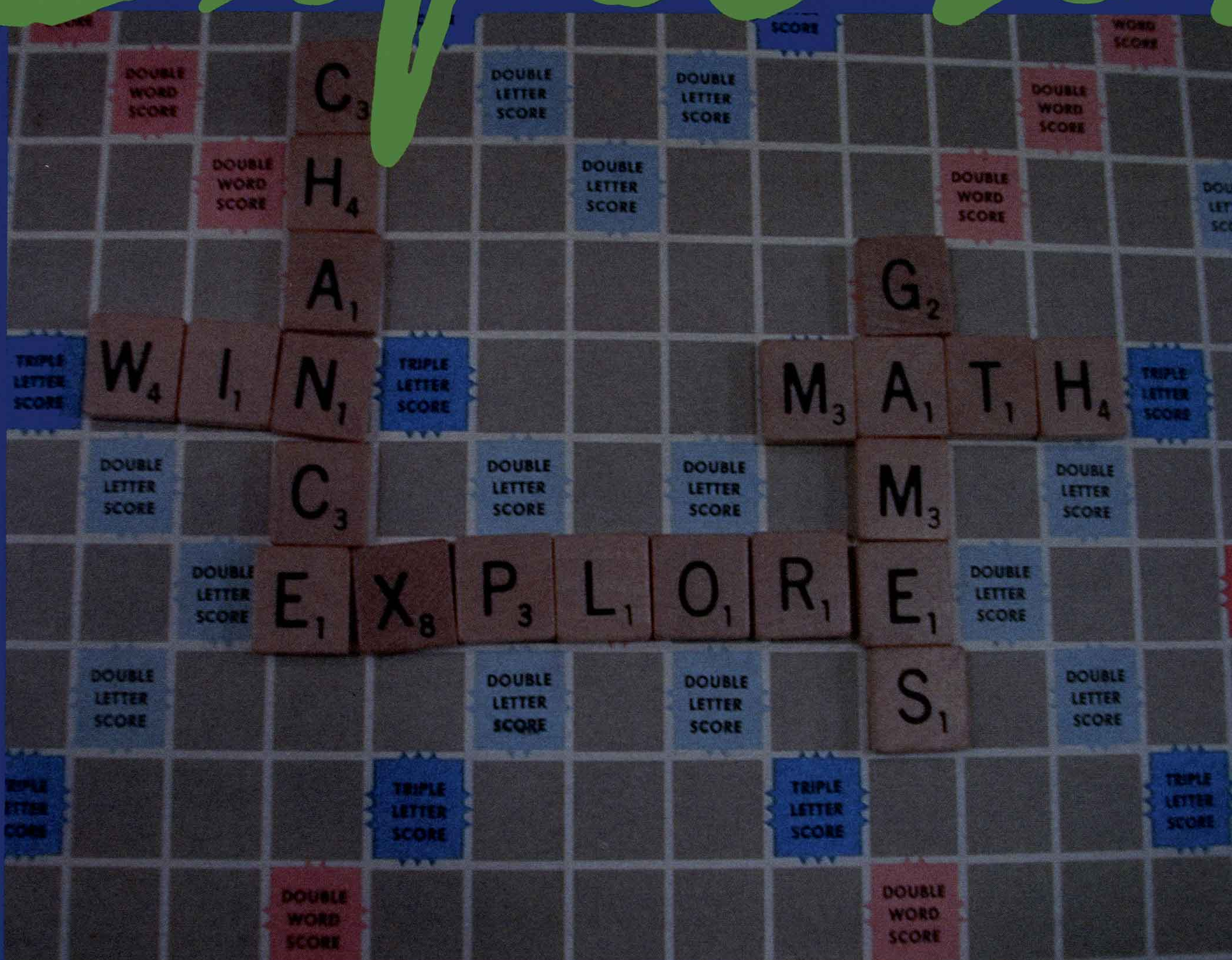
Math Explorer



MATH & GAMES

A winning Matrix!

von Neumann and the Doomsday Machine

Breaking the Code!!

Vol. 3.3 Spring 2001

Math Explorer
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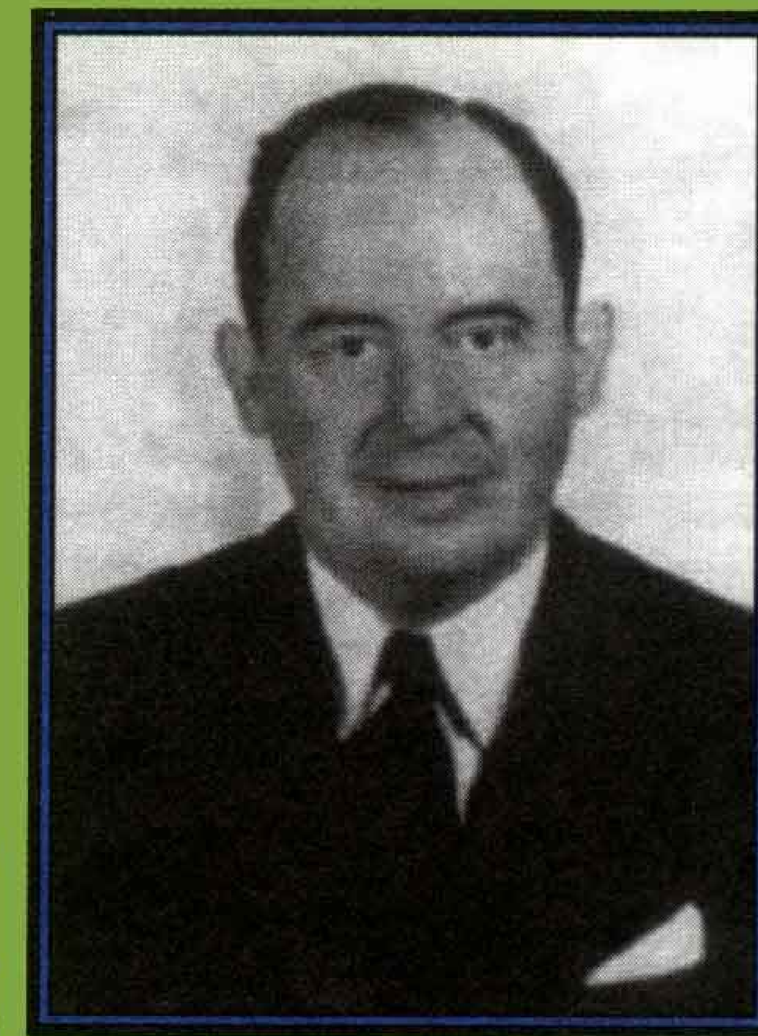
SWT

John von Neumann



by Joyce Fischer

John von Neumann was born in Budapest, Hungary on December 28, 1903. He showed an incredible talent for learning while he was still a young child. At age 6 he could divide 2 eight-digit numbers without using pencil or paper and tell jokes in classical Greek. By age 8, he had learned calculus, and by age 12 he was doing a very advanced level of mathematics. At age 23, he received a Ph.D. in mathematics and a degree in chemical engineering from cities in 2 different countries (Budapest, Hungary and Zurich, Switzerland). In 1930, at age 27, he moved to the US to teach at Princeton University.

Even before the US entered WWII, von Neumann was busy with the design of the Atomic bomb and the Hydrogen bomb. At first, his work was calculated manually, but in 1944, he was invited to the University of Pennsylvania to see the ENIAC (electronic numerical integrator and computer). The ENIAC was bigger than a dinosaur but had virtually no memory capacity. Von Neumann designed a way to store and access memory electronically, and by 1945 he had received enough money to develop his own computer, which he called the MANIAC. He was a champion in the fight to keep the use of computers available to all people, so that they couldn't become a tool for only governements and wealthy people. Advancing the work of Alan Turing, his creation, the "von Neumann machine," still makes up most of the computing devices of today, from the simple chips for Furbies and cell telephones to highly advanced super computers.

In the early 1950s, he used his logical abilities to develop military strategy for the cold war between the US and Russia. His strategic advice to President Eisenhower, the von Neumann defense strategy, was to assure the Soviet Union that if a country dropped a nuclear bomb on the US, the Soviet Union could be certain that the entire earth would be destroyed. This policy, nicknamed the "Doomsday Machine," determined the defense of the US into the 1970s and helped von Neumann secure his second Presidential Award.

Von Neumann was a true inter-disciplinary genius, advancing and originating work in mathematical logic, game theory, set theory, physics, quantum theory, statistics, and many other fields.

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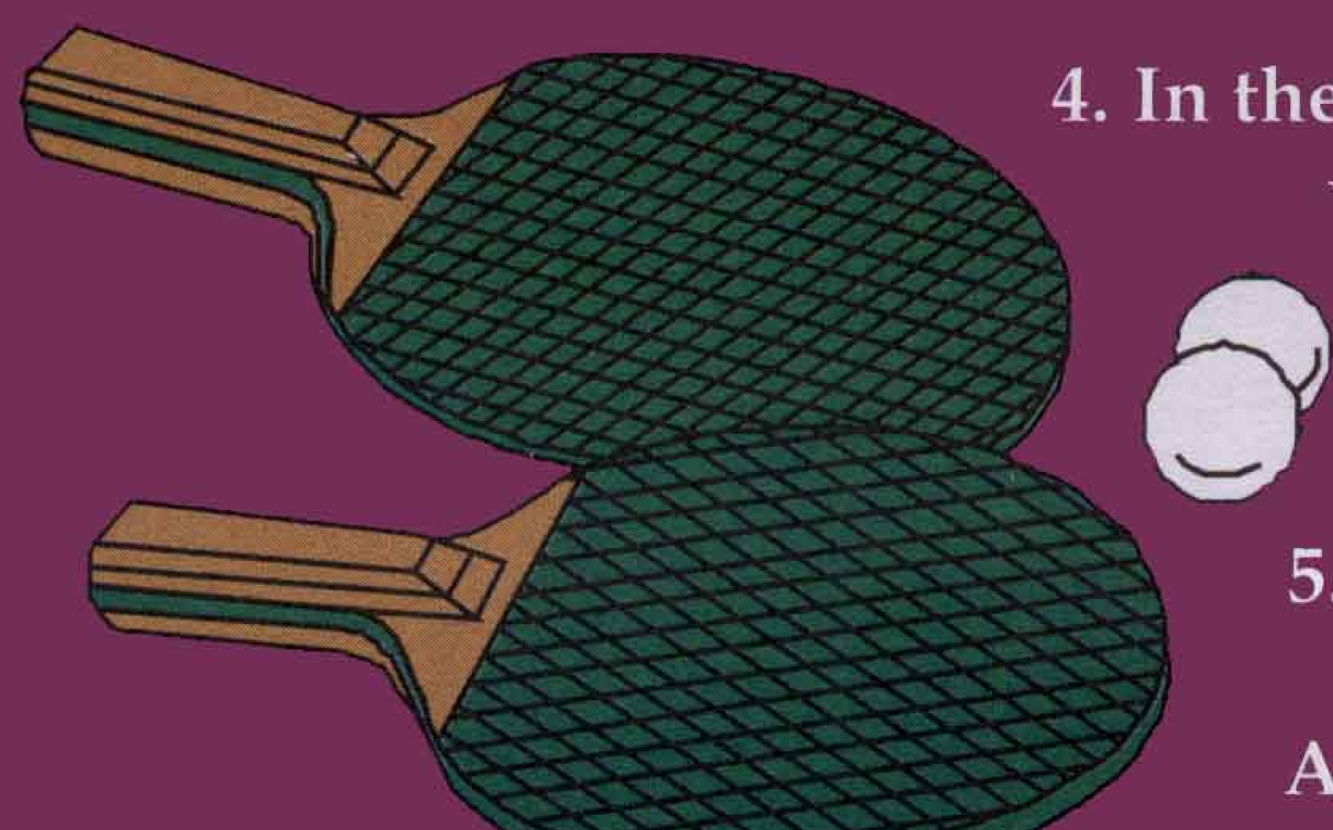
PROBLEMS OF THE MONTH

1. Tom, Ben, Jennifer, Lydia, and Leo are lined up from tallest to shortest. Ben is beside Tom and Leo. Jennifer is shorter than Tom. Lydia is taller than Tom. Who is the tallest?

2. A set of numbers is sum-free if no two of the numbers add up to a third in the set. For example, {2, 4, 5, 8, 11} is sum-free, but {2, 4, 5, 9} is not because 4 + 5 = 9.
   a. Find a set of 5 sum-free numbers from {1, 2, 3, 4, 5, 6, 7, 8}.
   b. Find a way to divide {1, 2, 3, 4, 5, 6, 7, 8} into two sets of four, both of which are sum- free.

3. A number raised to the power of 11 has last digit 2. What is the last digit of the number?



4. In the ping-pong club this month, each boy has played a game with exactly 3 of the girls, and each girl has played with exactly 4 of the boys. There are 8 boys in the club. How many girls are there?

5. You are offered a choice between the following deals:

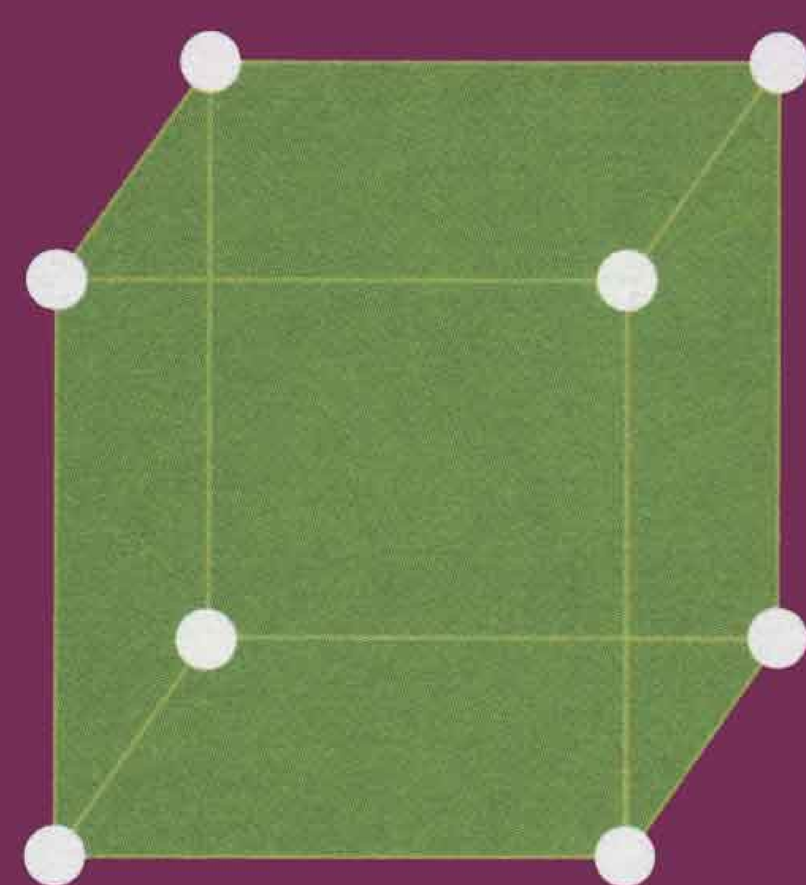
A: You can get $1 million on February 28.

B: You get 1 cent on February 1, 2 cents on February 2, 4 cents on February 3 and so on, doubling the amount of money you get each day up to and including February 28.

Which choice yields more money for you?

6. If you add up 5 consecutive integers you always get a multiple of 5. For example, 2 + 3 + 4 + 5+ 6 = 20 = 4 x 5. Find 5 consecutive integers which add up to 100.

7. On the first three math tests a student made the scores 76, 88, and 68. What does she need to score on the fourth test to make an average of 80 for the four tests?



8. Number the eight vertices of a cube 1 to 8 so that the sum of the numbers on each face is the same for every face.

9. How many colors do we need to paint the vertices of a cube, if no two vertices on the same edge can have the same color?

10. Juan, Mike, Alana, and Jessica were playing a card game. They used the Jack, Queen, King, and Ace of each suit in the deck, so the deck had 16 cards, 4 of each kind. They each were dealt 4 cards from the deck. Juan got 3 cards of the same kind, and an Ace. So did Mike and Alana. What cards did Jessica get?

Send us your solutions! Every month, we will publish the best solutions on our website: www.mathexplorer.com. If we print your solutions, we will send you and your teacher free *Math Explorer* pens!

MATRIX GAMES

by Greg Passty

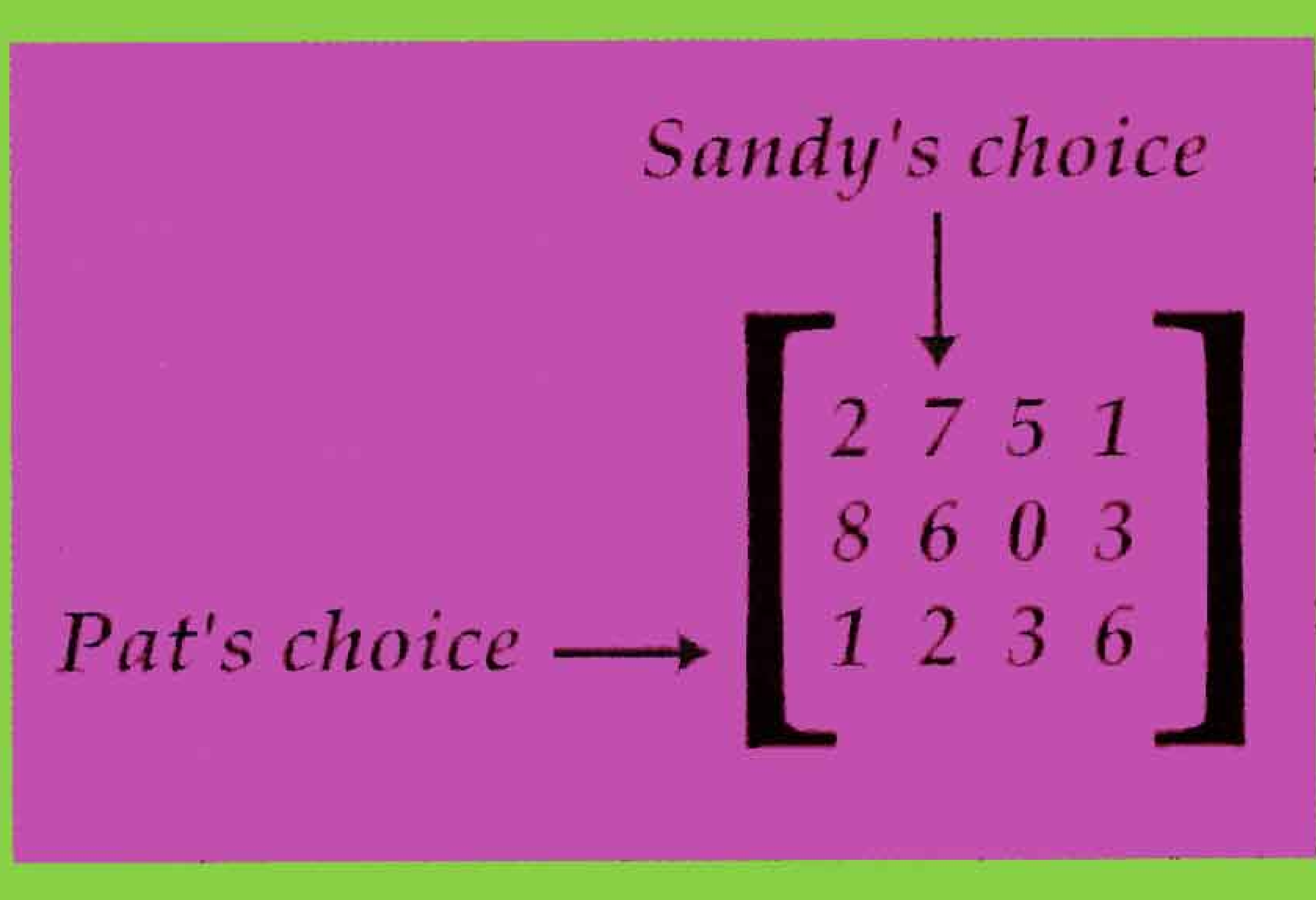
A *matrix* is a rectangular arrangement of numbers, like this:

$$\begin{bmatrix} 2 & 7 & 5 & 1 \\ 8 & 6 & 0 & 3 \\ 1 & 2 & 3 & 6 \end{bmatrix}$$

This matrix has three *rows*, which run from left to right, and four *columns*, which go up and down. The *shape* of this matrix is said to be three by four, listing the number of rows first and then the number of columns.

Mathematicians and scientists use *matrices*, as the plural is written, for a variety of applications, ranging from processing information downloaded by satellites to determining how to allocate resources for national chains of grocery stores. We will do something slightly different with matrices in this article: we will learn how to play a game.

To demonstrate how a matrix game is played, we introduce our two players, Pat and Sandy. Pat will choose one of the three rows. Sandy will choose one of the four columns. The players keep their choices secret until the showdown, when they reveal their choices at the same time. Let's say that Pat chooses Row 3, the bottom row, while Sandy chooses Column 2, the second column counting from the left. The result of the game is the intersection of the two choices. In this case, it's the 2 in the third row and second column.



Since 2 is the result, this play of the matrix game ends with Sandy paying Pat 2 tokens. The column player always pays the row player the number that is the result of the game. This number is called a *payoff*, and the matrix is referred to as a payoff matrix.

To summarize, beginning with a given payoff matrix, the row player chooses a row, the column player chooses a column, and then the column player pays the row player the number which is located at the intersection of the chosen row and column.

A matrix game is meant to be played many times. Keep track of the results of each play, and then add them up at the end of the session. For example, if Pat and Sandy play this matrix game ten times, here is a chart showing what the results might look like. (We are imagining what the players' choices are for each play of the game.)

Play Number	Pat's Row Choice	Sandy's Column Choice	Sandy's Payoff to Pat
1	2	4	3
2	1	3	5
3	3	4	6
4	2	3	0
5	2	2	6
6	2	2	6
7	3	3	3
8	1	1	2
9	1	2	7
10	3	4	6
Total			**44**

For this run of ten plays, Sandy pays Pat a total of 44 tokens. Since there were ten plays of the matrix game, Sandy pays Pat an average per play of 44/10 = 4.4 tokens per play.

Take a look at the choices made by the two players. If you were Sandy, could you have avoided paying out so much? If you were Pat, could you have found a sequence of choices where you would be paid even more? Since each payoff depends on choices made by *both* players, neither one alone can decide anything for certain. But each player can formulate a plan once they find out what the opponent is likely to do.

In this game, Sandy would probably get tired of paying Pat all the time. One way of making the game fairer is to have the two players switch roles after every ten plays. For the next ten plays, Sandy would choose a row, and Pat would choose a column, and then Pat would pay Sandy the result of their choices.

The players could be thought of as being on "offense" and "defense": The row player, or offense, tries to achieve as large a payoff as possible, while the column player, or defense, always tries to minimize the payoff.

Sometimes in football, the offensive team actually loses yardage on a play. This can be thought of as a negative payoff. Something similar happens when negative numbers appear in a payoff matrix, such as

$$\begin{bmatrix} 10 & 2 \\ -4 & 8 \end{bmatrix}$$

In this matrix game, if Pat, the row player, chooses Row 2 and Sandy, the column player, chooses Column 1, then the payoff is -4, a negative number. This means that Sandy pays Pat -4 tokens. Since the payoff is negative, however, for Sandy to pay Pat a negative number of tokens means that Pat must actually pay Sandy +4 tokens. So the column player is the one who actually gets paid.

So why wouldn't Sandy choose Column 1 every time? Well, lurking at the top of Column 1 is the payoff of 10. As soon as Pat discovers that Sandy is playing Column 1, Pat will play Row 1 every time, and get a payoff of 10 units for each play of the game. This is why it's important for Sandy to keep her choice of column secret, until both players reveal their moves at the same moment. It's also important for Pat to keep her choice secret. Otherwise, Sandy could take advantage of knowing what Pat's choice will be. This particular game is very exciting, since the results for each player can vary greatly, depending on what the other player does.

The following problems have some exciting games like this one, and some other games where players will find themselves fairly set in their choices, after a little bit of experimentation. Try all the games by playing ten times and then switching roles and playing another ten times. Keep track of your results using either tokens or a chart, and try to outguess your opponent!

Problems

The first five problems concern the 3 by 5 matrix game shown here:

$$\begin{bmatrix} -1 & 0 & 2 & 0 & -1 \\ 9 & 3 & 2 & 6 & 3 \\ -3 & -2 & -3 & -1 & 7 \end{bmatrix}$$

1. If Pat chooses Row 2 and Sandy chooses Column 5, how much does Sandy pay Pat?

2. If Pat chooses Row 1 and Sandy chooses Column 2, how much does Sandy pay Pat?

3. If Pat chooses Row 3 and Sandy chooses Column 4, who actually gets paid, and how much?

4. If Pat found out that Sandy was going to choose Column 1, what would Pat's best move be?

5. If Sandy found out that Pat was going to choose Row 2, what would Sandy's best move be?

Answers on page 7.

Greg Passty is professor of mathematics and associate dean to the college of science at Southwest Texas State University.

Puzzle Page

Math Explorers: We want to print your work! Send us original math games, puzzles, problems, and activities. If we print them, we'll send you and your math teacher free *Math Explorer* pens.

In this addition, the same letters mean the same digits, and different letters mean different digits. What number is (ABCDACE) on top?

	A	B	C	D	A	C	E
		B	C	D	A	C	E
			C	D	A	C	E
				D	A	C	E
					A	C	E
						C	E
+							E
E	E	E	E	E	E	E	2

How many ways can you cut up a square into 10 smaller squares?

Fill in the fields of the first row and first column using the numbers 1, 2, 3, 4, 5, and 6 so that the given results are true.

+			
			9
		6	
	6		

Bulletin Board

Mathematics Behind the Game

http://www.math.toronto.edu/mathnet/games/ has several computer generated mathematical strategy games.

Did You Know?

111,111,111 x 111,111,111 = 12,345,678,987,654,321?

GirlStart

Girlstart, an Austin based non-profit organization, offers summer camps for girls ages 11-18. Learn technology, math and science hands-on at their Girls' Technology Center.For more information visit them on the web at **http://www.girlstart.org/summer.htm** or contact Amy at 1-877-768-4775

Words of Wisdom

The function of mathematics in providing answers is often less important than its function in providing understanding.

-- Ben Noble

Matrix Games (cont'd)

Play each of the following matrix games twenty times, switching roles after the first ten plays. Keep track of your results using either tokens or a chart such as the one in the article.

$$\begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix} \qquad \begin{bmatrix} 2 & 8 & 1 \\ 5 & 6 & 4 \\ 3 & 9 & 2 \end{bmatrix}$$

Answers:

1. Sandy pays Pat 3 tokens.
2. Since the payoff is 0, Sandy pays Pat nothing.
3. Since the payoff is -1, Pat pays Sandy 1 token.
4. Choose Row 2, to get the payoff of 9 tokens.
5. Choose Column 3, to pay out the minimum of 2 tokens.

MATH ODYSSEY

by Jean Davis

The Navajo Code Talkers-America's Secret Weapon.

A code is a group of letters, words, or symbols used to represent information. Codes have many uses, but are tremendously important in military campaigns. All intelligence information concerning combat operation, troop movement, and damage estimates is communicated in code to prevent intercepted messages from being understood by the enemy. Though in peace time we need not worry about the enemy, security in communicating sensitive or private material is still an important issue. Often when we write a message on the computer, it is encrypted, or sent by code, then decoded for the receiver.

In World War II, before the days of the sophisticated technology we have now, the Japanese were able to break every code the Allied Forces used-except one. The undeciphered code involved using the Navajo language, a complex, spoken language. A group of Navajo Indians were recruited by the US military and were called the Navajo Code Talkers. When a code talker received a message, what he heard was a string of seemingly unrelated Navajo words. He would translate each word into English, then use only the first letter of the English equivalent to spell a word.

For example, either of the Navajo words "moasi" (cat) or "ba-goshi" (cow) could be used to represent the letter "c". The word "sea" could be encoded as "klesh (snake) ah-jah (ear) be-la-sana (apple)." Even a Navajo who understood the language would hear "snake ear apple"-aparently meaningless gibberish, unless you knew the code.

The Navajo Code Talkers were first and foremost excellent soldiers. Each was part of a team that would infiltrate behind enemy lines and radio back important information about the position of enemy forces and possible routes for invasion. They contributed significantly to the American victory in the Pacific. Navajo Code Talk was top secret and was not declassified until 1968, nearly 25 years after the war ended. If you visit the Pentagon, in Washington, DC, you can see the Navajo Code Talker exhibit.

References: www.execpc.com/~shepler/codetalkers

Jean Davis teaches mathematics at Southwest Texas State University.

Games are great fun and give us a chance to form strategies, think creatively, make decisions, and then see the outcome of our actions. Games can be board games like chess and backgammon, electronic video games, or simply a set of rules played with pencil and paper. The matrix game is just one of the fun and challenging games that uses mathematical ideas. We hope you and your friends will enjoy finding the winning moves and solutions to the games, puzzles and problems in this month's *Math Explorer*.

Sincerely,

Hiroko K. Warshauer

Hiroko K. Warshauer, editor