I plan to discuss a new topic in mathematics called fusion systems, which has applications to finite group theory and algebraic topology. One application, the Martino-Priddy Conjecture in topology, gives a strong connection between a certain topological space associated with a finite group and a fusion system associated with the group. This statement and some extensions have been proved, partly by assuming the classification of finite simple groups. Recently, J. Lynd and I have removed this assumption.