

Probability, Randomness, and Uncertainty

Classical Probability

Probability experiment (trial): Any process in which the result is random in nature (coin, dice, etc)

Outcome: each individual result that is possible

Sample Space- set of all possible outcomes for a given probability experiment

Event- subset of outcomes of the sample space

Subjective Probability- an educated guess regarding the chance that an event will occur, accuracy depends on the expertise of the person giving

Empirical Probability

$$P(E) = \frac{\text{number of times event } E \text{ occurs}}{\text{total number of times the experiment is performed}} = \frac{f}{n}$$

f = frequency n = sample size

Law of large numbers- greater the number of trials, the closer the empirical probability will become to the true probability

Classical Probability (Theoretical):

$$P(E) = \frac{n(E)}{n(S)}$$

*only when outcomes are equally likely

n(E) = number of outcomes of an event n(S) = number of outcomes in the sample size

Probability Rules

$$0 \leq P(E) \leq 1$$

P(S) = 1 an outcome from a sample space must occur

P(∅) = 0 no outcomes from an empty set

Event + complement (opposite of event) = entire sample space

$$P(E) + P(E') = 1$$

$$P(E) = 1 - P(E')$$

Addition rule for probability

$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$$

Mutually Exclusive Events- events that share no outcomes

Addition rule for mutually exclusive events

$$P(E \text{ or } F) = P(E) + P(F)$$

With repetition- outcomes may be repeated

With replacement-placing objects back into consideration

Without repetition-outcomes may not be repeated

Without replacement-first choice is not put back in for consideration

Independent-one event happening does not influence the probability of the other event

Multiplication rule for probability for two independent events

$$P(E \text{ and } F) = P(E) * P(F)$$

Conditional probability- without replacement

Example: P(2nd king | 1st king) → probability of drawing a king second given that a king was drawn first = 3/51

Multiplication rule for dependent events

$$P(E \text{ and } F) = P(E)*P(F | E)$$

Counting Rules

Fundamental counting principle

Can multiply together the number of possible outcomes for each stage in an experiment in order to obtain the total number of outcomes for that experiment

Example: 5 sandwiches and 4 drinks

$5*4=20$ different outcomes

How many ways to rewrite “smith”

$5*4*3*2*1=120$

Factorials

$$n! = n(n-1)(n-2)\dots(2)(1)$$

$$0! = 1$$

Excel: =fact(x)

Permutation

Order in which the objects are chosen are important

$${}_n\text{Pr} = \frac{n!}{(n-r)!}$$

Excel: =permut (n,r)

Combination

Order is not important

$${}^n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Excel: =combin(n,r)

Special permutation

$$\frac{n!}{k_1!k_2!\dots k_i!}$$

Example: how many ways to rearrange the word "Mississippi"

M = 1

I = 4

S = 4

P = 2

$$\frac{11!}{1!4!4!2!} = 34650$$