MATRICES

A **matrix** is a rectangular array of numbers.
The numbers in the array are called the **elements of the matrix**. The array is enclosed with brackets.

- If **m** and **n** are positive integers, then an ***m*** x ***n*** **matrix** (read "m by n") is a matrix that contains m rows (horizontal lines) and n columns (vertical lines).

- An array composed of a single row of numbers is called a **row matrix**.

- An array composed of a single column of numbers is called a **column matrix**.

- A matrix with the same number of rows and columns is said to be a **square matrix**.

- Two matrices are **equal matrices** if they have the same order (***m*** x ***n***) and their corresponding components/elements are equal.

- An *n* x *n* matrix that consists of 1's (ones) on its main diagonal and 0's (zeros) elsewhere is called the **identity matrix of order *n*** and is denoted by I or I_n.

$$I_{3x3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Note: An identity matrix must be square. If A is an *n* x *n* matrix, then the identity matrix has the property that AI = A and IA = A.

- If A is a square matrix and if there exists a matrix A^{-1} such that $AA^{-1} = I = A^{-1}A$, then A^{-1} is called the **inverse** of A.

- Matrix operations include:
  - addition/subtraction of matrices of the same order.
  - scalar multiplication—multiplication of a matrix by a number (usually called "scalar").
  - matrix multiplication—multiplication of a matrix by another matrix.

<u>Examples of Matrices:</u>

2×3 matrix

$$E = \begin{bmatrix} 4 & -4 & 3 \\ 7 & -2 & -5 \end{bmatrix}$$

Row matrix

$$E = \begin{bmatrix} 1 & \frac{3}{7} & -\frac{1}{2} \end{bmatrix}$$

Column matrix

$$T = \begin{bmatrix} .5 \\ .75 \\ .92 \end{bmatrix}$$

Square matrix

$$U = \begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix}$$

Identity matrices

$$I = [1], \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

MATRIX OPERATIONS

1. Matrix Addition and Subtraction

Two matrices can only be added and a matrix can only be subtracted from another if the matrices have the same number of rows and columns.

To add two matrices simply add the elements in corresponding positions.

$$\begin{bmatrix} 5 & 3 \\ 2 & 6 \\ 9 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ -1 & 3 \\ -5 & -6 \end{bmatrix} = \begin{bmatrix} 5+5 & 3+6 \\ 2+(-1) & 6+3 \\ 9+(-5) & 4+(-6) \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 9 \\ 1 & 9 \\ 4 & -2 \end{bmatrix}$$

Note that the sum $\begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 5 & 4 \\ 2 & 7 \end{bmatrix}$ is undefined.

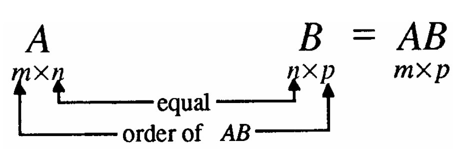
To subtract a matrix from another proceed similarly:

$$\begin{bmatrix} 5 & 3 \\ 2 & 6 \\ 9 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 6 \\ -1 & 3 \\ -5 & -6 \end{bmatrix} = \begin{bmatrix} 5-5 & 3-6 \\ 2-(-1) & 6-3 \\ 9-(-5) & 4-(-6) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -3 \\ 3 & 3 \\ 14 & 10 \end{bmatrix}$$

2. Matrix Multiplication

The product of two matrices is defined only if the number of columns of the first matrix equals the number of rows of the second matrix. That is,



Example 1: Find the product AB where

$$A = \begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix} \quad and \quad B = \begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix}$$

Solution:
First check if the product AB is defined. Since the order of A is 3 x 2 and the order of B is 2 x 2 and the number of columns of A is equal to the number of rows of B we know that the product AB is defined and is of order 3 x 2.

$$\begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix}$$

To find c_{11} we multiply corresponding entries in the first row of A and the first column of B and add these products.

$C_{11} = (-1)(-3) + (3)(-4) = -9$

$$\begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} [-9] & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix}$$

To find c_{12} we multiply corresponding entries in the first row of *A* and the second column of *B* and add these products.

$C_{12} = (-1)(2) + (3)(1) = 1$

$$\begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} -9 & [1] \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix}$$

Continuing this pattern, we find the values

$C_{21} = (4)(-3) + (-2)(-4) = -4$
$C_{22} = (4)(2) + (-2)(1) = 6$
$C_{31} = (5)(-3) + (0)(-4) = -15$
$C_{32} = (5)(2) + (0)(1) = 10$

This product is

$$AB = \begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} -9 & 1 \\ -4 & 6 \\ -15 & 10 \end{bmatrix}$$

Example 2: Find the product CE where

$$C = \begin{bmatrix} 18 & 3 & 25 \end{bmatrix} \quad \text{and} \quad E = \begin{bmatrix} 12 & 15 \\ 45 & 38 \\ 15 & 17 \end{bmatrix}$$

Solution:
The product CE is defined since the number of columns of C is equal to the number of rows of E.

$$CE = \begin{bmatrix} 18 & 3 & 25 \end{bmatrix}\begin{bmatrix} 12 & 15 \\ 45 & 38 \\ 15 & 17 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \end{bmatrix} = \begin{bmatrix} 726 & 809 \end{bmatrix}$$

$$b_{11} = (18)(12) + (3)(45) + (25)(15) = 726$$
$$b_{12} = (18)(15) + (3)(38) + (25)(17) = 809$$

Example 3: Find the product AB where

$$A = \begin{bmatrix} 5 & 2 \\ 4 & 7 \\ 9 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 6 & 1 \\ 9 & 4 \\ 3 & 2 \end{bmatrix}$$

Solution:

The product AB is not defined.

The order of A is 3 x 2 and the order of B is 3 x 2.

Since the number of columns of A is not equal to the number of rows of B, we cannot multiply A by B.

Source: Larson, Roland E. and Hostetler, Robert P. College Algebra, 2nd edition. D.C. Heath and Co., Lexington, 1989.

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