TEXAS STATE UNIVERSITY

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Discrete Mathematics Seminar

Time: Friday, March 23, 2018, 2:15-3:15 PM
Room: 237 Derrick Hall
Title: Primal-Dual Weak Galerkin Finite Element Methods for PDEs
Speaker: Dr. Chunmei Wang, Department of Mathematics, Texas State University

Abstract:
Weak Galerkin (WG) is a newly developed finite element technique for partial differential equations (PDEs) where the differential operators (e.g., gradient, divergence, curl, Laplacian, etc.) in the variational forms are reconstructed by using a framework in the theory of distributions. The framework yields discrete weak differential operators such as weak gradients and weak curls. The computation of the discrete differential operators involves the solution of inexpensive problems defined locally on each element. Appropriately defined stabilizers are employed to provide weak continuities that are necessary for the original formulations. The fundamental difference between the weak Galerkin finite element method and other existing finite element methods (FEM) is the use of weak functions and weak derivatives (i.e., locally reconstructed differential operators) in the design of numerical schemes based on conventional weak forms for the underlying PDE problems. Weak Galerkin is a natural extension of the classical conforming and non-conforming finite element method and has advantages over FEM in many aspects. Due to its great structural flexibility, the weak Galerkin finite element method is well suited to most PDEs by providing the needed stability and accuracy in approximations. In this talk, the speaker will introduce the basic ideas and a general framework for WG methods by using the second order elliptic equation as a model problem. The speaker will then discuss a recent development of WG, known as "Primal-Dual Weak Galerkin (PD-WG)". The essential idea of PD-WG is to interpret the numerical solutions as constrained minimization of some functionals with constraints that mimic the weak formulation of the PDEs by using the weak derivatives. The resulting Euler-Lagrange equation offers a symmetric finite element scheme involving both the primal and the dual variable (also known as the Lagrange multiplier). The primal-dual weak Galerkin methods will be applied to several challenging problems for which existing methods have difficulty in applying; these problems include the second order elliptic equations in nondivergence form, Fokker-Planck equation, first order convection equations, and elliptic Cauchy problems. Finally, the speaker will introduce an abstract framework for the PD-WG method and discuss its great potential in other scientific applications.