TEXAS STATE UNIVERSITY

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Discrete Mathematics Seminar

Time: Friday, January 29, 2020, 2:15 - 3:15 PM (Central Time)
Title: The Determinant of $\{\pm 1\}$-Matrices and Oriented Hypergraphs
Speaker: Dr. Lucas Rusnak, Department of Mathematics, Texas State University
Zoom Link: Meeting ID: 999 2462 8868, Password: 753321

Abstract:

Signed graphs can be studied through their bidirected orientations where each edge-end receives a value of +1 or -1. An edge is positive if the edge-end orientations have different signs, and negative if the edge-end orientations have the same sign. An oriented hypergraph is a multi-directed generalization of bidirected graphs where each incidence of a hyperedge receives an orientation. Signed graphic techniques are then applied locally within the oriented hypergraph; that is, the adjacencies receive signs, not the edges. Any integer matrix can be regarded as the incidence matrix of an oriented hypergraph with the inclusion of parallel incidences. Hadamard's maximum determinant problem aims to find the maximum determinant of a matrix $\mathbf{H}$ of size n with entries $+1$ and -1.

I will introduce a set of highly symmetric families that each calculate the determinant of a given $\{\pm 1\}$-matrix by using the oriented hypergraphic Laplacian and the incidence-based notion of cycle-covers. Any non-edge-monic family of cycle-covers is shown to vanish in every determinant, while any one of the $n!$ remaining edge-monic families is equivalent to determining the absolute value of the determinant. Hadamard's maximum determinant problem is shown to be equivalent to optimizing the number of locally signed graphic circles of a given sign in any one of these families or across all of these families. Various symmetries regarding orthogonality, equivalence to $\{0, +1\}$-matrices, and their relation to these edge-monic families are shown to be different fundamental circles related by theta-subgraphs.