

GRE MATH REVIEW #7

Specific Techniques for Attacking Arithmetic, Algebra, and Geometry Problems on the GRE

Now that we have reviewed the basic arithmetic, algebra, and geometry required for the GRE, we can discuss the specific techniques used to attack these types of problems. Following these suggestions rather than your algebra class methods will make you much more efficient on the GRE. Many people find it easier to improve their math scores than their verbal scores just by practicing and trusting these techniques that follow.

Before taking the GRE, make sure you are familiar with the **instructions** for each type of math question. Do not waste any of your test time reading the instructions or the examples. Here is a list of information contained in the math instructions on the GRE. Once you understand what it means here, do not bother reading it during the test.

1. All numbers used are real numbers.
2. Position of points, angles, regions, etc., can be assumed to be in the order shown; angle measures can be assumed to be positive.

These two statements are for math whizzes who try to argue about their answers. Don't worry about them.

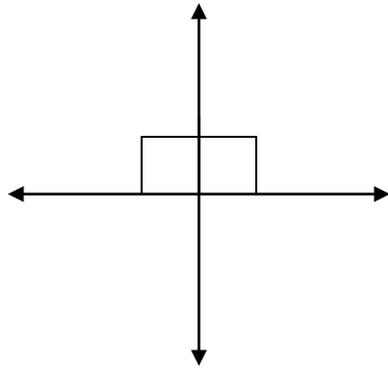
3. Lines shown as straight can be assumed to be straight.
4. Figures can be assumed to lie in a plane unless otherwise indicated.
5. Figures accompanying questions are intended to provide information useful in answering the questions. However, unless a note states that a figure is drawn to scale, do NOT solve these problems by estimating sizes by sight or measurement, but by your knowledge of mathematics.

This information simply means you can't look at a drawing and figure out if one side of a triangle is longer than another or about how large an angle is. Everything about drawings on the GRE is accurate except the dimensions.

Example 1: How many different positions can there be for a square that must have corners at both $(0, 1)$ and $(0, 0)$?

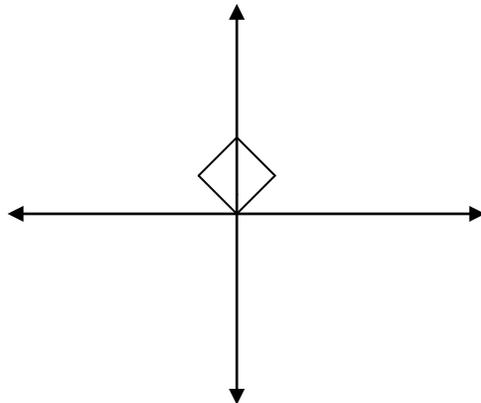
- (a) One (b) Two (c) Three (d) Four (e) Five

Solution: The most obvious, easy solution to this question is the two squares shown below with corners at each of the given points.



This question would be at the difficult level on the GRE; therefore, since two is the first answer that comes to mind for an average person, eliminate choice (b). We can also eliminate (a) since we've already found two squares. So, we've improved our odds of choosing the correct answer to 1 in 3, and this question is the difficult one of its section. Only about 8% are expected to answer this question correctly!

Now, we need to look for another way in which a square could have corners at the two given points. In the diagram below, we see that there is one more square, so the answer is three. Careless students might choose four by assuming there could be a square in each quadrant.



Average students like to find answers by using **easy arithmetic** that they understand. On difficult questions, these choices can be eliminated.

Example 2: A suit is selling for \$100 after a 20% discount. What was the original selling price?

- (a) \$200 (b) \$125 (c) \$120 (d) \$80 (e) \$75

Solution: A careless student will choose (c) since \$120 is 20% more than \$100. But that's not what the question asked. So, eliminate choice (c). You can also eliminate \$80

since that is 20% less than \$100. A careless person will choose (c) and (d) first since those answers are arrived at by easy manipulation of the numbers in the problem. Choice (e) can be eliminated because we're looking for an amount *greater* than \$100. In addition, \$200 is much too high since 20% of \$200 is \$40. Hence, the answer must be (b).

A careless student also likes answers that remind him/her of the problem. On difficult questions, do not be attracted to answers that simply repeat the numbers in the problem or are easily derived from the numbers in the problem. So, eliminate these types of answer choices.

Example 3: After 6 gallons of water are transferred from container A to container B, there are 10 gallons more water in container A than in container B. Container A originally had how many more gallons than container B?

- (a) 0 (b) 6 (c) 10 (d) 16 (e) 22

Solution: Careless students are immediately attracted to choices (b) and (c) because they repeat numbers that are in the problem. So, eliminate these two choices. Eliminate (d) as well since it is easily derived from numbers in the problem: $6 + 10 = 16$. Obviously, choice (a) makes no sense, so the answer is (e).

On some difficult questions, you will be asked to find the **least or greatest possible value** that satisfies certain conditions. On such problems, immediately eliminate the least or greatest value among the choices.

When **“It cannot be determined from the information given”** is an answer choice on easy or medium questions, it could possibly be the correct answer. However, when this is an answer choice on difficult questions, it is almost never the answer. Keep in mind that we are not talking about quantitative comparison questions where this is an answer choice on every question!

Working backward is a powerful technique on the GRE although it is not recommended in an algebra class. Instead of setting up an equation and solving it, simply plug in the answer choices until you find the one that works. At most, you will have to try four of the choices, but you can usually find the answer in one or two tries. This technique always works **when all of the answer choices are numbers**.

Example 4: Which of the following values of x does not satisfy: $5x - 3 < 3x + 5$?

- (a) -2 (b) 0 (c) 2 (d) 3 (e) 4

Solution: When the answer choices are numbers, start plugging in with the middle answer and work outward because the answer choices on the GRE are usually arranged in order of size. By starting in the middle, you may save time by eliminating choices that are

too large or too small. If you are quick at solving an inequality in your head and it is not a difficult question, it may be faster for you to do so. But be careful. Otherwise, plug in:

- (c) $5(2) - 3 < 3(2) + 5$, or $7 < 11$. True? Yes. Eliminate.
- (d) $5(3) - 3 < 3(3) + 5$, or $12 < 14$. True? Yes. Eliminate.
- (e) $5(4) - 3 < 3(4) + 5$, or $17 < 17$. True? No. The answer.

Example 5: The units digit of a 2-digit number is 3 times the tens digit. If the digits are reversed, the resulting number is 36 more than the original number. What is the original number?

- (a) 13
- (b) 26
- (c) 36
- (d) 62
- (e) 93

Solution: There are ninety 2-digit numbers. Eighty-five of these have already been eliminated because they are not answer choices. We need to eliminate four more by trying out the answer choices. Don't try to set up some kind of complicated equation; it would take too much time. The two conditions that the answer must satisfy are (1) the ones digit is 3 times the tens digit, and (2) the number resulting from reversing the digits must be 36 more than the original number. If any of the answer choices fails to satisfy either of these conditions, we can eliminate it. Just consider one condition at a time.

- (c) Is six 3 times 3? No. Eliminate.
- (b) Is six 3 times 2? Yes. A possible answer.
- (d) Is two 3 times 6? No. Eliminate.
- (a) Is three 3 times 1? Yes. A possible answer.
- (e) Is three 3 times 9? No. Eliminate.

So, the only two possibilities are (a) and (b). Now try the second condition.

- (a) Is $31 = 13 + 36$? No. Eliminate.

Hence, the correct answer is (b) since $62 = 26 + 36$. The technique of working backward from the answers can also be used on word problems instead of setting up an equation.

On the GRE, you must be aware of extra information in a problem that makes the problem a little more difficult.

Example 6: A restaurant owner sold 2 dishes to each of his customers at \$4 per dish. At the end of the day he had taken in \$180, which included \$20 in tips. How many customers did he serve?

- (a) 18
- (b) 20
- (c) 22
- (d) 40
- (e) 44

Solution: The information about tips is just extra information to make the problem a little more difficult. Subtract the \$20 in tips from the daily total to find out how much was

earned on just the dishes. Since each customer bought two 4-dollar dishes, each customer spent \$8 on food. The day's total, \$160, divided by \$8 is 20 people, answer choice (b).

If a problem has **variables in the answer choices** instead of just numbers, you can often find the answer by **making up numbers and plugging them in**. (1) First, pick a different number for each variable in the problem. (2) Solve the problem using your numbers. (3) Plug your numbers into the answer choices to see which one equals the solution in step 2. (4) Plug in working from the outside in, i.e. a, e, b, d, c, in that order.

Example 7: If $x + y = z$ and $x = y$, then all of the following are true EXCEPT

- (a) $2x + 2y = 2z$
- (b) $x - y = 0$
- (c) $x - z = y - z$
- (d) $x = z/2$
- (e) $x - y = 2z$

Solution: If you are pretty comfortable with algebra, you should recognize immediately that the first three are true. In that case, only use the following procedure on (d) and (e). Pick values for x , y , and z that are consistent with the two given equations. Since x and y are equal, let's pick them both to be 2. Since the sum of x and y is z , z must be 4. Remember we are looking for the answer choice which is false.

- (a) $2(2) + 2(2) = 2(4)$, or $4 + 4 = 8$. True. Eliminate.
- (e) $2 - 2 = 2(4)$, or $0 = 8$. False.

If you have enough time, you may want to verify that the other answer choices are true just to make sure you didn't make a mistake. $2 - 2 = 0$, or $0 = 0$. True.

- (d) $2 = 4/2$, or $2 = 2$. True.
- (b) $2 - 4 = 2 - 4$, or $-2 = -2$. True.

When plugging numbers into a problem, **choose numbers that are easy to work with**. This depends, however, on the problem. Usually small numbers are easier to work with than large numbers, especially if there are exponents involved. When exponents are involved, choose numbers like 2 or 3; 0 and 1 have special properties that may make it more difficult to recognize the answer.

Small numbers are not always the best though. When you're working with percentages, 10 and 100 are easy numbers to work with. In a problem involving minutes or seconds, 60 may be the easiest number to work with. Look for clues in the problem to determine which numbers to use.

Example 8: A street vendor has just purchased a carton containing 250 hot dogs. If the carton cost x dollars, what is the cost in dollars of 10 of the hot dogs?

- (a) $x/25$
- (b) $x/10$
- (c) $10x$
- (d) $10/x$
- (e) $25/x$

Solution: An easy number to pick for x would be 250 so that the hot dogs cost \$1 each. Hence, 10 of the hot dogs cost \$10. Now we look for the answer choice that yields 10 when we plug 250 in for x .

(a) $x/25 = 250/25 = 10$. The answer.

If you need more proof that you will get this answer with any number, try a few more numbers for x and see if you get the same answer. But don't do this during the test! You can also solve this problem using ratios and proportions.

Sometimes you may have to **plug in more than once** to find the correct answer. Here's an example.

Example 9: The positive difference between the squares of any two consecutive integers is always:

- (a) the square of an integer
- (b) a multiple of 5
- (c) an even integer
- (d) an odd number
- (e) a prime number

Solution: The word *always* in the question means we only have to find one instance in which the condition is false. Let's choose 2 and 3 as our two consecutive integers. Their squares are 4 and 9 and the difference is 5.

- (a) 5 is NOT the square of an integer. Eliminate.
- (b) 5 IS a multiple of 5. A possibility.
- (c) 5 is NOT an even integer. Eliminate.
- (d) 5 IS an odd integer. A possibility.
- (e) 5 IS a prime number. A possibility.

We have improved our odds to 1 in 3. But we need to eliminate 2 more choices, Let's choose 0 and 1 as our consecutive integers. The difference between the squares is 1. Now we need to check (b), (d), and (e).

- (b) 1 is NOT a multiple of 5. Eliminate.
- (d) 1 IS an odd integer. A possibility.
- (e) 1 is NOT a prime number. Eliminate.

So, the answer is (d).

Plugging in more than once is very often necessary **on inequalities**. This is especially true on inequalities which have squared variables, such as the following example, or

fractions with variables in the denominators. Do not try to solve for x on these types of inequalities.

Example 10: What are all the values of x such that $x^2 - 3x - 4$ is negative?

- (a) $x < -1$ or $x > 4$
- (b) $x < -4$ or $x > 4$
- (c) $1 < x < 4$
- (d) $-4 < x < 1$
- (e) $-1 < x < 4$

Solution: Just plug in arbitrary numbers that are in the regions described in each answer choice:

- (a) Plug in a number < -1 or > 4 lets say (5): $(5)^2 - 3(5) - 4$ is positive; so (a) is NOT the answer.
- (b) Try (5) again! (b) is NOT the answer.
- (c) Try (2): $(2)^2 - 3(2) - 4$ is negative; (c) might be the answer.
- (d) Try (0): $(0)^2 - 3(0) - 4$ is negative; (c) might be the answer.

Similarly, (e) might be the answer. So let's plug in another number that does NOT overlap the intervals we have in (c), (d), and (e). Say (-1): $(-1)^2 - 3(-1) - 4$ is negative; so (c) is NOT the answer. Finally try a number that doesn't overlap the intervals (d) and (e). Say (-2): $(-2)^2 - 3(-2) - 4$ is positive so the answer is (e).

If the inequality is a simple one containing no squared terms or algebraic fractions, it may be quicker to just solve it as we mentioned in Example 4.

Example 11: If $-3x + 6 \geq 18$, which of the following is true?

- (a) $x \leq -4$
- (b) $x \leq 6$
- (c) $x \geq -4$
- (d) $x \geq -6$
- (e) $x = 2$

Solution: Clearly, this inequality is simple to solve. Solving it would be faster than plugging in the answer choices on a problem like this.

$$\begin{aligned} -3x + 6 &\geq 18 \\ -3x &\geq 12 \\ -x &\geq 4 \\ x &\leq -4 \end{aligned}$$

Hence, the answer is (a).

Some problems in the GRE will give you the value of one of the terms in an expression, and then ask you for the value of that expression. **Never** bother to solve for the variable on a problem like this. It is **much faster** to plug in and there is less room for error.

Example 12: If $3x = -2$, then $(3x - 3)^2 = ?$

- (a) -25 (b) 13 (c) -5 (d) 25 (e) 9

Solution: Don't bother to solve for x and mess with fractions. Since the value of $3x$ is -2 , just plug -2 into the expression $(3x - 3)$:

$$(3x - 3) = (-2 - 3) = (-5) = 25$$

Plugging in simpler numbers for the numbers in the problem can really simplify a problem.

Example 13: Which if the following numbers is the closest approximation to the value of

$$\frac{0.507(507)}{5.07} ?$$

- (a) 1 (b) 5 (c) 10 (d) 50 (e) 100

Solution: Rather than spend time manipulating these numbers, just approximate each number since we are only looking for an approximate value anyway. Replace the last two digits of each number with zeros:

$$\frac{0.5(500)}{5}$$

Now we can see that this number is just half of 500 divided by 5. Half of 500 is 250, and 250 divided by 5 is 50. The answer is (d).

Example 14: $2^8 - 2^7 =$

- (a) 2 (b) $2^{7/8}$ (c) 2^7 (d) 2^8 (e) 2^{15}

Solution: Do not make the mistake of subtracting the exponents on this problem. Remember that you can only add and subtract exponents when multiplying and dividing. Instead of figuring out the values of 2^8 and 2^7 , let's try this problem first with smaller numbers:

$$2^4 - 2^3 = 2^3$$

Since 2^4 is 16 and 2^3 is 8, then $2^4 - 2^3 = 16 - 8 = 8 = 2^3$. Note that the solution is the same as the second number in the problem. This makes sense since 2^4 is 2 times 2^3 or $2^3 + 2^3 = 2^4$. Using this same principle on the original problem, the answer must be 2^7 or answer (c). Had you not known what to do on this problem, you could have immediately eliminated (e), since the difference can't be greater than either of the two numbers, and (d), since the difference can't be the same as the larger number unless the second number was zero. Choices (a) and (b) are attractive to careless people who think they can use the division rule for exponents.

Watch out for **tricky wording** or changes in wording on the GRE. If there is an incorrect answer that would result from misreading the problem, you can bet it will be an answer choice.

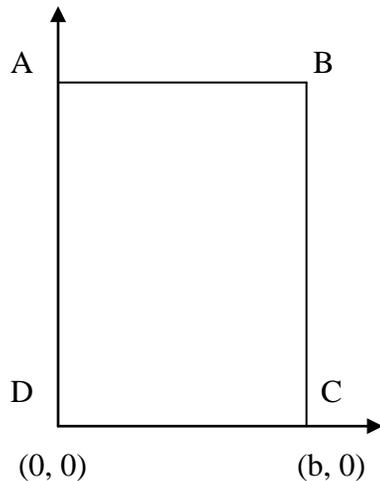
Example 15: $(PQ)(PQ) = 144$

If P and Q represent the digits in the two-digit numbers above, the ratio of Q to P is

- (a) 1 : 2 (b) 1 : 1 (c) 2 : 1 (d) 3 : 1 (e) 12 : 1

Solution: Since the square root of 144 is 12, then PQ must be 12. Hence, $P = 1$ and $Q = 2$. So, the ratio must be 1 : 2, right? Wrong! The question asked for the ratio of Q to P, so the answer is 2 : 1, or (c). Be careful!

On **geometry problems**, you can **plug in values for angles or lengths** as long as the values do not contradict the problem statement or geometry laws. So, you must remember the geometry laws we learned.



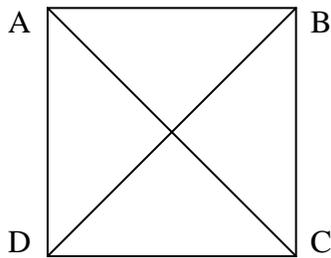
Example 16: The area of the rectangle ABCD is $3b^2$. The coordinates of C and D are given. In terms of b, $BD =$

- (a) b (b) 2b (c) $\sqrt{5} b$ (d) 3b (e) $\sqrt{10} b$

Solution: The distance from D to C is b (the width). If the area is $3b^2$, then the length is $3b$ (Why)? Looking at triangle ABD the length of the hypotenuse is $\sqrt{(b)^2 + (3b)^2} = \sqrt{10} b$ (e). Remember that $c^2 = a^2 + b^2$.

Another way to approach this problem is to eliminate answer choices that don't make common sense based on the geometry rules we've learned. Suppose again that we plug in 2 for b . The length of DC is 2, and the length of BC is 6 (area = length \times width). Recall from our geometry review that the longest side is opposite the largest angle and that the hypotenuse of a right triangle is always longer than either of its other two sides. Hence, the length of the hypotenuse BD must be more than 6. The hypotenuse must also be shorter than the sum of the other two sides. So, the length of the hypotenuse must be less than 8. Now look at the answer choices. Plugging in 2 for b , we see immediately that (a), (b), and (d) are all too small. Remember that the square root of 5 is approximately 2.2 (you were supposed to commit this to memory!). Hence, $2\sqrt{5}$ is approximately 4.4 which is also too small. The answer must be (e).

Example 17: In square ABCD below, what is the value of $\frac{(AD)(AB)}{(AC)(BD)}$?



- (a) $1/2$ (b) $\sqrt{2}/2$ (c) 1 (d) $\sqrt{2}$ (e) 2

Solution: Since the diagonal of a square (or rectangle like in the previous problem) is always larger than the sides, then $(AC)(BD)$ must be larger than $(AD)(AB)$. Since the numerator is smaller than the denominator, we are looking for a fraction between 0 and 1. That eliminates (c), (d), and (e). Now recall the side relationships of a 45:45:90 triangle. If one side of the 45:45:90 triangle is 1, then the hypotenuse is $\sqrt{2}$. In other words, if AD and AB have length 1, then AC and BD must have length $\sqrt{2}$. Hence:

$$\frac{(AD)(AB)}{(AC)(BD)} = \frac{(1)(1)}{\sqrt{2} \sqrt{2}} = \frac{1}{2}; \text{ which is (a).}$$

$$(AC)(BD) = \sqrt{2} \sqrt{2}$$

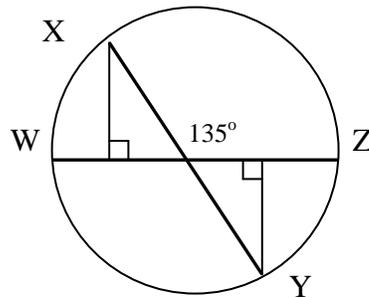
Some math problems on the GRE don't seem to fit any of the shortcuts that we've discussed. However, rather than solving it by some lengthy operation you learned in algebra class, look for a shortcut. Virtually all GRE math problems can be solved without time-consuming mathematical techniques. Improve your score by teaching yourself to look for the easy way out.

Example 18: If $x^2 - 4 = (18)(14)$, then x could be

- (a) 14 (b) 16 (c) 18 (d) 26 (e) 32

Solution: Solving for x or plugging in would take way too much time. Remember the expression $x^2 - y^2$ from our algebra review? Notice that the left side of the equation in this problem fits this form. Also, remember that $x^2 - y^2$ factors into $(x + y)(x - y)$. Hence, $x^2 - 4$ factors into $(x + 2)(x - 2)$. Since $(x + 2)(x - 2) = (18)(14)$, then $x + 2 = 18$ and $x - 2 = 14$. Since $16 + 2 = 18$ and $16 - 2 = 14$, then x could be 16 which is answer choice (b).

You can also find **shortcuts on geometry problems** on the GRE. You must remember however that diagrams are not drawn to scale, so you must not try to guess the answer by eyeballing or measuring a diagram. However, diagrams on the GRE do provide a lot of useful information which may provide a shortcut to the answer.



Example 19: In the circle above, if segment WZ and segment XY are diameters of the circle with length 12, what is the area of the two right triangles?

- (a) 36 (b) 33 (c) 30 (d) 18 (e) 12

Solution: Since the diameter of the circle is 12, the radius must be 6. Therefore, the hypotenuse of each of the two right triangles is 6. The small angle (either one) next to the $135^\circ = 180^\circ - 135^\circ = 45^\circ$. Hence, the two right triangles are 45:45:90 triangles, so the other two sides of the triangles must be equal. Recalling the ratio of the sides of a 45:45:90 triangle ($1:\sqrt{2}$), the other sides must have length $6/\sqrt{2}$.

The area of one triangle =

$$1/2 \cdot \text{base} \cdot \text{height} = 1/2 \cdot 6/\sqrt{2} \cdot 6/\sqrt{2} = 36/2(\sqrt{4}) = 36/2 \cdot 2 = 36/4 = 9$$

The total area = $2 \cdot 9 = 18$ (d).

Compiled by Robyn Wright, 1992

Revised by Mosbah Dannaoui, 1992; Ziad Diab, 1993; John Everett, 1999

Reference:

Robinson, Adam and John Katzman. The Princeton Review – Cracking the System: The GRE 1992 Edition. New York: Villard, 1991. 105-201.