

## Working With Straight Lines

### Important Information & Formulas

1.  $ax + by = c$  -- **Standard form of a line**, where  $a$ ,  $b$ , and  $c$  are real numbers, and  $a$  and  $b$  are not both zero.
2.  $y = mx + b$  -- **Slope-intercept formula**, where  $m$  is the slope and  $b$  is the  $y$ -intercept.
3.  $(y - y_1) = m(x - x_1)$  -- **Point-slope formula**, where  $m$  is the slope and  $(x_1, y_1)$  is a point given.
4.  $m = \frac{y_2 - y_1}{x_2 - x_1}$  -- **Slope formula**.
5. A line is **parallel** to another line if their slopes are equal.

For example if:

$$1. \quad m_1 = \frac{3}{4}; \quad \text{parallel} \quad m_2 = \frac{3}{4}$$

6. A line is **perpendicular** to another line if their slopes are negative reciprocals.

For example if:

$$m_1 = 2; \quad \text{perpendicular} \quad m_2 = -\frac{1}{2}$$

7. The **x - intercept** is a point when  $y = 0$ ;  $(x, 0)$ .
8. The **y - intercept** is a point when  $x = 0$ ;  $(0, y)$ .

### Examples

1. Given two points  $(3, 5)$  and  $(2, -3)$   
 $(x_1, y_1) \quad (x_2, y_2)$

Find the equation of the line and write in standard form:

a. Find the slope. Remember:  $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{-3 - 5}{2 - 3} = \frac{-8}{-1} = \frac{8}{1} = 8$$

b. Use the point-slope formula. Remember:  $(y - y_1) = m(x - x_1)$

$$y - 5 = 8(x - 3)$$

$$y - 5 = 8x - 24$$

$$\begin{array}{r} -8x \quad -8x \\ \hline -8x + y - 5 = -24 \end{array}$$

$$\begin{array}{r} \quad \quad +5 \quad +5 \\ \hline \hline 8x + y = -19 \end{array}$$

2. Given  $m = -\frac{3}{2}$  and a  $y$ -intercept of  $(0, -3)$ . Write an equation for a line.

(Hint: Use slope-intercept formula)  $y = mx + b$

$$y = -\frac{3}{2}x + (-3)$$

$$y = -\frac{3}{2}x - 3$$

3. Find an equation through  $(5, 3)$  which is parallel to the line  $4x + 5y = -10$ .

a. Place the standard form equation into  $y = mx + b$  form (slope-int form) so that we can identify the slope.

$$4x + 5y = -10$$

$$\begin{array}{r} -4x \quad -4x \\ \hline \end{array}$$

$$\frac{5y}{5} = -\frac{4x}{5} - \frac{10}{5}$$

$$y = -\frac{4}{5}x - 2$$

$$\text{therefore } m_1 = -\frac{4}{5}$$

b. If  $m_1 = -\frac{4}{5}$ , the slope of the parallel line is  $m_2 = -\frac{4}{5}$ . Given the point

$(5, 3)$ , use the point-slope formula:  $(y - y_1) = m(x - x_1)$

$$y - 3 = -\frac{4}{5}(x - 5)$$

$$y - 3 = -\frac{4}{5}x + \frac{20}{5}$$

$$y - 3 = -\frac{4}{5}x + 4$$

$$\begin{array}{r} +3 \quad +3 \\ \hline \hline \end{array}$$

$$y = -\frac{4}{5}x + 7$$

4. Find an equation through (4, 3) which is perpendicular to the line  $4x + 5y = -2$
- a. Place the standard form equation into  $y = mx + b$  form so that we can identify the slope.

$$\begin{array}{r}
 4x + 5y = -2 \\
 -4x \quad -4x \\
 \hline
 5y = \frac{-4x}{5} - \frac{2}{5} \\
 y = -\frac{4}{5}x - \frac{2}{5}
 \end{array}$$

- b. If  $m_1 = -\frac{4}{5}$ , the slope of the perpendicular line is  $m_2 = \frac{5}{4}$ . Given this point (4, 3), use point-slope formula:  $(y - y_1) = m(x - x_1)$

$$\begin{array}{r}
 y - 3 = \frac{5}{4}(x - 4) \\
 y - 3 = \frac{5}{4}x - \frac{20}{4} \\
 y - 3 = \frac{5}{4}x - 5 \\
 \hline
 +3 \quad +3 \\
 y = \frac{5}{4}x - 2
 \end{array}$$

**Note:** To write in standard form, multiply equation by common denominator

- c. Common denominator is 4

$$\begin{array}{r}
 4 * \left( y = \frac{5}{4}x - 2 \right) \\
 4y = 5x - 8 \\
 \hline
 -5x \quad -5x \\
 -5x + 4y = -8 \quad \text{or} \quad 5x - 4y = 8
 \end{array}$$