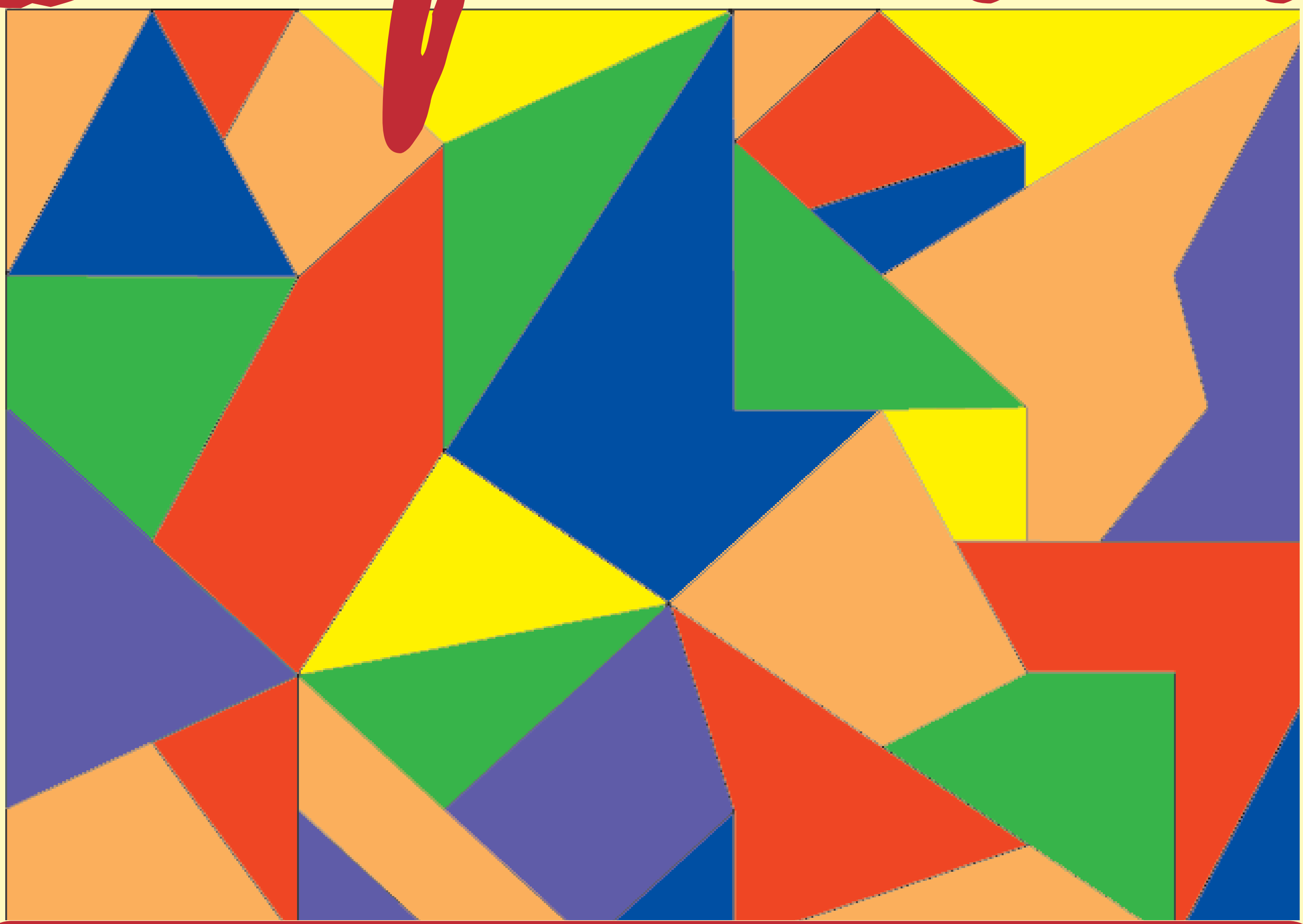
Math Explorer



MATHEMATICS AND AREA

Pick Looks at the Boundary

Splendid Sierpinski Shapes!

Triangles Once Removed

Vol. 5.3 Spring 2003

Math Explorer

Contents

Executive Editor: Hiroko K. Warshauer
Senior Editors: Terry McCabe, Max Warshauer, Eugene Curtin
Special Writers: Eugene Curtin, Joyce Fischer
Design: Amanda Eliasson
Final Editing and Proofreading: David Nelson, Michael Kellerman
Administration: Lydia Carbuccia
Circulation: Amanda Eliasson, Lydia Carbuccia, Jennifer LeGrévellec
Webmaster: Jennifer LeGrévellec

Math Explorer
Southwest Texas State University
San Marcos, TX 78666
Phone: (512) 245-3439, Fax: (512) 245-1469
e-mail: max@swt.edu.
Visit our website: www.swt.edu/mathworks

Math Explorer is published by
Southwest Texas State University
Mathworks.

Waclaw Sierpinski

by Joyce Fischer

Waclaw Sierpinski was born in Warsaw, Poland on March 14, 1882. Though his natural ability in mathematics was quickly recognized, history worked against his receiving a challenging education in Poland. Russia's occupation and domination of Poland was designed to stifle education, so he was forced to learn the Russian language and culture in order to survive.

Sierpinski's determination and adaptability paid off when he entered the Department of Mathematics and Physics at what is now the University of Warsaw in 1899. The university was then called the Russian Czar's University, and all classes were taught in Russian by Russian professors and staff. He graduated in 1904 despite subtly maintaining his Polish heritage by refusing to pass his Russian language examination. He earned a gold medal for his dissertation essay on his Russian professor Varonoy's achievement in number theory. In 1908, he received his doctorate from Jagiellonian University in Krakóv and was appointed to the University of Lvov.

Sierpinski first became interested in set theory in 1907 and after only two years of study conducted the first course devoted solely to that subject. During World War I, he was rescued from imprisonment by two Russian mathematicians, Egorov and Luzin, who took him to Moscow where he stayed until the war ended in 1918. He was offered a position at the University of Warsaw and remained there until his retirement in 1960. Before his death on October 21, 1969, he was awarded honorary degrees from nine universities in various countries, received numerous other honors and was elected to 14 prestigious scientific academies from all over the world.

Despite Russian occupation of his homeland during his youth; Russian internment during World War I; German destruction of his home, library, and personal papers; and the execution of many of his Polish colleagues during World War II, Sierpinski survived to write 724 papers and 50 books. His student Rotkiesicz aptly described him as "the greatest and most productive of all Polish mathematicians."

References: www-groups.dcs.st-and.ac.uk/~history/BiogIndex.html

Joyce Fischer teaches mathematics at Southwest Texas State University.

1. What is the next term in the sequence: 2, 3, 5, 9, 17... ?

2. In the 5th grade there are 45 students who have played soccer and 35 who have played basketball. There are 20 students who have played both sports, and 15 who have played neither. How many students are in the 5th grade?

3. Alana's age is a square number, and 17 years from now it will again be a square number. What is Alana's age?

4. How many integers between 1 and 2003 have exactly two zeroes?

5. How many integers between 100 and 999 have a larger digit in the 100's place than in the 10's place?

6. Jim's birthday is January 8th, which is a Wednesday in 2003. In what year will his birthday next fall on a Wednesday?

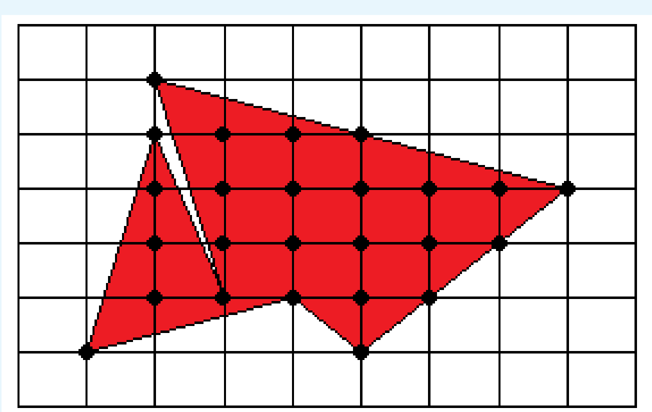
You may wish to read the article on Pick's Theorem before working the remainder of the problems. A 50 x 100 lattice rectangle is drawn on a grid with sides which are grid lines.

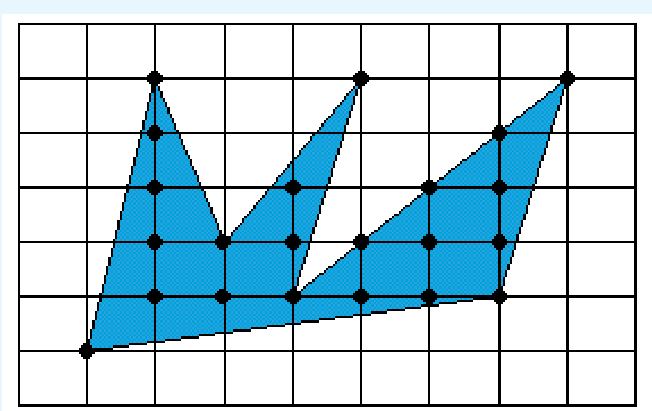
7. How many boundary lattice points are on the rectangle?

8. How many interior lattice points are on the rectangle?

9. Find the area of the following red polygon.

10. Find the area of the blue polygon.





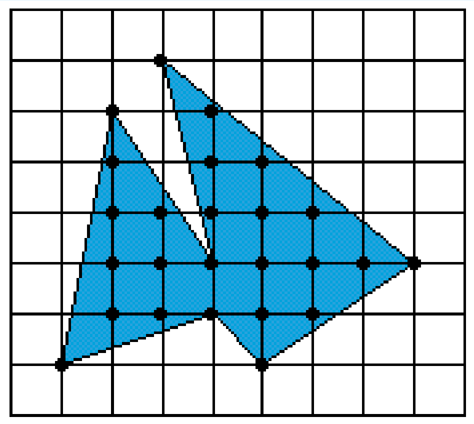
Answer: 1) 33 2) 75 3) 64 4) 39 5) 450 6) 2014 7) 300 8) 4851 9) 19 10) 16

Pick's Theorem:

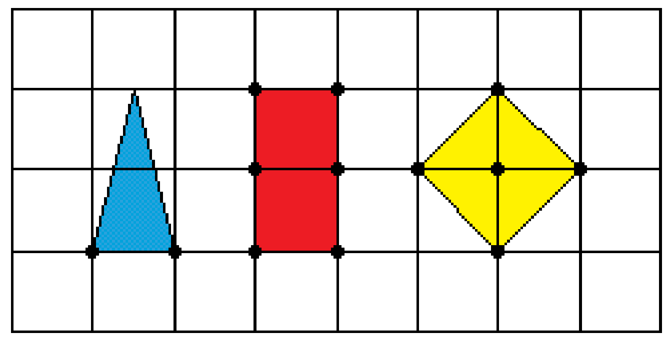
A method for finding the area of lattice polygons.

by Eugene Curtin

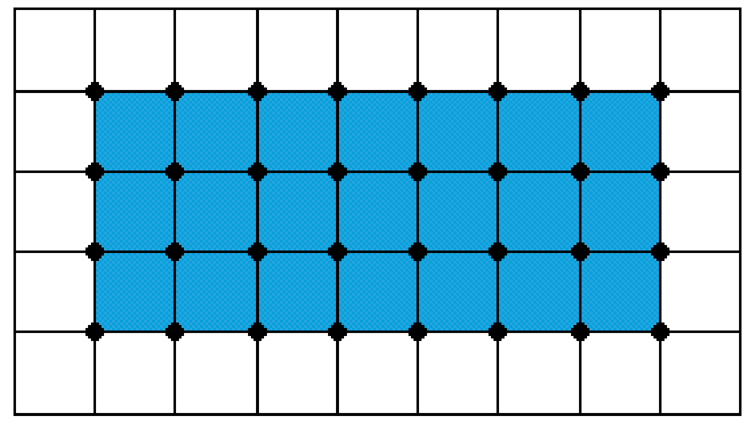
Let's start with a problem: find the area of the blue polygon below. Each square in the grid has area 1. Before reading further, you might like to try to find the area on your own by any method you choose.



First, we examine an approach based on counting points where grid lines intersect. These are called lattice points. This method applies only to lattice polygons, that is polygons whose vertices are all lattice points. We start with a simple example below.



The blue triangle above is not a lattice polygon, as its highest vertex is not a lattice point. Our method will not apply to this triangle. The red rectangle has 6 lattice points and the yellow square has 5, but both have area 2 square units. Should boundary points count less than interior points? It seems reasonable to count them as 1/2 each. Then the counts are 6 x (1/2)= 3 (red) and 1 + 4 x (1/2)= 3 (yellow). The count exceeds the area by 1 for these figures. Let's consider a larger example:

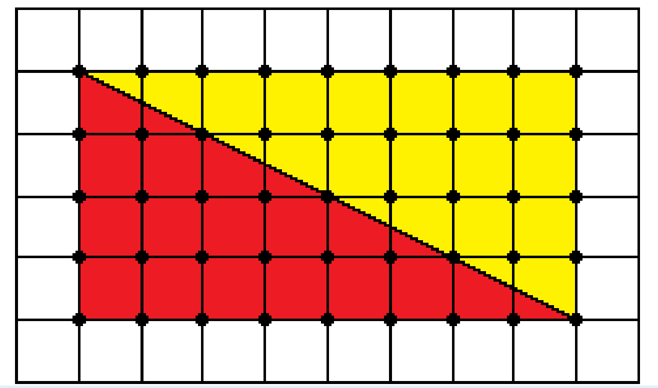


The area of the rectangle is 3 x 7 = 21. The number of interior lattice points is 2 x 6 = 12, and there are 7+3+7+3=20 lattice points on the boundary for total count of 12 + 20 x (1/2)= 22, again 1 more than the area.

In each example so far we have $A = I + B/2 - 1$ where A is the area, I is the number of internal lattice points, and B is the number of boundary lattice points. We do not yet know if $I + B/2 - 1$ will give the area for all lattice polygons, so let's call this quantity the value (V) of the polygon. Check some more lattice rectangles and verify that A = V for your examples. If you have some experience with algebra try this:

Challenge: Show that if you draw an M x N lattice rectangle in the grid with sides which are grid lines, then the value of the rectangle is M x N.

In the picture below, the red area plus the yellow area will give the area of a 5x8 rectangle.

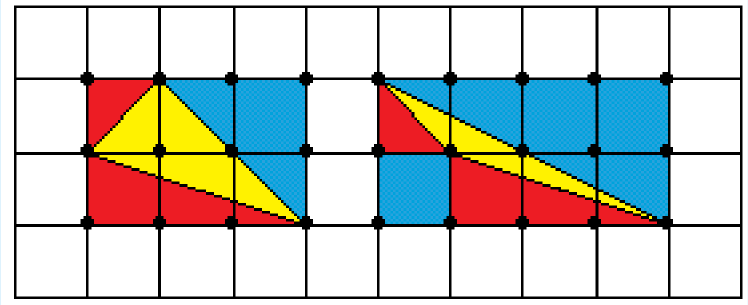


Does the red value plus the yellow value also give us the rectangle value? The interior points count 1 for the rectangle, and either 0+1=1 (yellow interior), 1+0=1 (red interior) or 1/2 + 1/2 = 1 (red-yellow boundary) for the red-yellow total value. The lattice points on the boundary score 1/2 for the rectangle, and 1/2 + 0 or 0 + 1/2 for the red-yellow team, except for the 2 points at either end of the dividing line. These each score a total of 1/2 for the rectangle and 1/2 + 1/2 = 1 for the red-yellow team. So far the red-yellow team is up by 1, but remember $V = I + B/2 - 1$. We reduce the count for the rectangle by 1, and the red-yellow team by 2 (1 for the red and 1 for the yellow). The answer is yes! If we add the red and yellow values, we get the rectangle value. Try some more examples on your own and make sure you understand why lattice polygon values add just like areas.

Principle: *If a lattice polygon is divided into 2 or more smaller lattice polygons, the value of the large polygon is the sum of the values of the smaller polygons.*

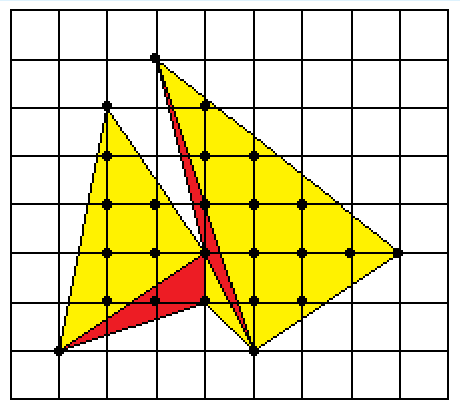
In the diagram above the red and yellow values are equal as the figure is symmetric about the center point of the rectangle. The red value is therefore 1/2 the value of the rectangle. Remember the rectangle value and area are equal. So the red value is 1/2 the area of the rectangle, which also equals the red area! Any lattice triangle with two sides along grid lines will be half of a rectangle in the same manner, and its area and value will be the same.

Are area and value the same for other triangles? Given any triangle we can box it



inside a rectangle, as we have done for the two yellow triangles below.
Can you see why area and value must be equal for the yellow triangles? You already know value and area are equal for the red and blue regions, and for the rectangles. Now just use addition and subtraction. Remember our blue polygon? It can be divided up into lattice triangles in many ways and we show one method below:

polygon area = sum of triangle areas
= sum of triangle values
= polygon value.



Our polygon has 18 interior lattice points, 7 boundary lattice points, and an area of $18 + 7/2 - 1 = 20.5$. Any lattice polygon can be divided into lattice triangles in this manner, so we always have $A = I + B/2 - 1$. This result is known as Pick's Theorem, after the Austrian mathematician George Alexander Pick.
Check out the web-page:
http://www.cut-the-knot.com/ctk/Pick.shtml
for more further information.

Eugene Curtin is a professor of mathematics at Southwest Texas State University.

Puzzle Page

Math Explorers: We want to print your work! Send us original math games, puzzles, problems, and activities to swtMathworks, 601University Dr., San Marcos, TX 78666

Word Search

Forwards or backwards, up, slanted, or down.
Where can the words in this puzzle be found?

- Lattice
- Vertex
- Polygon
- Boundary
- Interior
- Pick
- Gasket
- Equilateral
- Midpoint

E	O	I	S	H	R	M	I	S	H	P	W	L	E
O	Q	H	T	E	S	E	Q	U	E	N	A	E	A
N	B	O	U	N	D	A	R	Y	I	R	N	E	F
F	O	A	D	T	R	T	U	D	E	O	A	R	U
D	I	E	Q	L	E	G	C	T	G	S	B	Y	N
A	N	F	E	T	A	E	A	Y	E	C	E	T	E
Z	T	X	B	C	A	L	L	S	T	G	C	I	D
G	E	E	A	N	I	O	E	L	K	R	I	R	U
E	R	T	X	U	P	Y	I	C	T	E	T	A	T
N	I	R	Q	E	B	L	E	D	N	L	T	L	I
T	O	E	M	I	D	P	O	I	N	T	A	I	Q
Q	R	V	N	T	T	I	R	N	S	I	L	M	N
H	T	C	I	T	U	C	S	O	A	H	C	I	S
J	E	E	L	O	O	K	I	E	R	O	C	S	L

Find a 4-digit number ABCD so that if a decimal is placed between B and C, the new number is the average of AB and CD.

Four men check their coats at a cloak room. If the coats are returned at random, what is the probability that exactly two people will get the correct coat back?

How can you plant fourteen trees in seven rows of four trees each?

Bulletin Board

Celebrate Pi Day, March 14

π Day begins precisely at 1:59 p. m. on March 14 (3/14/1:59…) in celebration of the special number and to foster creativity and enjoyment of mathematics by students. It is also Albert Einstein's birthday! Visit the following Websites to learn how you and your school might celebrate Pi Day:
Mathmuseum.org/piday.htm
mathwithmrherte.com/pie_day.htm
www.exploratorium.edu/pi/pi98

Space Day 2003 Design Challenges

Space Day 2003…Celebrating the Future will take place on May 1. Want to work on this year's Space Day Design Challenges: Fly to the Future, Planetary Explorers, and Watt Power? Visit spaceday.com to learn more.

Junior Summer Math Camp 2003

Applications are now available for the SWT Junior Summer Math Camp. The camp will run June 2-13. For application information call 512-245-3439 or check our website at www.swt.edu/mathworks.

Group Photo From Junior Summer Math Camp 2002

Fun Reading!

Some books to check out: **MindGames: Number Games**, **MindGames: Probability Games** by Ivan Moscovich; And **The Warlord's Beads** by Virginia Walton Pilegard.

Order Form (Please type or print)

Send to: Number of subscriptions: ____

Name: ____ Method of Payment:

Address: ____ __Check (payable to SWT Mathworks)

City, State: ____ Zip: ____ __Purchase Order number: ____

School (optional): ____ __Master Card __Visa

Phone: ( ) ____ E-mail: ____ Card Number: ____-____-____-____ Exp. date: ____/____

Amount of Payment: $ ____ Cardholder's Signature ____

Type of Subscription: (Select one. All subscriptions include postage and handling for one year, 4 issues.)

__Individual ($8) __Group ($6, min. 25 subscriptions) __School ($4, min. 100 subscriptions)

Yes! I want to subscribe.

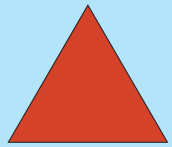
Math Explorer magazine (aimed at grades 4-8) is published four times a year. An annual subscription is $8.00 for individuals, $6.00 for group purchases of 25 or more, and $4.00 for school purchases of 100 or more. For subscriptions, fill out the order form above or contact *Math Explorer* at the address, phone, or e-mail on page 2.

by Joyce Fischer

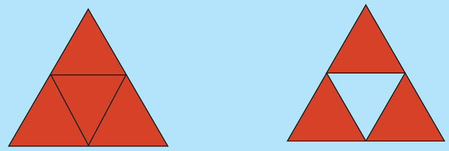
Triangles Once Removed - The Triangular Gasket

In 1916, Sierpinski combined number theory and geometry to portray unique two-dimensional objects, which are in general referred to as gaskets. Since the most basic two-dimensional figure is the triangle, we will begin with an equilateral triangle.

Draw an equilateral triangle.

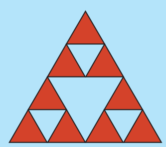


Step 1
Find the midpoint of each side of the original triangle. Connect these points. This will form 4 equilateral triangles. Remove the middle one. What portion of the original triangle have you removed? What portion of the original triangle is left?



Step 2
Find the midpoints of each of the sides of the remaining 3 triangles. Connect them. This produces 12 equilateral triangles. Remove each middle triangle formed. How many triangles did you remove? How many triangles did you leave? Can you describe the area of the remaining triangles?

Step 3
Continue the process.



Creating a table helps us to see some of the patterns in the Sierpinski triangle.

	Triangles Removed	Triangles Remaining
Step 1	1	3
Step 2	3	9
Step 3	9	27
Step 4	27	81

Can you see the powers of 3?

Extensions:
You can begin with any triangle and look for number patterns.
You can also begin with a square. Form 9 squares by dividing each side of the original square into 3 equal pieces and connecting these points. Remove the middle square. Continue this process and look for number patterns.

Dear Math Explorers,

Finding areas of shapes such as rectangles and triangles is not very difficult. However, areas of irregular shapes can be very challenging. Our main article looks at one way to find areas of polygons by counting dots. It sounds so simple! In the Math Odyssey you can see the formation of a beautiful pattern in Sierpinski's Triangle and read about who it's named after in the biography.

Good luck on solving the problems and puzzles and have fun celebrating Math Awareness Month in April!

Sincerely,

Hiroko K. Warshauer

Hiroko K. Warshauer, editor