STRAIGHT LINES

\[ y = -3x + 5 \]
\[ 3x - 4y = 9 \]
\[ y = \frac{-2}{3} x \]

Let’s look at some equations:

These are called first degree equations because the powers of the \( x \)'s are (1). All of the above equations can be written as: \( y = mx + b \), where \( m \) is the slope of the line and \( b \) is the y-intercept (where the line crosses the y-axis):

\[ y = -3x + 5 \quad \text{slope} = -3 \text{ and y – intercept} = 5 \]
\[ 3x - 4y = 9 \quad \text{slope} = \frac{3}{4} \text{ and y – intercept} = \frac{-9}{4} \]
\[ y = \frac{-2}{3} x \quad \text{slope} = \frac{-2}{3} \text{ and y – intercept} = 0 \]

SLOPE:

\[ \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{vertical movement; upward (positive) or downward (negative)}}{\text{horizontal movement; forward (to the right ALWAYS)}} \]

Example 1: Graph the line whose y-intercept = -1 and slope = 3/2

Notice that:
* y-intercept = -1
* go up 3 over 2
  (rise = 3, run = 2)
**Example 2:** Graph the line whose y-intercept = 4 and slope = -5/3

Notice that:
* y-intercept = 4
* go down 5 over 3

**Example 3:** Graph the line whose slope = ½ and passes through the (3, 2)

Notice that:
* point (3, 2)
* go up 1 over 2

To find the slope between two points \((x_1, y_1)\) and \((x_2, y_2)\) use the formula: 
\[ m = \frac{y_2 - y_1}{x_2 - x_1}. \]

**Example 4:** Sketch a line through each pair of points and find the slope of each line:

- (A) (-3, -4), (3, 2)
- (B) (-2, 3), (1, -3)
- (C) (-4, 2), (3, 2)
- (D) (2, 4), (2, -3)
Notice that:
- The slope (see above) can be one of four choices: (A) positive, (B) negative, (C) zero or (D) not defined.
- A horizontal line has a slope = 0.
- A vertical line has an undefined slope.

**EQUATIONS OF THE LINE:**

To find the equation of the line (if you have a point and slope) use the formula

\[ y - y_1 = m(x - x_1) \].
**Example 5:** Find the equation of the line that has slope = 3 and passes through (2, -5). Using the above formula with \( m=3 \), \( x_1 = 2 \) and \( y_1 = -5 \) gives:

\[
y - (-5) = 3 (x-2) \\
y + 5 = 3x - 6 \\
y = 3x - 11 \text{ is your solution.}
\]

**Example 6:** Find the equation of the line that passes through (-2, 1) and (6, -5).

To use the formula above we need a point and a slope just like example 5 above. To find the slope of the line we use the slope formula and then pick any of the points for the equation:

\[
m = \frac{-5 - 1}{6 - (-2)} = \frac{-6}{8} = -\frac{3}{4}
\]

if we pick (6, -5) as our point: if we pick (-2, 1) as our point:

\[
y - (-5) = \frac{-3}{4} (x - 6) \\
y + 5 = \frac{-3}{4} x + 18/4 \\
y = \frac{-3}{4} x + 9/2 -5 \\
y = \frac{-3}{4} x - 1/2
\]

Both answers are the same (it does not matter which point you choose).

**Example 7:** Find the equation of the line that passes through (3, -2) and has an x-intercept 4.

x-intercept 4 means the line passes through (4, 0):

\[
m = \frac{0 - (-2)}{4 - 3} = \frac{2}{1} = 2
\]

\[
y - (-2) = 2 (x-3) \quad \text{Point-slope form} \quad \text{point (3, -2), slope = 2} \\
y + 2 = 2x - 6 \\
y = 2x - 8 \quad \text{Slope-intercept form} \quad \text{slope = 2, y-intercept = -8} \\
y - 2x + 8 = 0 \quad \text{General form}
\]

**PERPENDICULAR AND PARALLEL LINES:**

Two lines are Parallel if and only if they have the same slope.
Two lines are Perpendicular if and only if they have negative reciprocal slopes.

**Example 8:** Find the equation of the line that passes through (5, 2) and:
(A) Parallel to the line passing through (4, -1) and (-8, 5).
(B) Perpendicular to the line passing through (4, -1) and (-8, 5).

\[
m = \frac{5 -(-1)}{-8 - 4} = \frac{6}{-12} = -\frac{1}{2}
\]

1. The two lines are parallel; they have the same slope; \( m = -1/2 \).
\[
\begin{align*}
y - (-2) &= -1/2 (x-5) \\
y + 2 &= -1/2 x + 5/2 \\
y &= -1/2 x + 1/2
\end{align*}
\]

2. The two lines are perpendicular; they have negative reciprocal slopes; 
m = +2/1 = 2.

\[
\begin{align*}
y - (-2) &= 2 (x - 5) \\
y + 2 &= 2x - 10 \\
y &= 2x - 12
\end{align*}
\]

**Example 9:** Find the equation of the line that passes through (-3, 5) and perpendicular to the line L: 3x – 2y = 5.

First find the slope of L by writing 3x – 2y = 5 in the equivalent slope-intercept form 
y = m x + b:

\[
\begin{align*}
3x - 2y &= 5 \\
-2y &= -3x + 5 \\
y &= 3/2 x - 5/2 \quad m = 3/2
\end{align*}
\]

Our line is perpendicular to L and has a slope of -2/3:
y – 5 = -2/3 (x + 3)
y = -2/3 x + 3

**Example 10:** Find the equation of the line that is the perpendicular bisector of (-3, 2) and (7, -4).

\[m = \frac{-4 - 2}{7 - (-3)} = \frac{-6}{10} = -\frac{3}{5}\]

Our line has slope of 5/3 (perpendicular) and passes through the midpoint which is:

\[
\left(\frac{-3 + 7}{2}, \frac{2 - 4}{2}\right) = \left(\frac{4}{2}, \frac{-2}{2}\right) = (2, -1)
\]

\[
\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2}{2}, \frac{-1}{2}\right)
\]

\[y + 1 = 5/3 (x - 2)\]
\[y + 1 = 5/3 x - 10/3\]
\[y = 5/3 x - 13/3\]

Revised: Spring 2004
Created by Ziad Diab, 1994

**STUDENT LEARNING ASSISTANCE CENTER (SLAC)**
Texas State University-San Marcos