Math Explorer

MATHEMATICS AND LOGIC

Vol. 6.3 Spring 2004

Think Rationally!!

Check Validity

Logician Bertrand Russell

Math Explorer

Contents

Math Explorer
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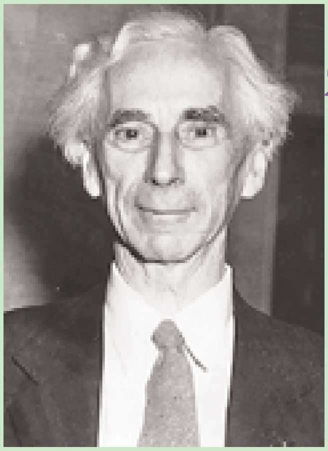


Bertrand Russell

by Joyce Fischer

Bertrand Russell, one of the best known logicians of our time, was born May 18, 1872 at Ravenscroft, Wales, where his aristocratic, conservative Whig family was considered one of the most distinguished in Britain at this time of heightened economic and political awareness. Within just 6 years of his birth, Russell's family was decimated by the loss of his mother, his sister, his father and his grandfather. His grandmother, who planned for him to follow family tradition and become the Prime Minister, raised him and attended to his formal education.

In 1890, Russell entered Trinity College at Cambridge University, where his achievements in mathematics and philosophy flourished, culminating with a first class B.A. in Mathematics in 1893. By 1908, he had become a fellow of the esteemed Royal Society. Two years later he began a 3-year collaboration with Alfred Whitehead, an English mathematician who had once taught Russell at Cambridge. The result was a 3-volume work, *Principia Mathematica*, a valiant attempt to prove that all of mathematics could be reduced to formal symbolic logic.

From 1910 until the end of his life, he was noted as an outspoken social and political activist with an eccentric lifestyle. During those years he married 3 times, had 3 children, was dismissed from several educational institutions for political and moral reasons. He also campaigned for women's suffrage and against nuclear armament, and was convicted several times (imprisoned twice) for his liberal beliefs. When his brother died in 1931, he became the Third Earl Russell.

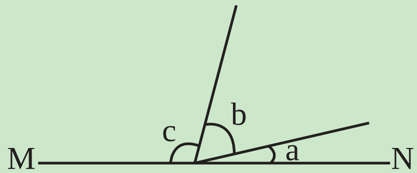
Just a few of the other honors and awards he received include the Butler Gold Medal for his work in philosophy (1915); the Order of Merit, complete with a certificate signed by King George VI (1949); and the Nobel Prize for Literature (despite the controversies surrounding his political and moral views) in 1950. A comprehensive bibliography of his major publications, released in 1994, is 3 volumes long. He died at age 97 in 1970.

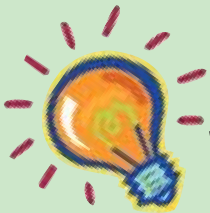
References:
http://plato.stanford.edu/entries/russell/
http://www.humanities.mcmaster.ca/~bertrand/awards.html

Some of the following problems appeared in the 7th Primary Math World Contest.

1. There are four kinds of bills valued at \$1, \$5, \$10, and \$50 respectively. There are a total of nine bills , with at least one bill of each kind. If the total of these bills is \$177, how many \$10 bills are there?

2. In the figure, MN is a straight line. The angles a, b and c satisfy the relations, $b : a = 2 : 1$ and $c : b = 3 : 1$. Find angle b.





3. Two bulbs flash at regular intervals of 30 and 36 seconds. Both bulbs flash together for the 1st time at 10:45 a.m. At what time will they flash together for the 13th time?

4. Jacob begins counting up from 100 by 7's (100, 107, ...) and Josephine begins counting down from 1000 by 8's (1000, 992, ...) at the same time. If they count at the same rate, what number will they say at the same time?

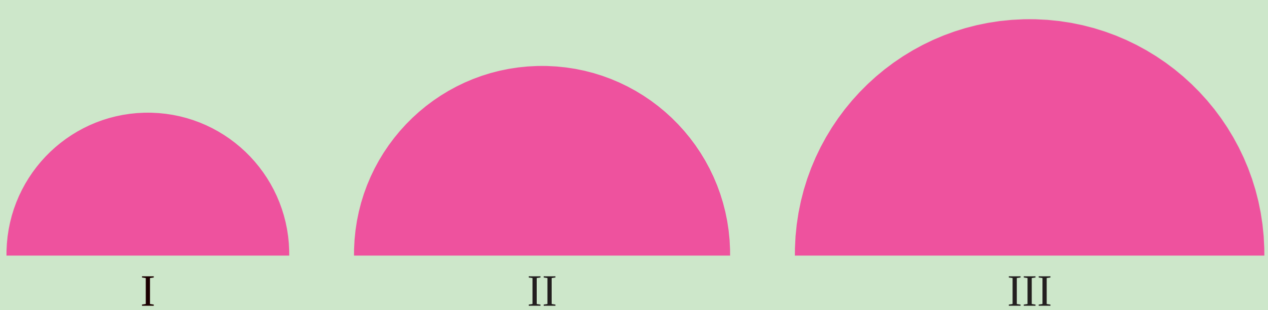
5. How many different isosceles triangles of perimeter 25 units can be formed where each side is a whole number (integer) of units?

6. The year number 2004 has digit sum 2+0+0+4 = 6. How many four digit numbers have the same digit sum as 2004?

7. Isaac won a ping pong game and the final score was 21-14. How many possibilities are there for the score in the game after 20 points were played?

8. Aaron cycled every day for fifty days. For the first forty days his average was 22 miles per day. What was his average for the last ten days if he cycled a total of 1,150 miles in the fifty-day period?

Ingenuity I, II, and III are three semi-circles of different sizes. If the ratio of the diameters of I, II, and III is 3 : 4 : 5, and the area of III is 24 cm^2, how many cm^2 is the sum of the areas of I and II?



Answer: **1)** 1 **2)** 40 **3)** 11:21 **4)** 520 **5)** 6 **6)** 56 **7)** 15 **8)** 27 miles per day **Ingenuity)** 24

WHY THINK LOGICALLY?

cogito ergo sum

by Joyce Fischer

Have you ever watched "Law and Order" or any television show that involves detectives who are trying to solve a crime? Like television detectives and real life detectives, when we reason (think), our objective is to come to a true conclusion (solve a case) about a topic (case) from statements about the topic that we already know to be true (the clues).

There are two basic ways to reason: the **Deductive** way, which starts with statements that are known to be true, progressing from these general truths to a specific truth; and the **Inductive** way, which starts with a number of observed cases that are true, progressing from these specific truths to a general truth. Deductive reasoning involves Logic, a branch of Philosophy, which, like Mathematics, a branch of Science, is both reflective and analytical. The primary goal of Logic is the analysis of the correctness of deductive reasoning.

Since good or sound deductive reasoning begins with true statements, we should first define a statement: a **statement** is a sentence that is either true or false.

Some examples of statements are:

a) The colors of the United States flag are red, white, and blue. (True)

b) John Steinbeck wrote the novel *Gone With the Wind*. (False)

c) Minnesota is called the Land of 10,000 lakes. (True)

d) Black bears run wild in Hawaii. (False)

Some examples that are not statements are:

1) Can a Volvo beat a Porsche on a straight one mile track?

2) Join the Army now!

3) Dogs, cats and people.

4) Basketball is a great sport.

5) I will read this book today.

Let's analyze why these are not statements, in other words, let's think logically about the definition of a statement. These examples are not statements because: number 1 asks a question, number 2 gives a command, number 3 is not a sentence, number 4 calls for a judgment about a topic, and number 5 involves knowing who the "I" refers to—which is a problem of self reference. So these examples are either not sentences or they do not have a true or false value that everyone can easily determine.

The next step in the process of deductive reasoning is to form a series of true statements that can lead to a true conclusion. Whenever a series of true statements leads to a true conclusion, the reasoning is considered good, and we say that a **valid argument** has occurred. Whenever a series of true statements leads to a false conclusion, the reasoning is considered bad and we say that a **fallacy** has occurred. There are many basic types of logical arguments, called forms, that can always lead to true conclusions when the statements are all true. We will examine three of these forms.

The first argument form, the **law of detachment**, says that when you have a true statement that describes a trait common to every item in some category and you know some specific thing is in this category, then you can conclude that this specific thing has the trait also. Let's look at an English sentence version and a symbolic version of this argument form.

1st statement All snakes are cold-blooded

Symbolic translation All p have q

2nd statement A rattler is a snake

Symbolic translation This is a p

Conclusion Therefore, a rattler is cold-blooded

Symbolic translation So, This has q

This first statement describes a trait, q = cold blooded, that all items in the original category, p = snakes, have. This second statement describes a specific type of item in the original category. The conclusion then follows that this specific item has the same trait as all of the items in the original category.

The second argument form, called the **law of the contrapositive**, says that when you have a true statement that describes a trait common to every item in some category and you know something specific does not have this trait, then you can conclude that the specific thing is not in the original category. Again, let's look at an English sentence version and a symbolic version of this argument form.

1st statement All snakes are cold-blooded

Symbolic translation All p have q

2nd statement A dog is not cold-blooded

Symbolic translation This does not have q

Conclusion Therefore, a dog is not a snake

Symbolic translation So, this is not a p

This first statement describes a trait that all items in the original category have.This second statement describes something specific which does not have this trait. The conclusion then follows that this specific thing is not in the original category.

The third argument form, called the **chain rule**, says that when you have a true statement that describes a trait common to every item in some category, and you know that whenever something specific has this trait, it must also possess some new trait; then you can conclude that the items in the original category must also possess this new trait. For instance, let's look at an English sentence version and a symbolic version of this argument form.

1st statement All snakes are cold-blooded

Symbolic translation All p have q

2nd statement All things that are cold-blooded do not sweat.

Symbolic translation All q have r

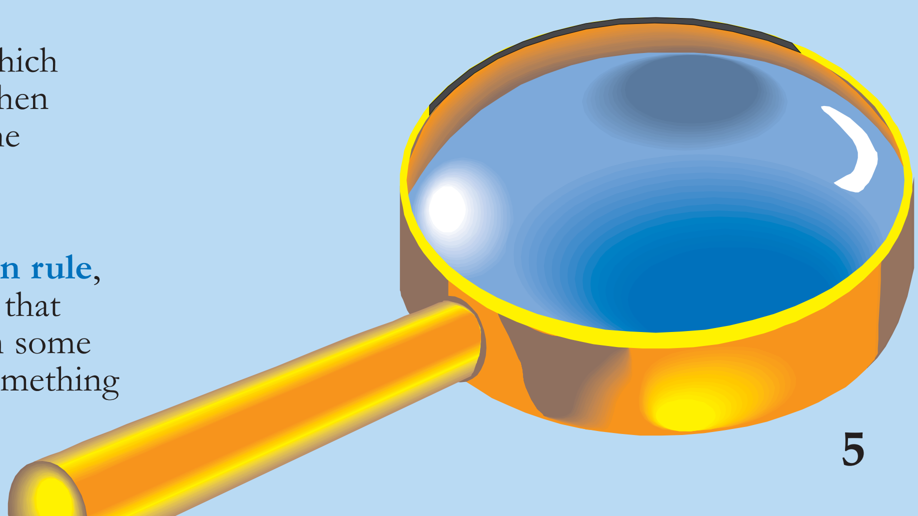
Conclusion Therefore, all snakes do not sweat.

Symbolic translation All p have r

This first statement describes a trait that all items in the original category have. This second statement describes a new trait that is true of all items that have the first trait. The conclusion then follows that the items in the original category must also have this new trait.

Just like detectives, we study and learn valid logical forms in order to recognize and avoid bad reasoning while at the same time improving our own reasoning by making clear and sound arguments. This increased knowledge about logic enhances reasoning in all aspects of our lives.

Joyce Fischer teaches in the Department of Mathematics at Texas State University-San Marcos. Her research concerns the relationship between logical reasoning and students' learning of mathematical concepts.



Puzzle Page

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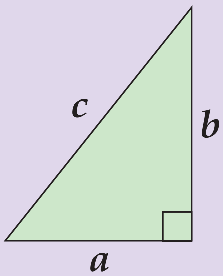
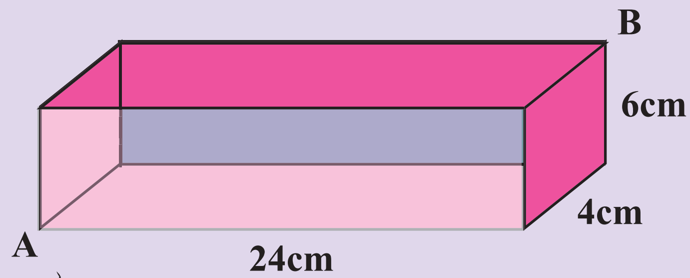
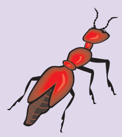


A right triangle has three sides of length a, b and c, where c is the longest side. Then you may assume the Pythagorean Theorem which states that:

$$a^2 + b^2 = c^2$$





Now consider a rectangular box pictured above with sides of length 24cm, 6cm, and 4cm. An ant, standing at point A, wants to go on the surface of the rectangular box to point B (diagonally opposite corner of the box). If the ant walks at a rate of 2 cm/min from A to B, find the minimum time the ant takes to complete the trip

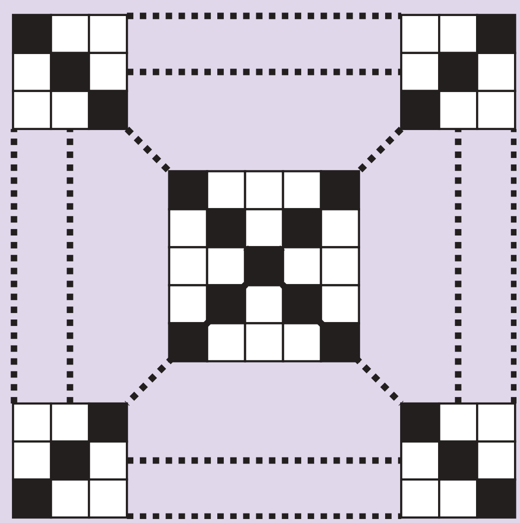
Word Search

Forwards or backwards, up, slanted, or down.
Where can the words in this puzzle be found?

Deductive
Inductive
Logic
Validity
Reasoning
Philosophy
Symbolic
Argument
Fallacy

J	B	E	Y	H	P	O	S	O	L	I	H	P	L
M	O	A	B	I	D	A	T	E	P	V	J	O	L
X	Q	L	F	B	E	A	R	U	L	E	T	Q	A
M	R	T	Y	U	D	N	W	A	X	F	A	I	K
V	P	O	M	C	U	S	J	G	S	A	R	V	B
A	I	N	D	U	C	T	I	V	E	O	G	W	S
L	Y	E	U	V	T	A	L	M	N	O	U	Y	G
I	E	T	C	L	I	D	F	L	O	P	M	L	A
D	F	S	I	A	V	Q	V	C	S	B	E	R	Y
I	V	W	G	D	E	K	H	J	O	L	N	J	P
T	M	N	O	Q	T	A	W	L	X	Z	T	D	F
Y	C	A	L	L	A	F	I	L	E	R	Y	T	G
L	K	J	F	T	E	C	W	Q	A	T	K	L	F
R	E	A	S	O	N	I	N	G	P	L	K	T	Y

A square floor is tiled with congruent square tiles. The tiles on the two diagonals of the floor are black. The rest of the tiles are white. If there are 101 black tiles, what is the total number of white tiles?



In a party, there are *n* persons. If everyone shakes hands once with each other person and there are a total of 231 handshakes, what is the value of *n*?

Bulletin Board

Mathworks Team competes in Hong Kong

In late December, Mathworks sent the third Mathworks team to the 7th Annual Po Leung Kuk Primary Mathematics World Contest (PMWC) in Hong Kong. The team, consisting of Pranay Kothari, E. Martin, Jeffrey Chen and Stephanie Chan, received the Po Leung Kuk Champion Cup, awarded to the top non-Asian team, for the second year in a row. Accompanying the students were team leader, Max Warshauer and deputy team leader, Amy Warshauer. For more pictures and information visit http://www.txstate.edu/mathworks

Meadows Foundation Awards Mathworks

Mathworks, in partnership with the Departments of Mathematics and Curriculum and Instruction at Texas State University-San Marcos, and the San Marcos CISD, has received a $244,000 grant to improve the mathematics abilities of middle school students and increase the number of certified mathematics teachers in the state. For more information about the Meadows Foundation, visit http://www.mfi.org

Celebrate Pi Day

March 14 (3.14) is π day. Visit the following websites and learn more about this intriguing, useful, and special number.
Archive.ncsa.uiuc.edu/edu/RSE/RSEorange/buttons.html
http://members.aol.com/loosetooth/pi.html

Get Ready for Math Camp 2004!

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MATH ODYSSEY

by Joyce Fischer

Logically Speaking

Reading “Why Think Logically” on page 4 will help you determine which of the following arguments are logically valid.
You can assume all of the statements are true.

Argument # 1
All babies are sweet things.
All sweet things bring people pleasure.
Therefore, all babies bring people pleasure.

Argument # 2
All fruits grow on trees.
All apples grow on trees.
Therefore, all apples are fruits.

Argument # 3
All dogs like to play ball.
Sandy does not like to play ball
Therefore, Sandy is not a dog.

Argument # 4
All dogs like to play ball.
Fido is not a dog.
Therefore, Fido does not like to play ball.

Argument # 5
All monkeys live in trees.
Tarzan lives in a tree.
Therefore, Tarzan is a monkey.

Argument # 6
All real numbers can be put on a real number line.
7 is a real number.
Therefore, 7 can be put on a real number line

The solutions are found below.

Here is a problem involving logical reasoning:

Tom and Marco are talking when Tom says, "How old are you?"
Marco replies: "Can you guess my age?"
"I'll try. Divide your age by 3 and tell me what the remainder is."
"2" "Now, divide your age by 5 and tell me what the remainder is."
"0" "One more time, divide your age by 7 and tell me what the remainder is."
"1" Can you figure out how old Marco is from these clues?

Solutions to the arguments.
Argument #1 is a valid form of the chain rule.
Argument #2 is an invalid form or a fallacy.
Argument #3 is a valid form of the law of the contrapositive.
Argument #4 is an invalid form or a fallacy.
Argument #5 is an invalid form or a fallacy.
Argument #6 is a valid form of the law of detachment.

Solution to the problem.
By using deductive reasoning you can realize that the number must be divisible by 5. One way to solve this problem is by writing all of the multiples of 5 up to 100 on a piece of paper; you can then use the process of elimination. First cross out the numbers that do not have a remainder of 2 when divided by 3, then cross out the numbers that do not have a remainder of 1 when divided by 7. Looking at the remaining number tells us Marcos is 50.

Dear Math Explorers,

Have you ever played a game where you were given a set of clues to solve a problem and you used logic to arrive at the answer? In mathematics, logical reasoning is the main tool we use in verifying or proving any statement or assertion. Read more about this important branch of both mathematics and philosophy as well as a biography of one of the most famous logicians of our times in our current issue.

April is national Math Awareness Month. Encourage your school to celebrate using this year’s theme about mathematics and networks, “Mathematics and Connections,” and please share your ideas with us!

Sincerely,
Hiroko K. Warshauer

Hiroko K. Warshauer, executive editor