Mathematical and Cognitive Fidelity, Technology Impacting Mathematical Achievement

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Abstract: The nature of fidelity in terms of technology driven mathematical objects leads to deeper awareness of how technology can affect mathematical understanding. Technology tools used in mathematics vary in the degree of mathematical and cognitive fidelity. In this paper we will take a closer look at the issue of fidelity and show a study that suggests mathematical and cognitive fidelity lead to greater mathematical achievement. We conclude, though new to the study of technology's role in studying mathematics, mathematical and cognitive fidelity need to be addressed in in-service and pre-service teacher technology courses.

"In order to function effectively as a representation of a mathematical "object," the characteristic of a technology-generated external representation must be faithful to the underlying mathematical properties of that object." (Zbiek, Heid, Blume, and Dick, 2007, p. 1174)

The idea of fidelity sheds new light on choosing good technology for the classroom. Dick (2007) points out that the tools used in computer technology may not be true to the mathematics intended to be represented. He also uses the terms mathematical fidelity and cognitive fidelity to define how computer generated objects differ in their ability to convey mathematical understanding. The intent of this paper is to look at mathematical fidelity and cognitive fidelity, narrowing the perspective to two attributes of each, to gain additional insights into making judgments as to what technology driven math objects lead to greater fidelity and mathematical understanding. Such a dialogue is crucial when instructing pre-service and in-service teachers on how to strengthen their mathematics programs through the use of technology. Teachers need to be able to evaluate technology for their mathematics classroom based on the quality of the representation in terms of mathematical accuracy and based on how the object's actions lead to cognitive development.

Mathematical fidelity, narrowly defined and in terms of technology, refers to an object's conformity to mathematical accuracy. The question is posed: is its mathematical representation correct? Cognitive fidelity, on the other hand, refers to whether a concept is better understood when the object is acted on. In other words, because of action performed on it does it make sense and add depth of understanding to the concept.

For an example of mathematical fidelity think of the graphical representation found on a typical graphing calculator after entering a polynomial function. What is often seen as a vertical line is mistaken by students as the vertical asymptote. The representation created by connecting pixels is not mathematically accurate and leads to misunderstandings and that needs to be clarified. The mathematical "object" does not have a high level of mathematical fidelity. Yet the representation is useful and if the calculator's setting is changed from continuous to discrete the vertical line disappears and the line looses its continuous characteristics. Mathematical fidelity, it could also be said, is compromised when the answer to a division problem is truncated, but the calculator is giving you the most feasible answer for the place values it allows. Technology's limitations are a constant concern for mathematical fidelity

Cognitive fidelity enables one to make connections by seeing developing patterns that were only possible in the mind. Through the use of technology the relationship is represented by the movement of the object. Think of a familiar applet where parameter changes are explored on a quadratic function with the aid of a slider which causes a series of values to increase or decrease, resulting in the graph moving up or down, right or left. The relationship is clear, and the mental process of putting together a series of graphs to hypothesis and predict is replaced with the visual effect of seeing the graph getting fatter or thinner. The action on the object shows a relationship that cognitively flows in a logical sequence.

To be effective math objects, the limitation of the software do not impact the mathematical content and the action exhibited by the objects leads to a conceptual understanding of a mathematical principle.

Examining technology's application in teaching a concept leads to better understand of the terms mathematical and cognitive fidelity. A recent study (---, 2007) explored the relationship between using a graphing calculator to compare and contrast various quadratic functions with different parameter changes, and an interactive math object (IMO) created by Texas Instruments InterActive software where the parameter changes were initiated by a slider. With the graphing calculator activity students were asked to use their calculator to graph various functions, record their graphs on their work sheet, and then compare and make generalizations. On the other hand, the interactive math object (IMO) showed the action of compressing and stretching the parabola as the values transitioned to lesser or greater values. After the investigation using the IMO, students were asked to input mathematically precise values for a, b, c values (values that affected the parameter changes) to see if the graph they created would pass through points embedded on graphs forming a parabola.

The object created on the calculator was used as a static representation, with its value resting on it transferability to paper. The graph on the calculator was accurate within the limitations of the technology and had mathematical fidelity. No doubt many students were not as accurate when transferring the image to paper. The activity relied heavily on students being able to make sense of the different graphs by comparing and contrasting their representation. Cognitive fidelity, or the sense making component, was limited by static nature of the representation. Though not directly embedded in the activity, motivated students could try other values and compare and contrast, and use their conjectures, if made, to predict the equation for other parabolas. However, much of the time was spent on recreating the graph on paper. Another option for using the graphing calculator may have been to enter all the given functions into the calculator to compare and contrast. But, because the screen is small and limited, the graphs would overlap and threaten the mathematical fidelity.

The interactive math object (IMO) activity put the student in the role of the investigator, looking for cause and effect. Students were able to use a wide range of sequential values and view their resulting graphical image as they moved in a distinctive, predictable pattern. Based on the students initial observations of the moving object, they were then asked to make predictions on what values would complete an equation whose graphic representation could pass through a set of given points. Specific values were entered to see if they would create a parabola that would pass through the given points. Different values could be explored to see their effect. The cause effect cognitive connections were much clearer when using the IMO. The graphing calculator lacked the cognitive assistance that was possible with the IMO where multiple sequential images were created.

Though much could be said to contrast the two uses of technology, the graphing calculator and the TI InterActive IMO, our focus is on their fidelity. The difference between them rests more on cognitive fidelity than mathematical fidelity. Both had accurate representations, though one was more interactive than the other and produced multiple sequential images, and therein lay the difference. Technology can have more cognitive value if mathematical objects interact in a predicable pattern that is discernable, replicable, and consistent. Producing a given outcome as with the graphing calculator created an accurate but static representation that lacked connection. It was left up to the student to see significance in the parabolas they graphed.

The assumption that the IMO, with mathematical and cognitive fidelity, would produce an environment that encouraged conceptual understanding was verified by the above mentioned study. The results of the study (Bos, 2007) revealed that the mathematical achievement as indicated by the quadratic functions objective on the state mandated mathematics test (Texas Assessment of Knowledge and Skills, TAKS) was statistically significantly better for students who used the IMO than those using the calculator and related activity. The results of the Analysis of Covariance run on the data showed the group using the interactive math object was statistically significant with a $F(1,95) = 11.56$, $df = 1$, and $p \leq .001$ and etal = .112. The results do not come as a surprise, knowing that the IMO has strong cognitive fidelity and consistent mathematical fidelity.

When looking at the wide selection in technology based mathematics products we find drill and practice packages of which many have highly interactive gaming features. Most rely on their mathematical fidelity and immediate feedback as their marketable features; yet cognitive fidelity, the sense making of mathematics, is noticeably lacking. Technology, however, can be highly effective in teaching and learning if it has cognitive fidelity. The challenge for teachers of mathematics is to find technology products that have both mathematical fidelity and cognitive fidelity. Because technology is highly interactive, it can be used to show patterns quickly, efficiently, and accurately allowing for exploration and conjecturing. The cognitive fidelity of mathematical objects is relatively unexplored and presents new opportunities to mathematicians and teachers. Because teachers are charged to create the best learning opportunities for their students they need to focus on technology that has cognitive and mathematical fidelity. Technology is not just for repeated practice, but for developing an understanding of concepts and making connections. What was only possible in our mind is now possible through the cognitive strength of technology.

References

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