Discrete Mathematics Seminar

Time: Friday, 25 September 2009, 1:00–2:00 PM
Location: 238 Derrick Hall
Title: Depth Bounds for Edge Ideals of Trees
Speaker: Dr. Susan Morey, Mathematics Department

Abstract:

There is a one-to-one correspondence between graphs and square-free monomial ideals generated in degree two. Using this correspondence, combinatorial properties of the graph associated to an ideal can be used to determine algebraic properties of the ideal. In this talk, a lower bound will be given for $\operatorname{depth}(R/I^t)$ for $t \geq 1$ when I is the edge ideal of a tree. A similar bound will be given for small powers t when I is the edge ideal of a graph. The bounds given will be in terms of the diameter of the tree or graph. It is well-known that $\operatorname{depth}(R/I^t)$ stabilizes for large powers t. As a consequence of the depth bound, a lower bound for where the stability occurs will also be given. Algebraic background and definitions will be provided throughout the talk.

This correspondence between degree two monomial ideals and graphs naturally extends to a one-to-one correspondence between square-free monomial ideals and simple hypergraphs, also called clutters, or facets of a simplicial complex. If time permits, conditions will be given under which $\operatorname{depth}(R/I^t)$ is positive for this more general setting. In addition, and time permitting, a combinatorial condition on a hypergraph, which extends results of Herzog and Hibi for edge ideals of bipartite graphs, will be given under which $\operatorname{depth}(R/I)$ is maximal, or R/I is Cohen-Macaulay.