Title: Mean-field games – models and theoretical challenges

Abstract: I will introduce some mean field game (MFG) equations through a couple of models, displaying the common mathematical feature of the subject: a Hamilton-Jacobi-Bellman (HJB) equation coupled with a continuity or Fokker-Planck equation, with their conditions prescribed at different times. Infinitely many (in fact, a continuum of) agents interact with one another but, due to their indistinguishability and the large number of them, each player interacts only with the distribution of players as a whole. Each one of them thus solves the same optimization problem, which leads to the first (the Bellman) equation, while the second equation (Fokker-Planck) describes the evolution of the players’ distribution, provided they have chosen their controls optimally. This makes the problem more tractable than dealing with a large number of discrete agents. Many models in finance, economics and population dynamics have been cast as MFGs. On the more theoretical side, associated to a MFG system is a so-called master equation, which is usually a single PDE written in an infinite-dimensional space without a typical differentiable structure (it isn’t even a linear normed space). The investigation of these master equations calls for different methods, depending on which side of the mathematical spectrum they are tackled.