I. Numeric Reasoning

A. Number representation

1. Compare real numbers.
   
   a. Classify numbers as natural, whole, integers, rational, irrational, real, imaginary, and/or complex.
   
   b. Use and apply the relative magnitude of real numbers by using inequality symbols to compare them and locate them on a number line.
   
   c. Order real numbers with and without a calculator using relationships involving decimals, rationals, exponents, and radicals.
   
   d. Represent any rational number in scientific notation.

2. Define and give examples of complex numbers.
   
   a. State the standard form used to represent complex numbers and describe their real and imaginary parts.
   
   b. Represent $i^a$ and square roots of negative numbers as complex numbers.
   
   c. Understand that to solve certain problems and equations, number systems need to be extended from whole numbers to the set of all integers (positive, negative, zero), from integers to rational numbers, from rational numbers to real numbers (rational and irrational numbers), and from real numbers to complex numbers; define and give examples of each of these types of numbers.

B. Number operations

1. Perform computations with real and complex numbers.
   
   a. Add, subtract, multiply, and divide real numbers accurately, including irrational numbers, numbers with exponents, and absolute value.
   
   b. Transform numerical expressions using field properties (especially the distributive property), order of operations, and properties of exponents.
   
   c. Solve problems involving rational numbers, ratios, percents, and proportions in context of the situation.
d. Calculate the sum, difference, product, and quotient of two complex numbers and express the result in standard form.

C. Number sense and number concepts
1. Use estimation to check for errors and reasonableness of solutions.
   a. Identify the most reasonable solution for a given problem from a list of possible solutions; justify the choice.
   b. Use mental estimates to detect potential errors when using a calculator.
   c. Justify the need for an exact answer or an estimate in a given problem (e.g., determining amount of paint needed for a room).

II. Algebraic Reasoning

A. Expressions and equations
1. Explain and differentiate between expressions and equations using words such as “solve,” “evaluate,” and “simplify.”
   a. Define what an expression or equation represents.
   b. Distinguish among and apply different uses of equations to: state a definition, represent a conditional statement, and represent an identity.

B. Manipulating expressions
1. Recognize and use algebraic (field) properties, concepts, procedures, and algorithms to combine, transform, and evaluate expressions (e.g., polynomials, radicals, rational expressions).
   a. Use the algebraic (field) properties (e.g., commutative, associative, distributive) and order of operations to transform expressions to equivalent expressions.
   b. Use the algebraic (field) properties and order of operations to evaluate variable expressions when given the value of the variables.

2. Explain the difference between the solution set of an equation and the solution set of an inequality.
   a. Represent the solution set of an equation or inequality in various ways (e.g., set notation, interval notation, graphical representation, including shading).
   b. Understand that the real solution to an equation can be represented as the x-coordinate of the point of intersection of two graphs.
   c. Understand the relationship between a solution of a system of two linear equations with two variables and the graphs of the corresponding lines.

c. Explain why the algorithms and procedures used to transform algebraic expressions are valid.

C. Solving equations, inequalities, and systems of equations
1. Recognize and use algebraic (field) properties, concepts, procedures, and algorithms to solve equations, inequalities, and systems of linear equations.
   a. Solve equations and inequalities in one variable (e.g., numerical solutions, including those involving absolute value, radical, rational, exponential, and logarithmic).
   b. Solve for any variable in an equation or inequality that has two or more variables (e.g., literal equations).
   c. Use equality and algebraic (field) properties to solve an equation by constructing a sequence of equivalent equations.
   d. Use the elimination, substitution, and/or graphing method to solve a linear system of equations with two variables.
   e. Use technology when using matrices to solve linear systems with two or three variables.
d. Graph a function and understand the relationship between its real zeros, roots, and the x-intercepts of its graph.

D. Representations

1. Interpret multiple representations of equations and relationships.
   a. Interpret graphical representations of equations.
   b. Understand how variables can be used to express generalizations and represent situations.
   c. Recognize the solution(s) to an equation from a table of values.
   d. Describe numerical patterns using algebraic expressions and equations in closed or recursive forms, such as arithmetic sequences.

2. Translate among multiple representations of equations and relationships.
   a. Explain the common information presented in multiple representations of a relationship.
   b. Translate one given representation to another representation (e.g., tabular to graphic, graphic to symbolic).
   c. Use multiple representations to determine rate of change.
   d. Determine if a relationship given in graphical, tabular, or symbolic form is linear or nonlinear.

c. Describe and use cross-sections and nets of three-dimensional figures to relate them to plane figures.

d. Describe the conic sections as intersections of a plane with a cone.

e. Recognize and describe orthographic (top, front, side) and isometric views of three-dimensional geometric figures.

2. Make, test, and use conjectures about one-, two-, and three-dimensional figures and their properties.
   a. Develop and verify attributes of lines and parts of lines in a plane and in space: parallel, intersecting, perpendicular, and skew lines; and angle relationships associated with transversals on parallel lines.
   b. Develop and verify angle relationships: vertical, complementary, supplementary, angles on parallel lines, angle-side relations in a triangle, interior/exterior angles on polygons, and angles on circles.
   c. Develop, verify, and extend properties of circles, including properties of arcs, chords, tangents, secants, and spheres.
   d. Develop and verify properties of triangles and quadrilaterals (e.g., triangle congruence conditions, properties of a parallelogram).
   e. Develop and verify properties of parts of prisms, cylinders, pyramids, and cones.
   f. Apply properties of geometric figures to solve problems.

III. Geometric Reasoning

A. Figures and their properties

1. Identify and represent the features of plane and space figures.
   a. Construct and use drawings, models, and coordinate representations of plane and space figures in order to solve problems by hand and using technology.
   b. Recognize and describe the plane-figure components of three-dimensional figures, such as prisms, pyramids, cylinders, and cones.

   a. Apply the Pythagorean Theorem and its converse to solve real-life situations in two and three dimensions.
   b. Apply Pythagorean triples and special right triangle relationships to solve problems.
   c. Solve right triangle situations using sine, cosine, and tangent.
B. Transformations and symmetry

1. Identify and apply transformations to figures.
   a. Identify whether a transformation is a reflection, rotation, translation, or dilation.
   b. Find the image or pre-image of a given plane figure under a congruence transformation (e.g., translation, reflection, rotation) or composition of these transformations in coordinate and non-coordinate plane settings.
   c. Find the image or pre-image of a given plane figure under a dilation or composition of dilations in coordinate and non-coordinate plane settings.
   d. Use transformations and compositions of transformations to investigate and justify geometric properties of a figure (e.g., the sum of the three angles inside any triangle is 180 degrees).

2. Identify the symmetries of a plane figure.
   a. Identify and distinguish between reflective and rotational symmetry in an object.
   b. Identify congruent corresponding parts in a figure with reflective or rotational symmetry.
   c. Identify lines of symmetry in plane figures to show reflection.

3. Use congruence transformations and dilations to investigate congruence, similarity, and symmetries of plane figures.
   a. Use congruence transformations to justify congruence among triangles and to identify congruent corresponding parts.
   b. Use dilations and scale factors to investigate similar figures and determine missing image or pre-image dimensions.
   c. Identify symmetries in design situations and describe transformations used to create the symmetry and design (e.g., tiling problems).

C. Connections between geometry and other mathematical content strands

1. Make connections between geometry and algebra.
   a. Describe lines in the coordinate plane using slope-intercept and point-slope form.
   b. Use slopes to describe the steepness and direction of lines in the coordinate plane and to determine if lines are parallel, perpendicular, or neither.
   c. Relate geometric and algebraic representations of lines, segments, simple curves, and conic sections [e.g., describe algebraically a circle centered at (h, k) with radius (r)].
   d. Investigate and justify properties of triangles and quadrilaterals using coordinate geometry.
   e. Relate the number of solutions to a system of equations of lines to the number of intersections of two or more graphs.

2. Make connections between geometry, statistics, and probability.
   a. Compute probabilities using lengths of segments or areas of regions representing desired outcomes.
   b. Construct a trend line or a regression line for a scatter plot and use it to make predictions.

3. Make connections between geometry and measurement.
   a. Determine perimeter and area of two-dimensional figures and surface area and volume of three-dimensional figures using measurements and derived formulas.
   b. Find the measures of the lengths and areas of similar figures and of the lengths, surface areas, and volumes of similar solids.
c. Find arc length and sector area for a given central angle on a circle.

D. Logic and reasoning in geometry
1. Make and validate geometric conjectures.
   a. Use drawings, manipulatives (e.g., paper folding, transformations) and constructions (e.g., compass/straight-edge, computer graphing utility) to investigate patterns and make conjectures about geometric properties of figures.
   b. Use counterexamples to verify that a geometric conjecture is false.
   c. Give a logical argument in a variety of formats to verify that a geometric conjecture is true.
   d. Use a conditional statement to describe a property of a geometric figure. State and investigate the validity of the statement’s converse, inverse, and contrapositive.
   e. Make the connection between a biconditional statement and a true conditional statement with a true converse.

2. Understand that Euclidean geometry is an axiomatic system.
   a. Distinguish among theorems, properties, definitions, and postulates and use them to verify conjectures in Euclidean geometry.
   b. Understand that non-Euclidean geometries exist.

IV. Measurement Reasoning
A. Measurement involving physical and natural attributes
1. Select or use the appropriate type of unit for the attribute being measured.
   a. Determine appropriate units of measurement needed for the object being measured in a given situation (e.g., unit analysis, degree, or radian measure of an angle).
   b. Select and accurately use an appropriate tool to make measurements.
   c. Recognize and use significant digits to determine the accuracy of a measurement in problem situations.
   d. Use the appropriate level of precision when providing solutions to measurement problems.
   e. Know when to estimate and approximate measurements for given problem situations.

B. Systems of measurement
1. Convert from one measurement system to another.
   a. Convert between basic units of measurement from one system to another system (e.g., inches to centimeters, kilometers to miles, pounds to kilograms).

2. Convert within a single measurement system.
   a. Convert between basic units of measurement within a system (e.g., inches to feet, square inches to square feet, grams to milligrams).

C. Measurement involving geometry and algebra
1. Find the perimeter and area of two-dimensional figures.
   a. Describe the difference between perimeter and area of two-dimensional figures and the units of measurement used in their calculation.
   b. Solve problems involving perimeter and area of two-dimensional simple and composite figures with some unknown dimensions (e.g., triangles, quadrilaterals, circles).
   c. Solve problems involving the distance between two points in the coordinate plane and make algebraic and geometric connections.
2. Determine the surface area and volume of three-dimensional figures.
   a. Describe the difference between surface area and volume of three-dimensional figures and the relationship in the units of measurement used in their calculation.
   b. Solve problems involving surface area and volume of three-dimensional simple and composite figures with some unknown dimensions, including prisms, pyramids, cylinders, cones, and spheres.

3. Determine indirect measurements of figures using scale drawings, similar figures, the Pythagorean Theorem, and basic trigonometry.
   a. Determine how changes in dimension affect the perimeter, area, and volume of common geometric figures and solids.
   b. Solve problems using proportional relationships in similar two-dimensional and three-dimensional figures to determine unknown measurements.
   c. Determine unknown sides and angles in a right triangle using the Pythagorean Theorem and basic trigonometry.

D. Measurement involving statistics and probability

1. Compute and use measures of center and spread to describe data.
   a. Select, compute, and justify measurements of center (e.g., mean, median, mode) based on the data set and other influential information.
   b. Select, compute, and justify measurements of variation (e.g., range, IQR, percentiles, variance, standard deviation) based on the data set and other influential information.
   c. Calculate weighted averages, indices, and ratings.

2. Apply probabilistic measures to practical situations to make an informed decision.
   a. Justify decisions made from probability measures from a set of data.
   b. Interpret given probability measures in a problem.
   c. Use and interpret a normal distribution as a mathematical model of measurement for summarizing some sets of data.

V. Probabilistic Reasoning

A. Counting principles

1. Determine the nature and the number of elements in a finite sample space.
   a. Make lists, tables, and tree diagrams to represent all possible outcomes in determining specifics of the sample space.
   b. Determine the number of ways an event may occur using combination and permutation formulas and the Fundamental Counting Principle.

B. Computation and interpretation of probabilities

1. Compute and interpret the probability of an event and its complement.
   a. Conduct an experiment or simulation to compute the empirical probability of an event and its complement.
   b. Compute and interpret the theoretical probability of a simple event and its complement.
   c. Compare the empirical and theoretical probabilities of an event (e.g., experimental probabilities converge to theoretical probability as the number of trials increases).

2. Compute and interpret the probability of conditional and compound events.
   a. Distinguish between independent and dependent events.
   b. Explain the meaning of conditional probability and know when to use it.
c. Compute conditional probability.
d. Compute the probability of compound events using tree diagrams, tables, and other methods.
e. Compute the probability for dependent or independent compound events.

VI. Statistical Reasoning

A. Data collection
1. Plan a study.
   a. Determine question(s) that can be answered with data.
   b. Explain the difference between observational and experimental studies.
   c. Design and employ a plan of study to collect appropriate data.
   d. Use a variety of sampling methods (e.g., census, systematic sampling, random vs. non-random sampling).
   e. Identify sampling techniques used in our world (e.g., political polls, medical studies) and determine possible sources of bias.
   f. Compare and contrast data variability using different sampling methods.

B. Describe data
1. Determine types of data.
   a. Recognize and describe the differences between quantitative and qualitative data.
   b. Recognize and describe univariate and bivariate data.

2. Select and apply appropriate visual representations of data.
   a. Organize and construct graphical displays of data (e.g., line plots, bar graphs, histograms, box plots, scatter plots) to describe the distribution of data.
   b. Read and interpret graphical displays of data.

3. Compute and describe summary statistics of data.
   a. Calculate, describe, and use the appropriate measure of center (e.g., mean, median, mode) and spread (e.g., range, IQR, percentiles, variance, standard deviation).
   b. Describe the effect of outliers on summary statistics.

4. Describe patterns and departure from patterns in a set of data.
   a. Describe any natural variability evident in the results within the context of the situation.
   b. Describe any influences that may have induced variability within the context of the situation.

C. Read, analyze, interpret, and draw conclusions from data
1. Make predictions and draw inferences using summary statistics.
   a. Make a prediction about long-run behavior (e.g., coin toss).
   b. Draw conclusions from analyzing a set of data.

2. Analyze data sets using graphs and summary statistics.
   a. Analyze and compare distributions by describing similarities and differences of centers and spreads within and between data sets.
   b. Analyze and describe similarities and differences by comparing graphical distributions (e.g., parallel box plots, back-to-back stem-leaf plots) within and between data sets.

3. Analyze relationships between paired data using spreadsheets, graphing calculators, or statistical software.
a. Describe relationship and trend of paired data observed from scatter plots in the context of the situation.

b. Choose an appropriate linear or non-linear regression model to fit paired data based on graphical analysis.

c. Make a prediction using the appropriate regression model and describe any limitations to the calculated prediction.

4. Recognize reliability of statistical results.

a. Evaluate media reports by analyzing the study design, data source, graphical representation of data, and analyzed data results reported (or not reported).

b. Describe generalizations and limitations of results from observational studies, experiments, and surveys.

c. Identify and explain misleading uses of data.

d. Describe the reliability of statistical results from a set of data.

B. Analysis of functions

1. Understand and analyze features of a function.

a. Understand functional notation and evaluate a function at a specified point in its domain.

b. Determine the domain and range of a function defined by a table of values, graph, symbols, or verbal description.

c. Approximate or determine the x- and y-values of a function given in tabular, graphical, symbolic, or verbal form.

d. Determine and explain if a function, defined verbally or given in tabular, graphical, or symbolic form, is one-to-one.

2. Algebraically construct and analyze new functions.

a. Determine the domain and range of a combination or composition of two functions.

b. Formulate the composition of two functions.

c. Apply basic transformations to parent functions [e.g., af(x), f(x)+b, f(x+c)] and interpret the results verbally and graphically.

d. Analyze the effects of parameter changes of basic functions, [e.g., f(x)=mx+b, where m and/or b changes].

e. Analyze and apply piece-wise defined functions (e.g., step functions).
f. Determine the inverse function of a given function in tabular, symbolic, or graphical form, if it exists (e.g., the inverse of an exponential function is a logarithmic function).

g. Use properties of inverse functions to solve problems (e.g., inverse trigonometric functions to find angles in a right triangle).

C. Model real world situations with functions

1. Apply known function models.
   a. Apply a linear model for a situation represented by a constant rate of change.
   b. Apply given quadratic models to solve problems (e.g., area, velocity, projectile motion).
   c. Apply exponential models (e.g., compound interest, growth and decay models) to solve problems.
   d. Apply proportional or inverse variation models to solve problems.
   e. Recognize and solve problems that can be modeled using a system of two equations in two variables, such as mixture problems.

2. Develop a function to model a situation.
   a. Analyze a situation algebraically or graphically and determine if the relationship suggests a linear trend.
   b. Use technology to determine a linear regression model for a given situation.
   c. Identify real world situations that can be modeled by functions (e.g., situations in science, business, economics).

VIII. Problem Solving and Reasoning

A. Mathematical problem solving
   1. Analyze given information.
      a. Extract needed facts and relationships from given information.

b. Identify what is known, not known, and what one wants to know in a problem.

c. Distinguish relevant from irrelevant information in a given situation.

d. Determine the problem(s) to be solved.

e. Identify additional information needed to reach a solution.

f. Test ideas with specific cases.

2. Formulate a plan or strategy.
   a. Select or develop an appropriate problem-solving strategy (e.g., drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, working backwards).
   b. Identify needed algorithms or formulas.
   c. Determine the nature of a possible solution and the degree of precision required.

3. Determine a solution.
   a. Make and test conjectures.
   b. Find an approximate solution with or without technology.
   c. Identify and solve sub-problems.
   d. Use multiple representations (e.g., analytic, numerical, verbal, and graphical) to support a solution.

4. Justify the solution.
   a. Provide a clear explanation of the reasoning used to determine a solution.
   b. Evaluate the reasonableness of the solution in the context of the original problem.
   c. Verify a general solution in special cases.
   d. Review and check strategies and calculations, using an alternative approach when possible.
   e. Demonstrate an understanding of the mathematical ideas behind the steps of a solution, not just the solution.
5. Evaluate the problem-solving process.
   a. Reflect on the problem-solving process and use mathematical knowledge to evaluate its effectiveness.
   b. Recognize that a mathematical problem can be solved in a variety of ways.
   c. Consider extensions and generalizations of the problem, process, or solution.

B. Logical reasoning
1. Develop and evaluate convincing arguments.
   a. Use examples to formulate conjectures.
   b. Use counterexamples to refute conjectures.
   c. Determine the validity of a conditional statement, its converse, its inverse, and its contrapositive.

2. Use various types of reasoning.
   a. Use inductive reasoning to formulate a conjecture.
   b. Use deductive reasoning to prove a statement or validate a conjecture.
   c. Use geometric and visual reasoning.
   d. Use multiple representations (e.g., analytic, numerical, verbal, graphical) to support an argument.

C. Real world problem solving
1. Formulate a solution to a real world situation based on the solution to a mathematical problem.
   a. Make simplifying assumptions about a real world situation to formulate and solve an idealized mathematical problem.
   b. Convert given information into an appropriate mathematical model.
   c. Interpret results of the mathematical problem in terms of the original real world situation.

2. Use a function to model a real world situation.
   a. Choose a function suitable for modeling a real world situation presented using words or data.
   b. Determine and interpret the meaning of rates of change, intercepts, zeros, extrema, and trends.
   c. Use an appropriate linear or non-linear function (e.g., quadratic, exponential).
   d. Use a sequence expressed in recursive or closed form.

3. Evaluate the problem-solving process.
   a. Evaluate a real world solution for accuracy and effectiveness.
   b. Compare and analyze various methods for solving a real world problem.

IX. Communication and Representation

A. Language, terms, and symbols of mathematics
1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.
   a. Use variables to represent quantities in contextual situations.
   b. Analyze problem situations and represent them using algebraic expressions and equations.
   c. Use and understand the many ways an "=" sign is used (e.g., to state a definition or formula; to represent an identity; to express a conditional equation; to identify constant and variable terms in expressions, equations, and inequalities).
   d. Understand and use interval, set, and function notation.
   e. Understand that certain symbols and words can have multiple meanings [e.g., (1, 2) can represent a point or an interval].
2. Use mathematical language to represent and communicate the mathematical concepts in a problem.
   a. Represent information in a problem using algebraic expressions, equations, and inequalities.
   b. Recognize contextual problems represented by linear and non-linear models.

   a. Use inductive and deductive reasoning to reach valid conclusions.
   b. Write the converse, inverse, and contrapositive of any given conditional statement.

B. Interpretation of mathematical work

1. Model and interpret mathematical ideas and concepts using multiple representations.
   a. Make tables of inputs and outputs for mathematical relations/functions.
   b. Write symbolic representations for a verbal description of a relationship.
   c. Construct visual representations (e.g., a graph) of relationships.
   d. Describe orally or in written format the behavior of a mathematical idea using graphs, diagrams, tables, and algebraic representations.
   e. Represent inequalities using graphs, interval notation, and set notation.
   f. Use multiple representations of rate of change.

2. Summarize and interpret mathematical information provided orally, visually, or in written form within the given context.
   a. Interpret mathematical information in an article from a media source.
   b. Summarize mathematical information given orally and visually in a media report.

C. Presentation and representation of mathematical work

1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and words.
   a. Communicate ideas mathematically using symbols (e.g., equal signs, parentheses, subscripts, superscripts, order relations, set notation).
   b. Develop geometric models to represent concepts and relationships (e.g., scatter plots).
   c. Recognize and explain the meaning of information presented using mathematical notation.

2. Create and use representations to organize, record, and communicate mathematical ideas.
   a. Use Venn diagrams to represent sets of real numbers, surveys, and other set relationships.
   b. Show solutions of equations and inequalities, and solutions of systems of equations and inequalities, using the real number line and rectangular coordinate system.
   c. Construct and use graphic organizers (e.g., tables, bubble maps, Venn diagrams, tree diagrams).

3. Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.
   a. Explain reasoning in both oral and written forms using notation, terminology, and logic.
b. Communicate reasons associated with performing steps in algebraic methods (e.g., explaining why a quadratic equation must be written in standard form first when solving by factoring).

c. Identify units associated with any variables and constants used in a problem solution.

a. Model a given real world situation using an appropriate combination of sketches, graphs, and algebraic expressions.

b. Describe a given real world situation in algebraic terms, use that description to produce a geometric description, and vice-versa.

c. Connect mathematically created tables, graphs, and functions to fit real life situations (e.g., download data from the Internet).

X. Connections

A. Connections among the strands of mathematics

1. Connect and use multiple strands of mathematics in situations and problems.

   a. Represent a geometric two-dimensional figure on the rectangular coordinate plane using a set of equations or inequalities.

   b. Connect the concepts of ratios, rates, proportions, and percents (e.g., show slope as constant rate of change using similar triangles).

   c. Compare and contrast different mathematical concepts and procedures that could be used to complete a particular task.

   d. Combine appropriate numeric, algebraic, geometric, and statistical/probabilistic methods to solve a given problem.

2. Connect mathematics to the study of other disciplines.

   a. Use mathematical models to solve problems in areas such as science, business, and economics.

   b. Use applications of mathematics (e.g., carbon dating, exponential population growth, amortization tables).

   c. Use geometric concepts and properties to solve problems in fields such as art and architecture.

B. Connections of mathematics to nature, real world situations, and everyday life

1. Use multiple representations to demonstrate links between mathematical and real world situations.