Key HW 7_3.

1. I demonstrated in the lecture a short cylindrical bar magnet being dropped through a vertical aluminum pipe and through a PVC pipe. When dropped through a PVC pipe, the total time it takes for the bar magnet to fall through 6 feet is a little bit over 0.5 second. When dropped through a copper pipe the total fall time is 10 seconds. Explain what is happening. What has this to do with the damped eddy current pendulum of figure 7.16?

The fact that the magnet falls so slowly and not really accelerates implies that in addition to the gravity force another force is acting on the magnet that will cancel out the gravity force. This other force is the magnetic force. The falling magnet will cause flux changes in the copper pipe, resulting in induction current through the copper. Those induction currents generate magnetic fields that oppose the motion of the falling magnet. The direction of the induction currents can be perpendicular to the z-direction or perpendicular to the s-direction depending on the orientation of the magnet while falling through the tube. Note that the physics of both is similar. The graphics below show the two configurations. The arrows indicate the direction of the induced magnetic field. The direction is so that the induced magnetic field above (below) the magnet will attract (repel) the magnet, to the magnetic field is opposing the gravity force. The configuration on the right side shows similarities with the magnetic brake illustrated in Fig. 7.16.
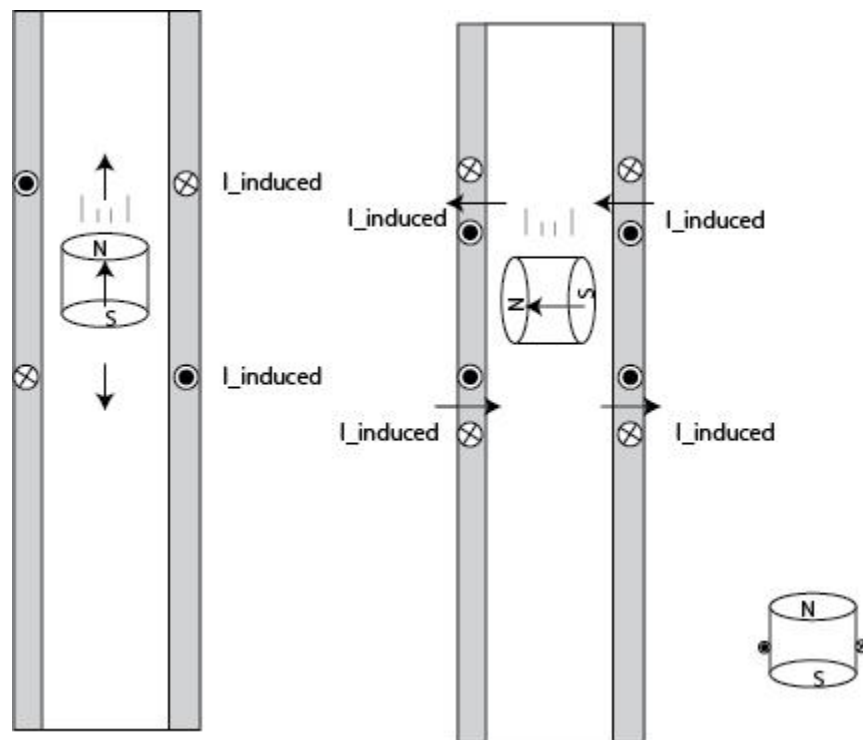
2. Estimate the induction current in the copper tube assuming the magnetic field of the permanent magnet to be 1.2 tesla, the weight of the permanent magnet to be 20 grams, the diameter of

the copper tube to be 0.5 inch and the height of the magnet to be 1 cm, the currents to be located 0.5 inch below the falling magnet and 0.5 inch above the falling magnet.

To estimate the induction currents we use the configuration on the left side. Note that also the permanent magnet has a current: not a free current but a bound current, but Maxwell's equations do not see the difference. The bound current is indicated in the graphics on the very far right. So the lower induced current in the copper tube and the bound surface current of the magnet are anti-parallel and repel each other i.e. magnetic force on magnet is working up. The upper induced current in the copper tube and the surface current of the magnet are parallel resulting in an attracting force, i.e. magnetic force on magnet is working up.

The force between two parallel currents depends on the distance between both currents as well as on the length of the wires. The force can be calculated from equation 5.16. Assuming that the induced current in the copper pipe and the bound current on the magnet can be modeled by two parallel circular currents each with a radius of r and separated by z, the force can be estimated from:

$$\vec{F} = 2I_{bound} \int d\vec{l} \times \vec{B} = 2I_{bound} \int B dl = 2I_{bound} B \int dl = 2MhB2\pi r$$

Where is the magnetic field caused by the induced current through the copper pipe and h is the height of the magnet. Note that I_{bound} is equal to $K_{bound}*h=Mh$, where M is the magnetization of the magnet. The magnetic field caused by the induced current in the copper pipe at the position of the magnet can be estimated from 5.38:

$$B = \mu_o \frac{I}{2\pi z}$$

Where z is the distance between the center of the magnet and the average position of the induction current through the copper pipe. Combining both equations gives:

$$F = 4\pi\mu_o \, rMh \frac{I}{2\pi z} = \frac{2\mu_o rMHI}{z} \therefore I = \frac{Fz}{2\mu_o rMh}$$

Note that we can now plug in all the numbers. The total magnetic force on the magnet should be close to the gravity force on it, i.e. mg=0.196 Newton. Assume r=0.5cm, h=1 cm, z=1 cm, $\mu_o M=\mu_o B/\mu_o=1.2$, M=945 kA/m. Plugging in give us a value of 16.3 ampere. I would not expect such high current! I wonder what we would expect based on the flux rule? Who wants to try?

3. A long solenoid with radius b and n turns per unit length carries a time-dependent current I(t) in the φ_head direction. Find the magnitude and direction of the electric field at a distance s from the axis of the solenoid.

4. Work problems 7.12, 7. 16, 7.18 and 7.22.

7.12

Problem 7.12

$$\Phi = \pi \left(\frac{a}{2}\right)^2 B = \frac{\pi a^2}{4} B_0 \cos(\omega t); \quad \mathcal{E} = -\frac{d\Phi}{dt} = \frac{\pi a^2}{4} B_0 \omega \sin(\omega t). \quad I(t) = \frac{\mathcal{E}}{R} = \boxed{\frac{\pi a^2 \omega}{4R} B_0 \sin(\omega t).}$$

7.16

Problem 7.16

(a) The magnetic field (in the quasistatic approximation) is "circumferential". This is analogous to the current in a solenoid, and hence the field is $\boxed{\text{longitudinal.}}$
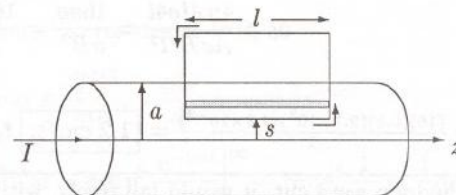
(b) Use the "amperian loop" shown.
Outside, $\mathbf{B} = 0$, so here $\mathbf{E} = 0$ (like $\mathbf{B}$ outside a solenoid).
So $\oint \mathbf{E} \cdot d\mathbf{l} = El = -\frac{d\Phi}{dt} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{a} = -\frac{d}{dt} \int_s^a \frac{\mu_0 I}{2\pi s'} l \, ds'$
$\therefore E = -\frac{\mu_0}{2\pi} \frac{dI}{dt} \ln\left(\frac{a}{s}\right)$. But $\frac{dI}{dt} = -I_0 \omega \sin \omega t$,

so $\boxed{\mathbf{E} = \frac{\mu_0 I_0 \omega}{2\pi} \sin(\omega t) \ln\left(\frac{a}{s}\right) \hat{\mathbf{z}}.}$

7.18

Problem 7.18

$$\Phi = \int \mathbf{B} \cdot d\mathbf{a}; \quad \mathbf{B} = \frac{\mu_0 I}{2\pi s} \hat{\phi}; \quad \Phi = \frac{\mu_0 I a}{2\pi} \int_a^{2a} \frac{ds}{s} = \frac{\mu_0 I a \ln 2}{2\pi}; \quad \mathcal{E} = I_{\text{loop}} R = \frac{dQ}{dt} R = -\frac{d\Phi}{dt} = -\frac{\mu_0 a \ln 2}{2\pi} \frac{dI}{dt}.$$

$$dQ = -\frac{\mu_0 a \ln 2}{2\pi R} dI \implies \boxed{Q = \frac{I \mu_0 a \ln 2}{2\pi R}.}$$

The field of the wire, at the square loop, is *out of the page*, and *decreasing*, so the field of the induced current must point out of page, within the loop, and hence the induced current flows $\boxed{\text{counterclockwise.}}$

Problem 7.19

7.22

Problem 7.20

(a) From Eq. 5.38, the field (on the axis) is $\mathbf{B} = \frac{\mu_0 I}{2} \frac{b^2}{(b^2+z^2)^{3/2}} \hat{\mathbf{z}}$, so the flux through the little loop (area πa^2)

is $\boxed{\Phi = \frac{\mu_0 \pi I a^2 b^2}{2(b^2+z^2)^{3/2}}.}$

(b) The field (Eq. 5.86) is $\mathbf{B} = \frac{\mu_0}{4\pi} \frac{m}{r^3}(2\cos\theta \, \hat{\mathbf{r}} + \sin\theta \, \hat{\theta})$, where $m = I\pi a^2$. Integrating over the spherical "cap" (bounded by the big loop and centered at the little loop):

$$\Phi = \int \mathbf{B} \cdot d\mathbf{a} = \frac{\mu_0}{4\pi} \frac{I\pi a^2}{r^3} \int (2\cos\theta)(r^2 \sin\theta \, d\theta \, d\phi) = \frac{\mu_0 I a^2}{2r} 2\pi \int_0^{\bar{\theta}} \cos\theta \sin\theta \, d\theta$$

where $r = \sqrt{b^2 + z^2}$ and $\sin\bar{\theta} = b/r$. Evidently $\Phi = \frac{\mu_0 I \pi a^2}{r} \left.\frac{\sin^2\theta}{2}\right|_0^{\bar{\theta}} = \boxed{\frac{\mu_0 \pi I a^2 b^2}{2(b^2 + z^2)^{3/2}}}$, the same as in (a)!!

(c) Dividing off I ($\Phi_1 = M_{12} I_2$, $\Phi_2 = M_{21} I_1$): $\boxed{M_{12} = M_{21} = \dfrac{\mu_0 \pi a^2 b^2}{2(b^2 + z^2)^{3/2}}}$.