CHAPTER 2: Standards for Introductory College Mathematics

These standards provide a new vision for introductory college mathematics—a vision whereby students develop intellectually by learning central mathematical concepts in settings that employ a rich variety of instructional strategies.
Mathematics and its applications should permeate the undergraduate curriculum. Mathematics programs must demonstrate connections both among topics within mathematics and between mathematics and other disciplines. Introductory college mathematics should link students' previous mathematical experiences with the mathematics necessary to be successful in careers, to be productive citizens, and to pursue lifelong learning.

Adult students entering introductory college mathematics programs today bring a rich diversity of experiences. This diversity challenges educators to define clear goals and standards, develop effective instructional strategies, and present mathematics in appropriate contexts. Institutions, departments, and individual faculty must take active roles in addressing the needs of diverse students, in providing a supportive environment, and in improving curricular and instructional strategies. The standards presented in this chapter unite many different mathematical experiences and guide the development of a multidimensional mathematics program.

The standards are based on research evidence and the best judgment of the educators who contributed to this document. They provide goals for introductory college mathematics programs and guidelines for selecting content and instructional strategies for accomplishing these goals. Given the diversity of students and institutions, it is expected that the methods used to implement the standards will vary across higher education and even within institutions.

**FRAMEWORK FOR MATHEMATICS STANDARDS**

The standards presented in this document are consistent with frameworks presented in other mathematics reform initiatives and are intended to affect every aspect of introductory college mathematics. The standards are in three categories:

- **Standards for Intellectual Development** address desired modes of student thinking and represent goals for student outcomes.

- **Standards for Content** provide guidelines for the selection of content that will be taught throughout introductory college mathematics.

- **Standards for Pedagogy** recommend the use of instructional strategies that provide for student activity and interaction and for student constructed knowledge.

This framework for mathematics instruction will enable all students to widen their views of the nature and value of mathematics and to become more productive citizens.

**STANDARDS FOR INTELLECTUAL DEVELOPMENT**

At the conclusion of their introductory collegiate studies, all students should have developed certain general intellectual mathematical abilities as well as other competencies and knowledge. Introductory college courses in English, psychology, chemistry, or history attempt to broaden an existing educational foundation. In a similar way, an introductory college mathematics program should help students see mathematics as an enriching and empowering discipline.

"Another obstacle to change is the belief held by many mathematicians that the ultimate result of the current movement to revise teaching methods and curricula will be a watered-down mathematics program that is neither effective nor rigorous. They believe that many of the students who leave these courses will not have the mathematical skills necessary for our society and that mathematics majors will not have the experiences necessary for further study in graduate-level mathematics. Research in mathematics education at the collegiate level should produce evidence that students can develop rich concepts from advanced mathematics as they use technology and learn mathematics in alternative settings."

Mary Kim Prichard,
Mathematics Teacher,

"Art and music students at all levels have the opportunity to be creative. Mathematics students should have that same opportunity."

Uri Treisman,
University of Texas, Austin
Standard I-1: Problem Solving

**Students will engage in substantial mathematical problem solving.**

Students will use problem-solving strategies that require persistence, the ability to recognize inappropriate assumptions, and intellectual risk taking rather than simple procedural approaches. These strategies should include posing questions; organizing information; drawing diagrams; analyzing situations through trial and error, graphing, and modeling; and drawing conclusions by translating, illustrating, and verifying results. The students should be able to communicate and interpret their results.

Emphasizing problem solving will make mathematics more meaningful to students. The problems used should be relevant to the needs and interests of the students in the class. Such problems provide a context as well as a purpose for learning new skills, concepts, and theories.

Standard I-2: Modeling

**Students will learn mathematics through modeling real-world situations.**

Students will participate in the mathematical modeling of situations from the world around them and use the models to make predictions and informed decisions. Swetz (1991) describes the modeling process as "(1) identifying the problem, including the conditions and constraints under which it exists; (2) interpreting the problem mathematically; (3) employing the theories and tools of mathematics to obtain a solution to the problem; (4) testing and interpreting the solution in the context of the problem; and (5) refining the solution techniques to obtain a 'better' answer to the problem under consideration, if necessary" (pp. 358-359). In some cases, faculty may select problem situations and ask students to collaborate on the development of models. In other cases, students may be asked to evaluate previously developed models. Does the model behave as intended in that the equations fit the assumptions of the model? How well does the model agree with the real world it is supposed to represent? Does the model perform well on a data set different from the one for which it was developed? Whether students develop their own models or evaluate models that are given to them, they should look beyond how well a proposed model fits a set of data and attempt to provide mathematical or scientific reasons for why the model is valid.

Standard I-3: Reasoning

**Students will expand their mathematical reasoning skills as they develop convincing mathematical arguments.**

Students will regularly apply inductive and deductive reasoning techniques to build convincing mathematical arguments. They will develop conjectures on the basis of past experiences and intuition and test these conjectures by using logic and deductive and inductive proof, by framing examples and counterexamples, and by probabilistic and statistical reasoning. They will explore the meaning and role of mathematical identities, support them graphically or numerically, and verify them algebraically or geometrically. Finally, students will judge the validity of mathematical arguments and draw appropriate conclusions.
Standard I-4: Connecting With Other Disciplines

Students will develop the view that mathematics is a growing discipline, interrelated with human culture, and understand its connections to other disciplines.

If students are to gain a sense that mathematics is a growing discipline, course content must include topics developed since the eighteenth century. Topics such as algorithms needed for computer-based solution processes, the use of probability in understanding chance and randomization, and the applications of non-Euclidean geometries lend themselves to a discussion of who developed the ideas, when they were developed, and what kind of human endeavors motivated their development. Students will need to research sources other than standard mathematics textbooks to determine how mathematics provides a language for the sciences; plays a role in art, music, and literature; is applied by economists; is used in business and manufacturing; and has had an impact on history.

Standard I-5: Communicating

Students will acquire the ability to read, write, listen to, and speak mathematics.

Students will acquire the skills necessary to communicate mathematical ideas and procedures using appropriate mathematical vocabulary and notation. Students will learn to read and listen to mathematical presentations and arguments with understanding. Furthermore, mathematics faculty will adopt instructional strategies that develop both oral and written communication skills within a context of real applications relevant to the particular group of students. As students learn to speak and write about mathematics, they develop mathematical power and become better prepared to use mathematics beyond the classroom.

Standard I-6: Using Technology

Students will use appropriate technology to enhance their mathematical thinking and understanding and to solve mathematical problems and judge the reasonableness of their results.

Students will develop an ability to use technology to enhance their study of mathematics in two ways. First, technology can be used to aid in the understanding of mathematical principles. Shoaf-Grubbs (1994) found that graphing calculators provide "a means of concrete imagery that gives the student new control over her learning environment and over the pace of that learning process. It relieves the need to emphasize symbolic manipulation and computational skills and supports an active exploration process of learning and understanding the concepts behind the mathematics" (p. 191). In general, students can use technology to test conjectures, explore ideas, and verify that theorems are true in specific instances. For example, students can solve quadratic equations and inequalities graphically and then use their knowledge of the graphical solution to clarify the algebraic approach (Hector, 1992).

Second, students will use technology naturally and routinely as a tool to aid in the solution of realistic mathematical problems. "Those who use mathematics in the workplace—accountants, engineers, scientists—rarely use paper-and-pencil

"Writing can help mathematics students in many different ways. Students who are required to write must do considerable thinking and organizing of their thoughts before they write, thus crystallizing in their minds the concepts studied. After completion of the written work, it is then available for their own use in later studies, and it may also be shown to other students who have difficulty with the same concept. Finally, the entire process will give students valuable practical experience in expressing their thoughts in writing, a skill that they will most certainly need in any future position of responsibility."

Marvin L. Johnson,
Mathematics Teacher,
February 1983, p. 117.
Technology should be used to enhance the study of mathematics but should not become the main focus of instruction. The amount of time that students spend learning how to use computers and calculators effectively must be compatible with the expected gain in learning mathematics.

"The view of mathematical knowing as a practice (not in the sense of drill-and-practice) is supported by recent trends in the philosophy of mathematics... to understand what mathematics is, one needs to understand the activities or practices of persons who are makers or users of mathematics."
Edward A. Silver in Selected Lectures from the 7th International Congress on Mathematical Education (Quebec, August 17-23, 1992), p. 317.

procedures anymore. . . . Electronic spreadsheets, numerical analysis packages, symbolic computer systems, and sophisticated computer graphics have become the power tools of mathematics in industry" (NRC, 1989, p.1). In addition, graphing calculators, dynamic geometry software, matrix software, and statistical packages should be included among the technology staples to be used by students.

Technology should be used to enhance the study of mathematics but should not become the main focus of instruction. The amount of time that students spend learning how to use computers and calculators effectively must be compatible with the expected gain in learning mathematics.

Standard I-7: Developing Mathematical Power

Students will engage in rich experiences that encourage independent, nontrivial exploration in mathematics, develop and reinforce tenacity and confidence in their abilities to use mathematics, and inspire them to pursue the study of mathematics and related disciplines.

All students will have opportunities to be successful in doing meaningful mathematics that fosters self-confidence and persistence. They will engage in solving problems that do not have unique answers but, rather, provide experiences that develop the ability to conduct independent explorations. At the same time, they will learn to abstract mathematical principles in order to promote transfer of problem-solving strategies among a variety of contexts (Druckman & Bjork, 1994) and to better appreciate mathematics as a discipline. Furthermore, they will develop an awareness of careers in mathematics and related disciplines and have a vision of themselves using mathematics effectively in their chosen fields.

STANDARDS FOR CONTENT

Mathematics education has traditionally focused on content knowledge. Within this tradition, "knowing mathematics" meant knowing certain pieces of subject matter. This document takes the position that knowing mathematics means being able to do mathematics and that problem solving is the heart of doing mathematics. The successful problem solver can view the world from a mathematical perspective (Schoenfeld, 1992).

Students gain the power to solve meaningful problems through in-depth study of specific mathematics topics. When presented in the context of applications, abstract topics grow naturally out of the need to describe or represent the patterns that emerge. In general, emphasis on the meaning and use of mathematical ideas must increase, and attention to rote manipulation must decrease.

The content standards that follow are not meant to outline a set of courses. Rather, they are strands to be included in an introductory mathematics program in whatever structural form it may take. The specific themes were selected so that adult students can develop the knowledge and skills needed to function as productive workers and citizens as well as be equipped to pursue more advanced study in mathematics and other disciplines.

Standard C-1: Number Sense

Students will perform arithmetic operations, as well as reason and draw conclusions from numerical information.
Number sense includes the ability to perform arithmetic operations, to estimate reliably, to judge the reasonableness of numerical results, to understand orders of magnitude, and to think proportionally. Suggested topics include pattern recognition, data representation and interpretation, estimation, proportionality, and comparison.

**Standard C-2: Symbolism and Algebra**

Students will translate problem situations into their symbolic representations and use those representations to solve problems.

Students will move beyond concrete numerical operations to use abstract concepts and symbols to solve problems. Students will represent mathematical situations symbolically and use a combination of appropriate algebraic, graphical, and numerical methods to form conjectures about the problems. Suggested topics include derivation of formulas, translation of realistic problems into mathematical statements, and the solution of equations by appropriate graphical, numerical, and algebraic methods.

**Standard C-3: Geometry**

Students will develop a spatial and measurement sense.

Geometry is the study of visual patterns. Every physical object has a shape, so every physical object is geometric. Furthermore, mathematical objects can be pictured geometrically. For example, real numbers are pictured on a number line, forces are pictured with vectors, and statistical distributions are pictured with the graphs of curves. Modern dynamic geometry software allows for efficient integration of geometric concepts throughout the curriculum using geometric visualization.

Students will demonstrate their abilities to visualize, compare, and transform objects. Students will develop a spatial sense including the ability to draw one-, two-, and three-dimensional objects and extend the concept to higher dimensions. Their knowledge of geometry will enable them to determine particular dimensions, area, perimeter, and volume involving plane and solid figures. Suggested topics include comparison of geometric objects (including congruence and similarity), graphing, prediction from graphs, measurement, and vectors.

**Standard C-4: Function**

Students will demonstrate understanding of the concept of function by several means (verbally, numerically, graphically, and symbolically) and incorporate it as a central theme into their use of mathematics.

Students will interpret functional relationships between two or more variables, formulate such relationships when presented in data sets, and transform functional information from one representation to another. Suggested topics include generalization about families of functions, use of functions to model realistic problems, and the behavior of functions.

"The context engages them [the students], and when they are engaged they think. . . . We have a lot of sterile problems, like 'add these monomials.' Well, they got all kind of weird answers on that because it doesn't mean anything to them. . . . Mathematics interfaces with ordinary life in so many ways we don't have to be stilted in formulating problems for students."

Gloria Gilmer, quoted in Multiculturalism in Mathematics: Historical Truth or Political Correctness by Allyn Jackson. Article appears in Heeding the Call for Change, MAA, 1992, pp. 121-132.
Standard C-5: Discrete Mathematics

Students will use discrete mathematical algorithms and develop combinatorial abilities in order to solve problems of finite character and enumerate sets without direct counting.

Problem situations in the social and behavioral sciences, business, computing, and other areas frequently do not exhibit the continuous nature so readily treated by techniques traditionally studied in introductory college mathematics. Rather, the problems involve discrete objects and focus on determining a count (Dossey, 1991; Hart, 1991). This standard does not imply that recently developed college courses in discrete mathematics are included in introductory college mathematics. Such courses commonly require precalculus or calculus as prerequisites. The standard echoes the recommendations made in the NCTM Standards (NCTM, 1989) and in Reshaping College Mathematics (Siegel, 1989); namely, the conceptual framework of discrete mathematics should be integrated throughout the introductory mathematics curriculum in order to improve students' problem-solving skills and prepare them for the study of higher levels of mathematics as well as for their careers. Topics in discrete mathematics include sequences, series, permutations, combinations, recursion, difference equations, linear programming, finite graphs, voting systems, and matrices.

Standard C-6: Probability and Statistics

Students will analyze data and use probability and statistical models to make inferences about real-world situations.

The basic concepts of probability and descriptive and inferential statistics should be integrated throughout the introductory college mathematics curriculum at an intuitive level. Students will gather, organize, display, and summarize data. They will draw conclusions or make predictions from the data and assess the relative chances for certain events happening. Suggested topics include basic sampling techniques, tabulation techniques, creating and interpreting charts and graphs, data transformation, curve fitting, measures of center and dispersion, simulations, probability laws, and sampling distributions.

Standard C-7: Deductive Proof

Students will appreciate the deductive nature of mathematics as an identifying characteristic of the discipline, recognize the roles of definitions, axioms, and theorems, and identify and construct valid deductive arguments.

The dependence of mathematics on deductive proof sets it apart as a unique area of human endeavor. While not being the main focus of instruction in introductory college mathematics, mathematical proofs, including indirect proofs and mathematical induction, will be introduced where they will enhance student understanding of mathematical concepts. Students will engage in exploratory activities that will lead them to form statements of conjecture, test them by seeking counterexamples, and identify and, in some instances, construct arguments verifying or disproving the statements.
STANDARDS FOR PEDAGOGY

One of the most widely accepted ideas within the mathematics community is that students should understand mathematics as opposed to thoughtlessly grinding out answers.

But achieving this goal has been like searching for the Holy Grail. There is a persistent belief in the merits of the goal but designing school learning environments that successfully promote understanding has been difficult. (Hiebert & Carpenter, 1992, p. 65)

Constructivism [see Crocker (1991)], which has become a popular theory for linking teaching to student learning, is based on the premise that knowledge cannot be "given" to students. Rather, it is something that they must construct for themselves. However, Resnick and Klopfer (1989) are quick to point out that constructivism does not imply that faculty should get out of the way and let students learn by themselves. All of the traditional questions remain: "how to present and sequence information, how to organize practice and feedback, how to motivate students, how to integrate laboratory activities with other forms of learning, and how to assess learning" (p. 4). "The goal is to stimulate and nourish students' own mental elaborations of knowledge and to help them grow in their capacity to monitor and guide their own learning and thinking" (p. 4).

While constructivist theories may be interpreted differently by different educators and accepted to varying degrees, Brophy and Good (1986) point out that educational research shows that instructional strategies, be they constructivist or not, have a dramatic impact on what students learn. Two themes cut across research findings: "One is that academic success is influenced by the amount of time that students spend on appropriate academic tasks. The second is that students learn more efficiently when their teachers structure new information for them and help them to relate it to what they already know" (p. 366).

The standards for pedagogy that follow are compatible with the constructivist point of view. They recommend the use of instructional strategies that provide for student activity and student-constructed knowledge. Furthermore, the standards are in agreement with the instructional recommendations contained in Professional Standards for Teaching Mathematics (NCTM, 1991).

Standard P-1: Teaching with Technology

Mathematics faculty will model the use of appropriate technology in the teaching of mathematics so that students can benefit from the opportunities it presents as a medium of instruction.

The use of technology is an essential part of an up-to-date curriculum. Faculty will use dynamic computer software to aid students in learning mathematics concepts and will model the appropriate use of technology as tools to solve mathematical problems. The effort spent on teaching students to use technology should be an investment in their future ability to use mathematics. Emphasis should be placed on the use of high-quality, flexible tools that enhance learning and tools they are likely to encounter in future work.

In addition, faculty will use technology as a medium of instruction. Instructional media such as videotapes and computers allow students to progress at their own pace and make mistakes without fearing peer or professional judgment.

The use of technology within the instructional process should not require

"Because the classroom is such a familiar and uncomplicated place for most people, it's hard to see that developing and executing a good curriculum is about as simple as composing and performing a good symphony."


"The graphing calculator is a wonderful tool for exploring and learning. It allows me to 'play around' by comparing graphs and changing variables. It gives an immediate picture of a particular function. It allows a lot of 'what if?' situations—that is great!"

A student's comment
more time. In fact, the use of technology, coupled with a decreased emphasis in some traditional content areas, should provide the time that is needed to implement the needed reforms in mathematics education.

Standard P-2: Interactive and Collaborative Learning

Mathematics faculty will foster interactive learning through student writing, reading, speaking, and collaborative activities so that students can learn to work effectively in groups and communicate about mathematics both orally and in writing.

Mathematical literacy is achieved through an understanding of the signs, symbols, and vocabulary of mathematics. This is best accomplished when students have an opportunity to read, write, and discuss mathematical problems and concepts (NCTM, 1989). The following types of experiences will be encouraged in college classrooms: cooperative learning (Crocker, 1992; Becker & Pence, 1994); oral and written reports presented individually or in groups; writing in journals; open-ended projects; and alternative assessment strategies such as essay questions and portfolios (Leitzel, 1991; NCTM, 1991).

Standard P-3: Connecting with Other Experiences

Mathematics faculty will actively involve students in meaningful mathematics problems that build upon their experiences, focus on broad mathematical themes, and build connections within branches of mathematics and between mathematics and other disciplines so that students will view mathematics as a connected whole relevant to their lives.

Mathematics must not be presented as isolated rules and procedures. Students must have the opportunity to observe the interrelatedness of scientific and mathematical investigation and see first-hand how it connects to their lives. Students who understand the role that mathematics has played in their cultures and the contributions of their cultures to mathematics are more likely to persevere in their study of the discipline. Making mathematics relevant and meaningful is the collective responsibility of faculty and producers of instructional materials. Administrators have the responsibility of supporting faculty in this effort.

Standard P-4: Multiple Approaches

Mathematics faculty will model the use of multiple approaches—numerical, graphical, symbolic, and verbal—to help students learn a variety of techniques for solving problems.

Mathematical power includes the ability to solve many types of problems. Solutions to complex problems require a variety of techniques and the ability to work through open-ended problem situations (Pollak, 1987). College mathematics faculty will provide rich opportunities for students to explore complex problems, guide them to solutions through multiple approaches, and encourage both oral and written responses. This will motivate students to go beyond the mastery of basic operations to a real understanding of how to use mathematics, the meaning of the answers, and how to interpret them (NRC, 1989).
Mathematics faculty will provide learning activities, including projects and apprenticeships, that promote independent thinking and require sustained effort and time so that students will have the confidence to access and use needed mathematics and other technical information independently, to form conjectures from an array of specific examples, and to draw conclusions from general principles.

Mathematics faculty will assign open-ended classroom and laboratory projects. In addition, they will help their institutions form partnerships with area business and industry to develop opportunities for students to have realistic career experiences (Reich, 1993). Such activities will enable students to acquire the confidence to access needed technical information, and independently use mathematics in appropriate and sensible ways.

GUIDELINES FOR ACHIEVING THE STANDARDS

Faculty who teach introductory college mathematics must increase the mathematical power of their students. This power increases the students' options in educational and career choices and enables them to function more effectively as citizens of a global community where the opportunities offered by science and technology must be considered in relation to human and environmental needs. In order to achieve these goals, mathematics education at the introductory level requires reform in curriculum and pedagogy.

The idea that mathematics competence is acquired through a curriculum that is carefully structured to include the necessary content at the appropriate time and the use of diverse instructional strategies is an underlying principle of this document. The following tables provide guidance for change in the content and pedagogy of introductory college mathematics. When items are marked for decreased attention, that does not necessarily mean that they are to be eliminated from mathematics education. Rather, it may mean that they should receive less attention than in previous years, or that their in-depth study should be moved to more advanced courses where they may be immediately applied. On the other hand, increased emphasis must be placed on the items listed in the increased attention column in order to achieve the goals set forth in this document.

“A large number of rich contexts for mathematics instruction is now available. . . . The main problem is that of implementation, which requires a fundamental change in teaching attitudes before it can be solved.”

<table>
<thead>
<tr>
<th>Increased Attention</th>
<th>Decreased Attention</th>
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</thead>
<tbody>
<tr>
<td>pattern recognition, drawing inferences</td>
<td>rote application of formulas</td>
</tr>
<tr>
<td>number sense, mental arithmetic, and estimation</td>
<td>arithmetic drill exercises, routine operations with real numbers</td>
</tr>
<tr>
<td>connections between mathematics and other disciplines</td>
<td>presentation of mathematics as an abstract entity</td>
</tr>
<tr>
<td>integration of topics throughout the curriculum</td>
<td>algebra, trigonometry, analytic geometry, and so forth, as separate courses</td>
</tr>
<tr>
<td>discovery of geometrical relationships through the use of models, technology, and manipulatives</td>
<td>establishing geometric relationships solely through formal proofs</td>
</tr>
<tr>
<td>visual representation of concepts; for example, probability as area under a curve; timelines for annuities and interest; tables for logic and electrical circuits</td>
<td>rote memorization and use of formulas</td>
</tr>
<tr>
<td>integration of the concept of function across topics within and among courses</td>
<td>separate and unconnected units on linear, quadratic, polynomial, radical, exponential, and logarithmic functions</td>
</tr>
<tr>
<td>analysis of the general behavior of a variety of functions in order to check the reasonableness of graphs produced by graphing utilities</td>
<td>paper-and-pencil evaluation of functions and hand-drawn graphs based on plotting points</td>
</tr>
<tr>
<td>connection of functional behavior (such as where a function increases, decreases, achieves a maximum and/or minimum, or changes concavity) to the situation modeled by the function</td>
<td>emphasis on the manipulation of complicated radical expressions, factoring, rational expressions, logarithms, and exponents</td>
</tr>
<tr>
<td>modeling problems of chance by constructing probability distributions or by actual experiment</td>
<td>“cookbook” problem solving without connections</td>
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<tr>
<td>theoretical development of probability theorems</td>
<td></td>
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<tr>
<td>Increased Attention</td>
<td>Decreased Attention</td>
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<tr>
<td>---------------------------------------------------------</td>
<td>----------------------------------------------------------</td>
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<tr>
<td>collection of real data for analysis of both descriptive and inferential statistical techniques</td>
<td>analysis of contrived data</td>
</tr>
<tr>
<td>exploratory graphical analysis as part of inferential procedures</td>
<td>“cookbook” approaches to applying statistical computations and tests which fail to focus on the logic behind the processes</td>
</tr>
<tr>
<td>use of curve fitting to model real data, including transformation of data when needed</td>
<td>reliance on out-of-context functions that are overly simplistic</td>
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<tr>
<td>discussion of the meaning of nonzero correlation and the independence of correlations from any implications of cause and effect</td>
<td>blind acceptance of ( r )</td>
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<tr>
<td>use of statistical software and graphing calculators</td>
<td>paper-and-pencil calculations and four-function calculators</td>
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<tr>
<td>problems related to the ordinary lives of students; for example, financing items that students can afford and statistics related to sports participated in by females as well as by males</td>
<td>problems unrelated to the daily lives of most students; for example, investments of large sums of money in savings or statistics related to sports only played by males</td>
</tr>
<tr>
<td>matrices to organize and analyze information from a wide variety of settings</td>
<td>requiring a system of equations to be solved by three methods</td>
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<tr>
<td>graph theory and algorithms as a means of solving problems</td>
<td>algebraically derived exact answers</td>
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<tr>
<td>Increased Use</td>
<td>Decreased Use</td>
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<td>--------------------------------------------------</td>
<td>------------------------------------</td>
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<tr>
<td>active involvement of students</td>
<td>passive listening</td>
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<tr>
<td>technology to aid in concept development</td>
<td>paper-and-pencil drill</td>
</tr>
<tr>
<td>problem solving and multistep problems</td>
<td>one-step single-answer problems</td>
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<tr>
<td>mathematical reasoning</td>
<td>memorization of facts and procedures</td>
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<tr>
<td>conceptual understanding</td>
<td>rote manipulation</td>
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<tr>
<td>realistic problems encountered by adults</td>
<td>contrived exercises</td>
</tr>
<tr>
<td>an integrated curriculum with ideas developed in context</td>
<td>isolated topic approach</td>
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<tr>
<td>multiple approaches to problem solving</td>
<td>requiring a particular method for solving a problem</td>
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<tr>
<td>diverse and frequent assessment both in class and outside of class</td>
<td>tests and a final exam as the sole assessment</td>
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<tr>
<td>open-ended problems</td>
<td>problems with only one possible answer</td>
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<tr>
<td>oral and written communication to explain solutions</td>
<td>required only short, numerical answers, or multiple-choice responses</td>
</tr>
<tr>
<td>variety of teaching strategies</td>
<td>lecturing</td>
</tr>
</tbody>
</table>

**Summary**

These standards provide a new vision for introductory college mathematics—a vision whereby students develop intellectually by learning central mathematical concepts in settings that employ a rich variety of instructional strategies. To provide a more concrete illustration of these standards, the Appendix contains a set of problems that brings them to life.