

### Chi-Square Test for Association

Ho: The two qualitative variables are independent (not related)

Ha: The two qualitative variables are dependent (related)

Example:

$$E(n_{1s}) = n * p_1 * p_s$$

n = sample size  
 $p_1$  = probability of the first row  
 $p_s$  = probability of the  $s^{th}$  column  
 $E(n_{1s})$  = expected value

### Chi-Square test statistic:

$$\chi^2 = \frac{(n_{1s} - E(n_{1s}))^2}{E(n_{1s})} + \frac{(n_{2s} - E(n_{2s}))^2}{E(n_{2s})} + \dots + \frac{(n_{xs} - E(n_{xs}))^2}{E(n_{xs})}$$

**OR**

$$\chi^2 = \frac{(\text{Observed value} - \text{Expected value})^2}{\text{Expected value}} + \dots + \frac{(\text{Observed value} - \text{Expected value})^2}{\text{Expected value}}$$

Degrees of freedom = (number of rows - 1) \* (number of columns - 1)

### Rejection Region:

Reject Ho if  $\chi^2 > \chi^2$  critical value

### Chi-Square Test for Goodness of Fit

Chi-Square distribution

$$\frac{(n - 1)s^2}{\sigma^2}$$

n = sample size  
 $s^2$  = sample variance  
 $\sigma^2$  = population variance

Degrees of Freedom = n-1

Example:

$$E(n_1) = n * p_1$$

### Chi-Square test statistic:

$$\chi^2 = \frac{(n_1 - E(n_1))^2}{E(n_1)} + \frac{(n_2 - E(n_2))^2}{E(n_2)} + \dots + \frac{(n_x - E(n_x))^2}{E(n_x)}$$

$$\chi^2 = \frac{(\text{Observed value} - \text{Expected value})^2}{\text{Expected value}} + \dots + \frac{(\text{Observed value} - \text{Expected value})^2}{\text{Expected value}}$$

Degrees of Freedom = (rows - 1) or (columns - 1)

If n is large enough =  $E(n_i) \geq 5$

### Rejection Region:

Reject Ho if  $\chi^2 \geq \chi^2$  critical with alpha and (rows - 1) or (columns - 1) degrees of freedom