Mathworks Math Contest
For Middle School Students
Mathworks at Texas State University
October 31, 2012

COVER SHEET

Student First Name:___________________________ Last Name:____________________________

Current Grade in School: ________

Home Address:____

City:_______________________ State: ________ Zip: ________________

Home Phone: (______) ____________________________

E-mail Address: __________________________________

School Name: ____

Teacher:____

Check Math Courses Taken:
□Pre-Algebra □Algebra 1 □Algebra 2 □Geometry

Student Birth date (MM/DD/YYYY): _______ / ________/ __________

Gender: □Male □Female

Are you a U.S. Citizen or Permanent Resident? □Yes □No

Return Completed Test by November 7th to:
Mathworks
601 University Dr., ASBS #110
Texas State Univ.
San Marcos, TX 78666

Test Directions

- You will be given 2 hours to work on the following 15 questions
- **NO** calculators allowed
- Show all your work and how you obtained each answer
- Please mark final answer in box provided for each question
- Attach additional paper to show your work as needed. Do your best and good luck!

1. How many distinct 4 digit numbers can be written using exactly two different digits in each? For example, if *a* is a non-zero digit, and *b* any other digit, then *aabb* is such a number.

2. Mixture *A* is 40% alcohol and 60% water, while Mixture *B* is 90% alcohol and 10% water. A chemist has 4 liters of Mixture *A*, but needs to create a new Mixture *C* that is 60% alcohol. How much of Mixture *A* **should be drained out** and replaced with Mixture *B* to create 4 liters of this new mixture?

3. There are two types of toys, *A* and *B*. Toys of type *A* cost $3 each, and toys of type *B* cost $7 each. If Mary spends $100, how many possibilities are there for the number of type *A* toys she purchased?

4. The number n is a positive integer and $n^2 + 689$ is a perfect square. Find n.

5. Suppose that A,B,C and D are integers 0-9,

$$\begin{array}{rcccc} & & & & A \\ & & & A & B \\ & & A & B & C \\ + & A & B & C & D \\ \hline & 2 & 0 & 1 & 2 \end{array}$$

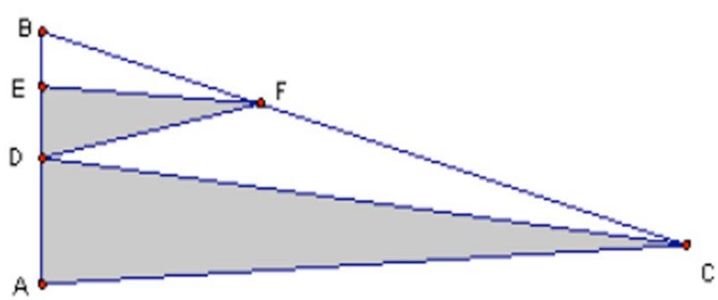
Find $A+B+C+D$

6. Let $a_n = \frac{1+2+3+4+\cdots+n}{1+3+5+7+\cdots+(2n-1)}$. Find the smallest integer n with $a_n - \frac{1}{2} < \frac{1}{2012}$

7. Find the smallest positive integer n such that 2^n+1 is divisible by 11 and 3^n-1 is divisible by 13.

8. Ann repeatedly rolls a pair of six sided dice, one red and one blue. After each toss she plots the point (a, b) in the plane where a is the number rolled on the red die and b is the number rolled on the blue die. She notices that there is no line which contains 6 of her plotted points. What is the largest possible number of points she can have plotted?

9. In the diagram below BF:BC=2:7, (the ratio of BF to BC is 2 to 7) D is the midpoint of AB, and EB:EA=2:9. The triangle DEF has area 12. Find the area of triangle ADC.



10. Isosceles triangle ABC has AC = BC and AB = 13 units. If the altitude from A to side BC is 12 units, what is the area of the triangle in square units?

11. Express 224 in a base (-3) number system. Your answer should only use the digits 0, 1, or 2.

12. The first three terms of a sequence of positive integers form an arithmetic progression and the second, third and fourth terms form a geometric progression. Find all possible sequences if the sum of the first and fourth terms is 28 and the sum of the second and third terms is 24.

13. In each of the 12 parking spots shown below, Tom the valet has to park either a red, white, or blue Toyota Camry. Each spot must have a car. No two cars horizontally across from each other can be the same color. Likewise, no two cars vertically adjacent can be the same color. It's OK for cars diagonally across from each other to be the same color. In how many different ways can Tom park the cars?

14. In triangle ABC, AB = 11, BC = 12 and CA = 13. Let M be the midpoint of AC and let N be the point on BC so that AN⊥BC. If segments BM and AN intersect at K, what is the ratio AK:KN?

15. Bob decides to take a road trip, and after looking at his map below notices there are several tunnels, shown as rectangles, along the way to his destination. He decides to go through exactly one of them, still taking only a shortest possible route (i.e., always driving south or east). How many different ways can he do this?