Title: Mean Value Theorems and the Geometry of Mean Value Sets for Divergence Form Elliptic PDEs

Abstract: One basic property of harmonic functions is the fact that at any point the value of the function is exactly equal to the average value of the function over a ball (or a sphere) centered at that point. This theorem can be used to establish other basic properties of harmonic functions including the maximum principle, the Harnack inequality, and some important a priori estimates. Standard proofs of the mean value theorem rely on smoothness and symmetry properties of the Laplacian and therefore do not generalize nicely to non-Euclidean spaces or more general elliptic operators. However, in his Fermi lectures on the obstacle problem in 1998, Luis Caffarelli indicated how one could use solutions of specific obstacle problems to construct an elegant proof of the mean value theorem which does not depend on either the smoothness or symmetry properties of the Laplacian. In this talk, I will explain that proof and show how my collaborators and I have used it to generalize the mean value theorem to other settings.

Interested faculty and graduate students are encouraged to attend.