FACTORING

Summary of steps for factoring a polynomial:

1. Factor out any common factors.
2. If the polynomial is a binomial, check whether it can be rewritten as the difference of two squares, the
difference of two cubes, or the sum of two cubes.
3. If the polynomial is a trinomial, check whether it is a perfect square trinomial. If not, factor by trial and error
(guessing), or apply the AC Method.
4. If the polynomial has more than three terms, try factoring by grouping.

Factoring Out a Common Factor:

Example 1: 5x + 15 = 5(x + 3)

Example 2: 12xy^4 − 27xy^2 + 15xy^3 = 3xy^2(4y^2 − 9 + 5y) = 3xy^2(4y^2 + 5y − 9)

The Difference of Two Squares:

Factoring formula: \(x^2 − y^2 = (x + y)(x − y)\)

Note: The sum of two squares \(x^2 + y^2\) is NOT factorable.

Example 1: \(x^2 − 9 = x^2 − 3^2\)
\[= (x + 3)(x − 3)\]
break down in to squares
factor using formula

Example 2: \(25x^2 − 81 = (5x)^2 − 9^2\)
\[= (5x + 9)(5x − 9)\]
break down in to squares
factor using formula

Example 3: \(x^4 − 81 = (x^2)^2 − 9^2\)
\[= (x^2 − 9)(x^2 + 9)\]
\[= (x^2 − 3^2)(x^2 + 9)\]
\[= (x + 3)(x − 3)(x^2 + 9)\]
simplify

The Difference and Sum of Two Cubes:

Factoring formulas: \(x^3 − y^3 = (x − y)(x^2 + xy + y^2)\)
\(x^3 + y^3 = (x + y)(x^2 − xy + y^2)\)

Example 1: \(x^3 − 27 = x^3 − 3^3\)
\[= (x − 3)(x^2 + 3x + 3^2)\]
\[= (x − 3)(x^2 + 3x + 9)\]
break down in to cubes
factor using formula
simplify

Example 2: \(27x^3 − 8y^3 = (3x)^3 − (2y)^3\)
\[= (3x − 2y)((3x)^2 + (3x)(2y) + (2y)^2)\]
\[= (3x − 2y)(9x^2 + 6xy + 4y^2)\]
break down in to cubes
factor using formula
simplify
Example 3: \((A + B)^3 - 8 = (A + B)^3 - 2^3 = ((A + B) - 2)((A + B)^2 + (A + B)(2) + 2^2) = (A + B - 2)(A^2 + 2AB + B^2 + 2A + 2B + 4)\)

Example 4: \(216x^{3m} - 27b^{9m+3} = (6x^m)^3 - (3b^{3m+1})^3 = (6x^m - 3b^{3m+1})(6x^m)^2 + (6x^m)(3b^{3m+1}) + (3b^{3m+1})^2 = (6x^m - 3b^{3m+1})(36x^{2m} + 18x^mb^{3m+1} + 9b^{6m+2})\)

Example 5: \(x^6 - 64 = x^6 - 2^6 = (x^3)^2 - (2^3)^2 = (x^3 + 2^3)(x^3 - 2^3) = (x + 2)(x^2 - 2x + 4)(x - 2)(x^2 + 2x + 4)\)

Example 6: \((x + y)^3 + 27 = (x + y)^3 + 3^3 = ((x + y) + 3)((x + y)^2 - (x + y)(3) + 3^2) = (x + y + 3)(x^2 + 2xy + y^2 - 3x - 3y + 9)\)

Example 7: \(8x^3 + 343c^3 = (2x)^3 + (7c)^3 = (2x + 7c)((2x)^2 - (2x)(7c) + (7c)^2) = (2x + 7c)(4x^2 - 14xc + 49c^2)\)

Example 8: \(27A^{9n} + 216B^{2n+6} = (3A^{3n})^3 + (6B^{n+2})^3 = (3A^{3n} + 6B^{n+2})(3A^{3n})^2 - (3A^{3n})(6B^{n+2}) + (6B^{n+2})^2 = (3A^{3n} + 6B^{n+2})(9A^{6n} - 18A^{3n}B^{n+2} + 36B^{2n+4})\)

**Perfect Square Trinomial:**

**Factoring formulas:**

- \(x^2 + 2xy + y^2 = (x + y)^2\)
- \(x^2 - 2xy + y^2 = (x - y)^2\)

Example 1: \(4x^2 + 4xy + y^2 = (2x)^2 + 2(2x)(y)^2 + (y^2)^2 = (2x + y)^2\)

Example 2: \(25a^4b^8 - 60a^2b^4c^3 + 36c^6 = (5a^2b^4)^2 - 2(5a^2b^4)(6c^3) + (6c^3)^2 = (5a^2b^4 - 6c^3)^2\)

**AC Method:** For an in-depth description of this procedure see SLAC handout: AC Method.

Example 1:

\[20x^2 - 31xy + 12y^2 = 20x^2 - 16xy - 15xy + 12y^2 = 4x(5x - 4y) - 3y(5x - 4y) = (5x - 4y)(4x - 3y)\]

\((20)(12) = 2^4 \cdot 3 \cdot 5 = -16 - 15 = -31\) factor out any common factors

\[apply\ factor\ by\ grouping\]

Example 2:

\[28a^2 - 51ac + 20c^2 = 28a^2 - 35ac - 16ac + 20c^2 = 7a(4a - 5c) - 4c(4a - 5c) = (4a - 5c)(7a - 4c)\]

\((28)(20) = 2^4 \cdot 5 \cdot 7 \rightarrow -35 - 16 = -51\) factor out any common factors

\[apply\ factor\ by\ grouping\]
Grouping:

Example 1: \(x^2 - 9y^2 - 3x - 6y = (x^2 - 9y^2) - (3x + 6y)\)
\[= (x^2 - 3y^2) - 3(x + 2y)\]
\[= (x - 3y)(x + 3y) - 3(x + 2y)\]

Example 2: \(a^2x^2 + b^2y^2 - a^2y^2 - b^2x^2 = (a^2x^2 - a^2y^2) - (b^2x^2 - b^2y^2)\)
\[= a^2(x^2 - y^2) - b^2(x^2 - y^2)\]
\[= (x^2 - y^2)(a^2 - b^2)\]
\[= (x + y)(x - y)(a + b)(a - b)\]

Practice Problems: Factor the following polynomials.

1. \(15x^2 + 41x + 14\)
2. \(3x^2 - 16x + 5\)
3. \(5x^4y^4 - 135xy\)
4. \(b^3 + a^2 + b^2 - a^2 - 2ab\)
5. \((a - b)^6 + 125\)
6. \(6zx^2 + 2yx - 3zyx - y^2\)

Answers:

1. \((3x + 7)(5x + 2)\)
2. \((x - 5)(3x - 1)\)
3. \(5xy(xy - 3)(x^2y^2 + 3xy + 9)\)
4. \((b - a)(b^2 + ab + a^2 + b - a)\)
5. \(((a-b)^2 + 5)((a-b)^4 - 5(a-b)^2 + 25)\)
6. \((2x - y)(3zx + y)\)