Math Explorations
Algebra I

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Preface and Introduction

Math Explorations follows several fundamental principles. It is important to carefully state these at the beginning, and describe how these are a perfect fit not only in educating the general student population, but also in teaching students whose native language is not English. These guiding principles will help the curriculum come alive for all students.

First, learning math is not a spectator sport. The activities that fill the text and accompanying workbooks encourage students to develop the major concepts through exploration and investigation rather than by given rules to follow. **A crucial element is to understand the importance of small-group work, and to appreciate the extent to which everyone can benefit from working together.** In fact, often the process of explaining how to work a problem helps the explainer as much or more than the person who asks the question. As every teacher knows, explaining an idea to someone else is one of the best ways to learn it for oneself. Some basic rules for discussion within a group include

1. **Encourage everyone to participate**, and value each person's opinions. Listening carefully to what someone else says can help clarify a question. The process helps the explainer often as much as the questioner.
2. If one person has a question, remember that the chances are someone else will have the same question. Be sure everyone understands new ideas completely, and **never be afraid to ask questions**.
3. **Don't be afraid to make a mistake.** In the words of Albert Einstein, "A person who never made a mistake never discovered anything new." Group discussion is a time of exploration without criticism. In fact, many times mistakes help to discover difficulties in solving a problem. So rather than considering a mistake a problem, think of a mistake as an opportunity to learn

more about the process of problem-solving.

4. **Finally, always share your ideas with one another**, and make sure that everyone is able to report the group reasoning and conclusions to the class. Everyone needs to know why things work and not just the answer. If you don't understand an idea, be sure to ask "why" it works. You need to be able to justify your answers. The best way to be sure you understand why something works is to describe your solution to the group and class. You will learn more by sharing your ideas with one another.

If an idea isn't clear, there are several things to try.

1. First, look for simpler cases. Looking deeply at simple cases can help to you see a general pattern.
2. Second, if an idea is unclear, ask your peers and teacher for help. Go beyond "Is this the right answer?"
3. Third, understand the question being asked. Understanding the question leads to to mathematical progress.
4. Focus on the process of obtaining an answer, not just the answer itself; in short become problem centered, not answer centered. One of the major goals of this book is to develop an understanding of ideas that can solve more difficult problems as well.
5. After getting help, work the problem yourself, and make sure you really understand. Make sure you can work a similar problem by yourself.

Some hints to help in responding to oral questions in group and class discussion: As you work through the Explorations in the book, working both individually and in groups can make understanding the material easier. Sometimes it is better to explore problems together, and other times try exploring first by yourself and then discuss your ideas with others. When you discuss the problems as a group, it is more productive if you try to remember these simple rules:

1. Try not to interrupt when someone else is talking.
2. In class, be recognized if you want to contribute or ask a

question.

3. Be polite and listen when others in your group or class are talking. This is one of the best ways to learn.
4. Finally, don't be shy. If you have a question, raise your hand and ask. Remember, there is almost always someone else with a similar or identical question.

Last, some general advice about reading and taking notes in math.

1. Reading math is a specific skill. Unlike other types of reading, when you read math, you need to read each word carefully. The first step is to know the mathematical meaning of all words.
2. It is often necessary to write definitions of new words and to include mathematical examples. Try to write definitions in your own words without changing the meaning or omitting any important point. When you write down a definition, look for an example that illustrates what you are learning. This will help you to relate what you are learning to real world situations.
3. Explaining new ideas and definitions that you read to your peers and teachers is very helpful. This will provide practice with any new definitions, and make sure that you are using the words correctly. Explaining a concept can help to correct any misconceptions and also reinforces learning.
4. If possible, try to draw a visual representation to make a difficult or new concept clear. It is really true that "a picture is worth a thousand words." Visual cues help you understand and remember definitions of new terms.

Throughout this book, students learn algebraic thinking and the precise use of mathematical language to model problems and communicate ideas. The communication that makes this possible can be in small groups, in class discussion, and in student notes. It is important to note that the use of variables and algebra is not an afterthought, but is woven throughout all of our books. By using language purposefully in small groups, class discussions, and in written work, students develop the ability to solve progressively more challenging problems.

The authors are aware that one important member of their audience is the parent. To this end, they have made every effort to create explanations that are as transparent as possible. Parents are encouraged to read both the book and the accompanying materials.

The authors have written a 3 volume set of books that is designed to take all students from pre-algebra through Algebra I. This includes students who may not have understood the previous years math. Students from 4th through 8th grade should enjoy the ingenuity and investigation problems at the end of each set of exercises. Math Explorations is intended to prepare all students for algebra, with algebraic concepts woven in throughout. In addition, ME Part II, and ME Algebra I cover all of the 7th and 8th grade Texas Essential Knowledge and Skills (TEKS). The Teacher Guide has been written to make the textbook and its mathematical content as easy and intuitive for teachers as possible. Answers to the exercises are readily available and readable in the teacher edition. The teacher edition and accompanying CD contain supplementary activities that the students might enjoy.

This text has its origins in the Texas State Honors Summer Math Camp (HSMC), a six-week residential program in mathematics for talented high school students. The HSMC began in 1990 modeled after the Ross program at Ohio State, teaching students to think deeply of simple things (A. E. Ross). Students learned mathematics by exploring problems, computing examples, making conjectures, and then justifying or proving why things worked. The HSMC has had remarkable success over the years, with numerous students being named Siemens- Westinghouse semi-finalists, regional finalists, and national finalists. Initially supported by grants from the National Science Foundation and RGK Foundation, the HSMC has also had significant contributions from Siemens Foundation, Intel, SBC Foundation, Coca-Cola, the American Math Society Epsilon Fund, and an active, supportive Advisory Board.

In 1996, two San Marcos teachers, Judy Brown and Ann Perkins, suggested that we develop a pipeline to the HSMC that would introduce all young students to algebra and higher-level mathematics. Following their suggestion, we began the Junior Summer Math Camp (JSMC) as a two-week program for students in grades 48. We carefully developed the JSMC curriculum by meeting regularly with Judy and Ann, who gave us invaluable feedback and suggestions.

With support from the Fund for the Improvement of Postsecondary Education (FIPSE), Eisenhower Grants Program, Teacher Quality Grants, and the Texas Education Agency, we developed the JSMC into a replicable model that school districts throughout the state could implement. The JSMC curriculum was designed to prepare all students for higher-level mathematics. In some districts the JSMC targeted gifted students; in other districts the program was delivered to a mixed group of students; and in other districts the program was used especially for ELL. In every setting, the program had remarkable results in preparing students for algebra as measured by the Orleans-Hanna algebra prognosis pre- and post-tests.

Over the years, we trained hundreds of teachers and thousands of students. Although we cannot thank each personally, we should mention that it has been through their suggestions and input that we have been able to continually modify, refine, and improve the curriculum.

The problem with the JSMC curriculum was that it was only supplementary material for teachers, and many of the state-required mathematics topics were not included. The 3 volume Math Explorations texts that we have written has taken the JSMC curriculum and extended it to cover all of the TEKS (Texas Essential Knowledge and Skills) for grades 6-8 while weaving in algebra throughout. In particular, this volume, Math Explorations, Algebra I, was developed especially for younger students. By learning the language of mathematics and algebra, young students can develop careful, precise mathematical models that will enable them to work multi-step problems that have been a difficult area for U. S. students on international tests.

This project had wonderful supporters in the Meadows Foundation, RGK Foundation Foundation, and Kodosky Foundation. A special thanks to our Advisory Board, especially Bob Rutishauser and Jeff Kodosky, who have provided constant encouragement and support for our curriculum project. The person who motivated this project more than any other was Jeff Kodosky, who immediately realized

the potential it had to dramatically change mathematics teaching. Jeff is truly a visionary with a sense for the important problems that we face and ideas about how to solve them. His kind words, encouragement, and support for our JSMC and this project have kept me going whenever I got discouraged.

Our writing team has been exceptional. The basis for the book was our junior summer math camp curriculum, coauthored with my wife Hiroko, and friend, colleague and coauthor Terry McCabe. We were very fortunate when we decided to extend that curriculum to cover all of algebra I to find an extraordinarily talented co-author, Alex White. Alex has taken the lead on the algebra book in working with our team of students, faculty, and teachers, while also doing the amazing job of both making edits and doing typesetting. His specialty is math education and statistics, which are important and often neglected parts of an algebra book. Our team of authors has many lively discussions where we debate different approaches to introducing a new topic, talk about different ways to engage students to explore new ideas, and carefully go through each new idea and how it should be sequenced to best guide student learning.

Over the summers of 2005-2012, we have been assisted by an outstanding group of former Honors Summer Math Camp students, undergraduate and graduate students from Texas State, as well as an absolutely incredible group of pilot teachers. While it would take a volume to list everyone, we would be remiss not to acknowledge the help and support from this past summer, as well as describe the evolution of this three-volume series.

Briefly, in 2008, the Math Explorations Book was only one volume. This was piloted by a group of 6th and 7th grade teachers in McAllen, San Marcos, and New Braunfels. The results of these pilots have been extremely encouraging. We are seeing young 6th and 7th grade students reach (on average) 8th grade level and above as measured by the Orleans-Hanna test by the end of 7th grade. However, there was a consensus that it would be beneficial to split the Math Explorations book into a separate 6th grade and 7th grade book.

After meeting with the McAllen teachers in the summer, 2009, we carefully divided the Math Exploration text into two volumes. Hiroko Warshauer led the team in developing this new book, Math Explorations Part I, assisted by Terry McCabe and Max Warshauer. Alex White from Texas State provided valuable suggestions, and took over the leadership of the effort for the third volume, Algebra 1. The complete set of 3 books covers all of the Texas Essential Knowledge and Skills (TEKS) for grades 6-8, while also covering Algebra I.

Although there is naturally some overlap, the new books, Math Explorations Part 1 and Part 2, much more closely align with what the teachers felt would work best with their students. Math Explorations Part 1 should work for any 6th grade student, while Math Explorations Part 2 is suitable for either an advanced 6th grade student or any 7th or 8th grade student. The complete set of 3 books covers all of the Texas Essential Knowledge and Skills (TEKS) for grades 6-8, while also covering all of the TEKS for Algebra I.

In this project, we have been incredibly fortunate to have the help of several talented teachers. Major contributors this past summer include Amanda Voigt and Ashley Beach from San Marcos, Patricia Serviere from McAllen, and Amy Warshauer from Austin. These teachers provided wonderful help in the development of a an accompanying workbook, that provides a template for how to teach the book, with new explorations and supplemental problems. They also made numerous suggestions and edits, while checking that we covered all of the state mandated topics for Algebra I.

Sam Baethge and Michael Kellerman gave the entire book a careful reading, which provided amazing support for editing and revising the book. Michael focused primarily on readability edits. Sam continued to develop new challenge and ingenuity problems which should engage and excite young students in mathematics. Cody Patterson made key contributions to the original design, problems and content of the text. Robert Perez developed additional resources for English Language Learners, including a translation of the glossary and key mathematical terms into Spanish.

As we prepared our books for state adoption, Bonnie Leitch came on board to help guide and support the entire project. Bonnie worked tirelessly to find where each of the Texas Essential Knowledge and Skills (TEKS) and English Language Proficiency Skills (ELPS) was covered in both the text and exercises. We added additional exercises and text to cover any TEKS that were not sufficiently addressed. Bonnie also edited these revisions and gave a final proofreading for each of the books, working with the authors to proofread every edit. However, in the end the authors take total responsibility for any errors or omissions. We do, however, welcome any suggestions that the reader might have to help make future editions better. In short, we had an incredible, hard-working team that did the work of an entire textbook company in a few short weeks! Without their help, the project could not have reached its present state.

Any curriculum will only be as effective as the teachers who use it, and without the support and encouragement of the administration and parents, this can never happen. In this, we have been very fortunate to be able to work with fabulous teachers and administrators from San Marcos, McAllen, New Braunfels, Midland, and Austin. The Mathworks staff gave invaluable help. Patty Amdende and Andrew Hsiau have provided support whenever needed. I hope you will join our team by giving us feedback about what works, what doesnt and how we can improve the book. By working together, I believe that we can develop a mathematics curriculum that will reach out to all students and that will engage students at a higher level than we have previously been able to achieve.

Max Warshauer

VARIABLES, EXPRESSIONS AND EQUATIONS

1

SECTION 1.1 CONSTRUCTING A NUMBER LINE

"What is Algebra?" Rather than give you an incomplete answer now, we hope that through learning, you will soon be able to answer this question yourself. As a preview, let's look at some questions that algebra can help us answer that we could not have answered before:

- If I drop a marble off a two-story building, how long will it take the marble to hit the ground?
- If I have $10 and go into a candy shop, where chocolate costs $.50 and licorice costs $.65, how many of each could I buy?

We begin by reviewing numbers and the ways you manipulate and represent them. We will develop collections of numbers in stages, building up smaller groups of numbers until we get all numbers.

We first encounter numbers as children by counting, starting with one, two, and three. We call the numbers that we use in counting the *natural numbers*, or sometimes the counting numbers. They include the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \ldots$, where the "$\ldots$" means that they go on forever in the same way. These numbers describe how many of something, for example, how many brothers or sisters you have, how many days in a week, the number of people in your town and even the number of grains of sand on a beach.

EXPLORATION 1

Make a number line on a large piece of paper. Put the number 1 in the middle of the line. Locate and label the first 20 natural numbers.

Including the number 0 in this set of numbers gives us the *whole numbers*. The whole numbers take care of many situations, but if we want to talk about the temperature, there are places on Earth that routinely have temperatures below zero, like -5 °C or -20 °C. We must expand our idea of number to include the negatives of the natural numbers. This larger collection of numbers is called the *integers*, it is denoted by the symbol $\mathbb{Z}$ and includes the whole numbers. Notice that every integer is either positive, zero or negative. The natural numbers are positive integers and denoted sometimes by $\mathbb{Z}^+$.

In this book we will try to be very precise in our wording, because we want our mathematics and words to be clear. Just as you learn new words in English class to express complicated concepts, we must learn new words and symbols in mathematics. We have discussed 3 collections of numbers so far: the integers, the whole numbers, and the natural numbers. In mathematics, we call collections of numbers (or other objects) *sets*. A set is defined by its members. We call these members *elements*. In order to write out what a set is, we want to describe its elements in *set notation*. This is done for example by listing the elements of a set inside braces. For example, the natural numbers are $\mathbb{Z}^+ = \{1, 2, 3, 4, \ldots\}$ and the integers could be written $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$. Not all sets go on forever, for example, the set of even natural numbers between 4 and 14 is $\{6, 8, 10, 12\}$.

Every natural number is also an integer. In our more precise language, this could be described as "every number in the set of natural numbers is also in the set of integers". For this reason, we call the natural numbers a *subset* of the integers. We will call a set a subset of another set when any element in the first set is also in the second. We use such precise wording in order to be clear in our discussion of mathematics. The use of precise language is more important in mathematics than it is in everyday life.

EXPLORATION 2

Continue to work on the number line from Exploration 1. Using a red marker, plot and label the negative integers from -1 to -20. What properties does the set of integers have that the set of whole numbers did not?

It does not take long to see the need for numbers that are not integers. You might hear in a weather report that it rained $2\frac{1}{2}$ inches or know that a person's normal body temperature is 98.6°Fahrenheit. So sometimes we need to talk about parts of whole numbers called fractions. This expanded set of numbers that includes fractions is called the set of *rational numbers*.

EXPLORATION 3

Using a different colored marker, plot and label 3 fractions between each of the following pairs of integers:

2 and 3 4 and 5 -1 and 0 -3 and -2

A rational number is the quotient of 2 integers and the denominator can not be zero. For example, both $\frac{3}{7}$ and $\frac{9}{4}$ are rational numbers. They are called rational numbers because they are the ratio of 2 integers. A rational number can be represented as quotient in more than one way. Also, every rational number can be written in decimal form. For example, $\frac{1}{2}$ is equivalent to $\frac{2}{4}$ and to 0.5, $\frac{3}{4}$ is the same as 0.75 and $2\frac{2}{5}$ is equal to 2.4 and $\frac{12}{5}$.

We asked you to find two fractions between 2 and 3. Could you find two fractions between the fractions you just found? How about two fractions between these two?

PROBLEM 1

How many fractions are there between 0 and 1? How many fractions are there between 2 and 3?

Notice that every integer is a rational number. There are, however, rational numbers that are not integers. This means that the set of integers is a subset of the set of rational numbers, but the set of rational numbers is not a subset of the set of integers.

PROBLEM 2

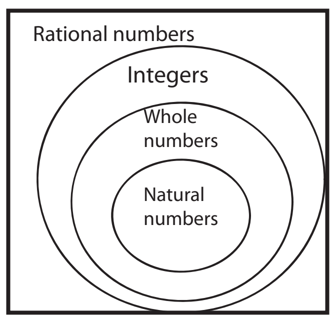
List 3 examples of rational numbers that are not integers and list 3 examples of integers that are not whole numbers. Locate these numbers on your number line.

EXAMPLE 1

Create a Venn Diagram to show the relationship between the following sets of numbers:

- rational numbers
- whole numbers
- integers
- natural numbers

Solution The sets of numbers are nested. For example, every integer is a rational number, but not every rational number is an integer.



Operations on the Number Line

One advantage of representing numbers on the number line is that it shows a natural order among its members. The location of the points representing 2 numbers reflects a relationship between those 2 numbers that we define as greater than, equal to or less than. You can also examine distances between numbers and model the operations of addition, subtraction, multiplication and division on the number line. In fact, we will frequently use examples on the number line to illustrate algebraic ideas.

Let's review how to do arithmetic with integers using a *linear model*, that is, by representing numbers on a number line. We will use the number line in discussing algebraic concepts. Let's get familiar with using this line.

EXPLORATION 4

1. Use the number line to illustrate the sum $3 + (-4)$ and the difference $3 - 4$. Explain how you arrived at your answer and location for each problem. Then, using the same pattern, explain how you compute the sum $38+(-63)$ and the difference $38 - 63$ without a detailed number line.
2. Use the number line to illustrate the difference $3 - (-5)$ and sum $3 + 5$. Then explain how you compute the difference $38 - (-63)$ without a detailed number line.
3. Summarize the rules for addition and subtraction of integers.
4. Use the number line to illustrate the product $3(-4)$ and $-3(4)$. Explain how you arrived at your answer and location for each problem. Then using the same pattern, explain how you compute the products $18(-6)$ and $-5(12)$ without a detailed number line.
5. Use the number line to illustrate the product $-3(-4)$. Explain how you arrived at your answer and location for each problem. Then using the same pattern, explain how you compute the product $-28(-3)$.
6. Summarize the rules for multiplication of integers.

The number line is also useful for thinking about operations with rational numbers and exploring the relationship between numbers.

EXPLORATION 5

1. Use the number line to illustrate the sums $1\frac{3}{4}+2\frac{3}{4}$ and $\frac{4}{5}+\frac{3}{5}$.

2. Starting at the point representing 3, determine and locate on the number line the following numbers. Explain how you arrived at your answer.
   a. The number that is 5 more than this number.
   b. The number that is 5 less than this number.
   c. The number that is 3 times this number.
   d. The number that is half as big as this number.
3. Locate and label three numbers that are greater than -5. Locate and label three numbers that are less than -6.

Distance on the Number Line

Another important concept to study on the number line is the *distance* between points.

EXPLORATION 6

Make a new number line from -15 to 15, labeling all of the integers between them. Locate the points 6 and 13 on the new number line. Determine the distance between 6 and 13.

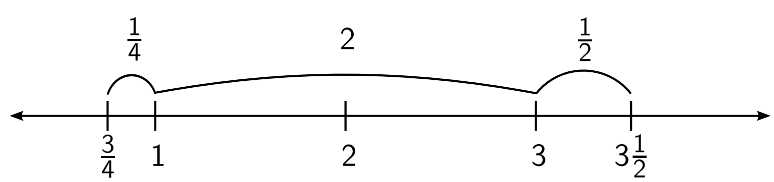
1. What is the distance from 12 to 4? Explain how did you got your answer.
2. What is the distance from -3 to -11? From -9 to -2? How did you get your answers?
3. What is the distance from -7 to 4? From 5 to -7? Explain.
4. Find the distance between $\frac{1}{2}$ and $3\frac{1}{2}$.
5. Find the distance between $\frac{1}{2}$ and $\frac{3}{4}$.
6. Find the distance between $\frac{3}{4}$ and $3\frac{1}{2}$.
7. What is the distance from $-\frac{1}{2}$ to $\frac{7}{8}$?
8. What is the distance between $4\frac{2}{3}$ and $1\frac{1}{2}$?

One way you might have found the distance between two points representing integers on a number line is to “count up” from the left most number until you reach the one on the right or to “count down” from the right most number until you reach the one on the left. For example from 6 you might have counted up and noted that it took 7 units to arrive at 13 and so concluded that the distance between 6 and 13 is 7. Or in the second question asking for the distance between 12 and 4, you might have counted down from 12 until you reached 4 and noted that it took 8 units, to conclude that the distance between 12 and 4 is 8. However, you might also have noticed that $12 - 4 = 8$ and $13 - 6 = 7$. The distance between two numbers is the difference of the lesser from the greater.

In part 6, you might want to break the distance from $\frac{3}{4}$ to $3\frac{1}{2}$ into three parts:

- the distance from $\frac{3}{4}$ to 1 is $\frac{1}{4}$,
- the distance from 1 to 3 is 2,
- the distance from 3 to $3\frac{1}{2}$ is $\frac{1}{2}$.

These parts add up to $\frac{1}{4} + 2 + \frac{1}{2} = 2\frac{3}{4}$.



The absolute value of a number is the distance from 0. We have a special symbol to represent absolute value. For example, we write $|6|$ and read it as absolute value of 6. We write $|-6|$ and read it as absolute value of -6. Since 6 and -6 are both 6 units from 0, we see that $|6| = |-6| = 6$. Since the absolute value is a distance, it is never negative. We often use absolute value when computing or representing distances between numbers. For example, if we want to compute the distance between -5 and 3, we can either subtract the lesser number from the greater number $3 - (-5) = 8$. Or we can take the absolute value of the difference, $|-5-3| = |-8| = 8$. The advantage of using the absolute value is that we can compute the difference in either order. Why is this true?

PROBLEM 3

Compute the distance between the following pairs of numbers.

1. -12 and 6
2. -52 and 27
3. -23 and -35
4. 1.75 and -1.25
5. $\frac{3}{4}$ and $-\frac{1}{3}$

EXERCISES

1. Compute the following sums or differences.
   a. $45 - 64$
   b. $42 + (-36)$
   c. $19 - (-33)$
   d. $17 - (-25)$
   e. $-13 + 26$
   f. $\frac{2}{3} + \frac{1}{5}$
   g. $\frac{3}{5} + \frac{2}{3}$
   h. $\frac{4}{5} - \frac{2}{3}$
   i. $\frac{5}{7} + \frac{1}{3}$
   j. $2\frac{3}{4} + 3\frac{1}{5}$
   k. $5\frac{3}{4} - 2\frac{2}{3}$
   l. $5\frac{1}{4} - 2\frac{2}{3}$

2. Compute the following products and quotients.
   a. $-2 \cdot 7$
   b. $5 \cdot (-5)$
   c. $-11 \cdot (-6)$
   d. $-24 \div 6$
   e. $-33 \div (-5.5)$
   f. $\frac{2}{3} \cdot \left(-\frac{4}{5}\right)$
   g. $-\frac{5}{7} \div \left(-\frac{15}{16}\right)$
   h. $-6 \div \frac{3}{5}$
   i. $3\frac{1}{2} \cdot 2\frac{2}{5}$

3. Evaluate the following expressions.
   a. $5 + 6 \cdot (-3)$
   b. $6 \cdot 7 - (-3) \cdot 7$
   c. $9 \cdot (-14 + 5)$
   d. $-13 - (-6 - 29)$
   e. $\frac{-2+20}{-3}$
   f. $\frac{2 \cdot 8 - 21}{-3 \cdot 10}$

4. Compute the distance between each of the following pairs of numbers.
   a. 8 and -3
   b. 4 and -5
   c. 1.1 and .9
   d. 3.4 and 2.95
   e. .26 and .3
   f. $\frac{2}{3}$ and 2
   g. $\frac{2}{3}$ and $\frac{1}{2}$
   h. $\frac{3}{5}$ and $\frac{3}{10}$
   i. 3.01 and 2.9
   j. 3.01 and 2.99
   k. 3.1 and 2.9
   l. 3.1 and 2.99

5. Using words, describe 3 subsets of whole numbers that are each infinite. Describe another infinite subset of whole numbers that is a subset of one of your first 3 subsets.

6. Copy the Venn Diagram from Example 1. For each condition given below, find a number that satisfies the condition and then place it on the Venn diagram.
   a. A whole number that is not a natural number.
   b. An integer that this not a whole number.
   c. A rational number that is not an integer.
   d. A rational number that is an integer, but not a whole number.
7. Find 3 numbers between each of the following pairs of numbers. Sketch a number line and plot the numbers on it.
   a. $\frac{2}{3}$ and 1
   b. $\frac{1}{4}$ and $\frac{1}{3}$
   c. $2\frac{3}{4}$ and $3\frac{1}{5}$
   d. $\frac{15}{7}$ and $\frac{5}{2}$
8. Compute the distance between each pair of the following list of numbers. Explain which pair is closest and which pair is the greatest distance apart.

   1.39 and 2.4   1.41 and 3.1 $1\dfrac{5}{6}$ and $3\dfrac{1}{3}$ $\dfrac{7}{4}$ and $\dfrac{11}{3}$
9. In each of the following problems, 3 numbers are given. Draw a number line and mark and label the 3 numbers. Pay attention to the distances between the numbers. Your picture should give an approximate sense of where the 3 numbers lie in relation to each other.
   a. 1, 4, and 7
   b. 5, 19, and 23
   c. 2, 4, and -5
   d. -2, -7, and -12
   e. -10, 20, and 30
   f. 6, 8, and -97