

# FACTORING

## Summary of steps for factoring a polynomial:

1. Factor out any common factors.
2. If the polynomial is a binomial, check whether it can be rewritten as the difference of two squares, the difference of two cubes, or the sum of two cubes.
3. If the polynomial is a trinomial, check whether it is a perfect square trinomial. If not, factor by trial and error (guessing), or apply the AC Method.
4. If the polynomial has more than three terms, try factoring by grouping.

## Factoring Out a Common Factor:

Example 1:  $5x + 15 = 5(x + 3)$

Example 2:  $12xy^4 - 27xy^2 + 15xy^3 = 3xy^2(4y^2 - 9 + 5y) = 3xy^2(4y^2 + 5y - 9)$

## The Difference of Two Squares:

Factoring formula:  $x^2 - y^2 = (x + y)(x - y)$

*Note: The sum of two squares  $x^2 + y^2$  is NOT factorable.*

Example 1:  $x^2 - 9 = x^2 - 3^2$   
 $= (x + 3)(x - 3)$

break down in to squares  
factor using formula

Example 2:  $25x^2 - 81 = (5x)^2 - 9^2$   
 $= (5x + 9)(5x - 9)$

break down in to squares  
factor using formula

Example 3:  $x^4 - 81 = (x^2)^2 - 9^2$   
 $= (x^2 - 9)(x^2 + 9)$   
 $= (x^2 - 3^2)(x^2 + 9)$   
 $= (x + 3)(x - 3)(x^2 + 9)$

break down in to squares  
factor using formula  
factor using formula  
simplify

## The Difference and Sum of Two Cubes:

Factoring formulas:  $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$   
 $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

Example 1:  $x^3 - 27 = x^3 - 3^3$   
 $= (x - 3)(x^2 + 3x + 3^2)$   
 $= (x - 3)(x^2 + 3x + 9)$

break down in to cubes  
factor using formula  
simplify

Example 2:  $27x^3 - 8y^3 = (3x)^3 - (2y)^3$   
 $= (3x - 2y)((3x)^2 + (3x)(2y) + (2y)^2)$   
 $= (3x - 2y)(9x^2 + 6xy + 4y^2)$

break down in to cubes  
factor using formula  
simplify

$$\begin{aligned}\text{Example 3: } (A + B)^3 - 8 &= (A + B)^3 - 2^3 \\ &= ((A + B) - 2)((A + B)^2 + (A + B)(2) + 2^2) \\ &= (A + B - 2)(A^2 + 2AB + B^2 + 2A + 2B + 4)\end{aligned}$$

$$\begin{aligned}\text{Example 4: } 216x^{3m} - 27b^{9m+3} &= (6x^m)^3 - (3b^{3m+1})^3 \\ &= (6x^m - 3b^{3m+1})((6x^m)^2 + (6x^m)(3b^{3m+1}) + (3b^{3m+1})^2) \\ &= (6x^m - 3b^{3m+1})(36x^{2m} + 18x^m b^{3m+1} + 9b^{6m+2})\end{aligned}$$

$$\begin{aligned}\text{Example 5: } x^6 - 64 &= x^6 - 2^6 \\ &= (x^3)^2 - (2^3)^2 \\ &= (x^3 + 2^3)(x^3 - 2^3) \\ &= (x + 2)(x^2 - 2x + 4)(x - 2)(x^2 + 2x + 4)\end{aligned}$$

$$\begin{aligned}\text{Example 6: } (x + y)^3 + 27 &= (x + y)^3 + 3^3 \\ &= ((x + y) + 3)((x + y)^2 - (x + y)(3) + 3^2) \\ &= (x + y + 3)(x^2 + 2xy + y^2 - 3x - 3y + 9)\end{aligned}$$

$$\begin{aligned}\text{Example 7: } 8x^3 + 343c^3 &= (2x)^3 + (7c)^3 \\ &= (2x + 7c)((2x)^2 - (2x)(7c) + (7c)^2) \\ &= (2x + 7c)(4x^2 - 14xc + 49c^2)\end{aligned}$$

$$\begin{aligned}\text{Example 8: } 27A^{9n} + 216B^{3n+6} &= (3A^{3n})^3 + (6B^{n+2})^3 \\ &= (3A^{3n} + 6B^{n+2})((3A^{3n})^2 - (3A^{3n})(6B^{n+2}) + (6B^{n+2})^2) \\ &= (3A^{3n} + 6B^{n+2})(9A^{6n} - 18A^{3n}B^{n+2} + 36B^{2n+4})\end{aligned}$$

### Perfect Square Trinomial:

Factoring formulas:  $x^2 + 2xy + y^2 = (x + y)^2$   
 $x^2 - 2xy + y^2 = (x - y)^2$

$$\begin{aligned}\text{Example 1: } 4x^2 + 4xy^2 + y^4 &= (2x)^2 + 2(2x)(y^2) + (y^2)^2 \\ &= (2x + y^2)^2\end{aligned}$$

$$\begin{aligned}\text{Example 2: } 25a^4b^8 - 60a^2b^4c^3 + 36c^6 &= (5a^2b^4)^2 - 2(5a^2b^4)(6c^3) + (6c^3)^2 \\ &= (5a^2b^4 - 6c^3)^2\end{aligned}$$

**AC Method:** For an in-depth description of this procedure see SLAC handout: AC Method.

Example 1:

$$\begin{aligned}20x^2 - 31xy + 12y^2 &= 20x^2 - 16xy - 15xy + 12y^2 && (20)(12) = 2^4 \cdot 3 \cdot 5 = -16 - 15 = -31 \\ &= 4x(5x - 4y) - 3y(5x - 4y) && \text{factor out any common factors} \\ &= (5x - 4y)(4x - 3y) && \text{apply factor by grouping}\end{aligned}$$

Example 2:

$$\begin{aligned}28a^2 - 51ac + 20c^2 &= 28a^2 - 35ac - 16ac + 20c^2 && (28)(20) = 2^4 \cdot 5 \cdot 7 \rightarrow -35 - 16 = -51 \\ &= 7a(4a - 5c) - 4c(4a - 5c) && \text{factor out any common factors} \\ &= (4a - 5c)(7a - 4c) && \text{apply factor by grouping}\end{aligned}$$

## Grouping:

$$\begin{aligned}\text{Example 1: } x^2 - 9y^2 - 3x - 6y &= (x^2 - 9y^2) - (3x + 6y) \\ &= (x^2 - (3y)^2) - 3(x + 2y) \\ &= (x - 3y)(x + 3y) - 3(x + 2y)\end{aligned}$$

$$\begin{aligned}\text{Example 2: } a^2x^2 + b^2y^2 - a^2y^2 - b^2x^2 &= (a^2x^2 - a^2y^2) - (b^2x^2 - b^2y^2) \\ &= a^2(x^2 - y^2) - b^2(x^2 - y^2) \\ &= (x^2 - y^2)(a^2 - b^2) \\ &= (x + y)(x - y)(a + b)(a - b)\end{aligned}$$

## Practice Problems:

 Factor the following polynomials.

1.  $15x^2 + 41x + 14$
2.  $3x^2 - 16x + 5$
3.  $5x^4y^4 - 135xy$
4.  $b^3 + a^2 + b^2 - a^3 - 2ab$
5.  $(a - b)^6 + 125$
6.  $6zx^2 + 2yx - 3zyx - y^2$

## Answers:

1.  $(3x + 7)(5x + 2)$
2.  $(x - 5)(3x - 1)$
3.  $5xy(xy - 3)(x^2y^2 + 3xy + 9)$
4.  $(b - a)(b^2 + ab + a^2 + b - a)$
5.  $((a - b)^2 + 5)((a - b)^4 - 5(a - b)^2 + 25)$
6.  $(2x - y)(3zx + y)$