Discrete Mathematics Seminar

Time: Friday, October 1, 2021, 2:15-3:15 PM (Central Time)
Title: Chromatic number of generalized Borsuk Graphs
Speaker: Mr. Francisco Martinez Figueroa (Ohio State University)
Zoom Link: https://txstate.zoom.us/j/99924628868?pwd=czdJWVpWOHZIZE0vbHBQL1pWell6QT09
ID: 999 2462 8868
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Abstract:

The Borsuk graph is the graph with vertex set the sphere $S^d$, and edges $\{x,y\}$ whenever $x$ and $y$ are epsilon-almost antipodal. It is well known that when epsilon is small, its chromatic number is $d+2$, which follows from the topology of $S^2$ via Borsuk-Ulam's Theorem. Given a finite group $G$ acting freely over a compact space $X$ via isometries, we define the $G$-Borsuk graph to be the graph with vertex set $X$ and edges $\{x,y\}$ whenever there exists a non-identity $g$ in $G$ such that $d(x,gy)<\epsilon$. The chromatic number of such graph is determined by the topology of $X$. In this talk we'll show some upper bounds for this chromatic number depending only on $|G|$ and $\dim(X)$. Our bounds are tight when $\dim(X)=1$, and we exhibit some tight results in dimensions 2 and 3 when $G=\mathbb{Z}_m$, for small values of $m$. We will also discuss random $G$-Borsuk graphs, which are finite induced subgraphs by choosing $n$ i.i.d. uniform vertices on $X$. For these, we exhibit a tight threshold for epsilon (up to a constant) such that a.a.s. the chromatic number of the random subgraph coincides with that of the whole $G$-Borsuk graph. This is joint work in progress with Matthew Kahle.