Texas Mathworks  
Texas State University – San Marcos  
Primary Mathematics World Contest (PMWC)  
Qualifying Test  
November 30, 2005  

COVER SHEET  

Name:_________________________________________________  
Street Address:__________________________________________  
City:__________________  State: ________  Zip: _____________  
Phone: (______) ___________________________  
School:________________________________________________  
Teacher: _______________________________________________  
Present Grade in School: ___________  
Math Courses Taken:  
Pre-Algebra_____  Algebra 1 _____  Algebra 2 __ Geometry _____  
Birthdate (Including year):  _______ – ________– _______  
Gender:      ______ Male              _____Female  
Social Security Number: ____________________  
Are you a U.S. Citizen? Yes_____   No ____
Directions: This test has 15 problems, with a time limit of 120 minutes. Do not use a calculator. Show all your work on the test, and how you obtained each answer. Partial credit will be given even if you do not obtain an answer. Do not worry if you cannot do all the problems. We are interested in how you approached each problem.

1. In ΔABC point P lies on side AC and point Q lies on ray AB so that the area of ΔABC = area of ΔPAQ. Find the length of BQ if AP = 3, PC = 5, BC = 11 and AB = 9.

Answer: _________________

Work:
2. Find the area of \( \triangle ABC \) if \( AB = 8 \) and the measures of angles \( A \) and \( B \) are respectively 75 and 60 degrees.

Answer: _________________

Work:
3. Seven letter “words” are formed using the letters A through G with no letters repeated in a “word”. If these “words” are placed in alphabetical order what is the 2005th “word”?

Answer: _________________

Work:
4. A positive three-digit integer is randomly chosen. What is the probability that it is the square of an integer or the cube of an integer?

Answer: _________________

Work:
5. If $a + b = 10$ and $a^2 + b^2 = 80$, find $a^3 + b^3$.

Answer: _________________

Work:
6. A numeral 1, 2, 3, or 4 is written on each of three balls. The twelve balls are placed into a jar and one is drawn and then replaced. After three draws and replacements what is the probability that the sum of the numerals on the three balls is a divisor of 30?

Answer: _________________

Work:
7. How many ways are there to choose 3 distinct numbers from \{1, 2, 3, \ldots, 19, 20\} with the sum divisible by 3?

Answer: ________________

Work:
8. How many positive integers are there in \{1,2,\ldots,1000\} that are not divisible by 2 or 3?

Answer: 

Work:
9. Find the smallest 4 digit number which is a factor of $2005^6 - 1$.

Answer: _________________

Work:
10. How many 10 digit numbers can be formed using all 10 digits 0-9 so that the 1 appears before the 2, the 2 appears before the 3, the 3 appears before the 4, but 0 does not appear before the 1.

Answer: _________________

Work:
11. Find the sum \[ \frac{1}{(1)(3)} + \frac{1}{(2)(4)} + \frac{1}{(3)(5)} + ... + \frac{1}{(98)(100)} \]

Answer: _________________

Work:
12. Some consecutive pages are missing from a book. The sum of the missing page numbers is 332. What is the smallest missing page number?

Answer: _________________

Work:
13. When a container is filled with water, 95% of the total weight of the water and container is due to water. Some water is poured out. Now 80% of the total weight (of water and container) is due to water. What fraction of the initial water remains in the container?

Answer: _________________

Work:
14. How many ways are there to make a 4 digit number abcd where
   \[ 1 \leq a \leq b \leq c \leq d \]?

Answer: _________________

Work:
15. How many integers between 10000 and 99999 use exactly 3 different odd digits, and no even digits, in their usual representation?

Answer: _________________

Work: