Math Explorer

MATHEMATICS AND TILING

Ross Leaves Legacy

The Tiling Challenge

Decimals Decimals Decimals......

Vol. 5.4 Summer 2003

Math Explorer

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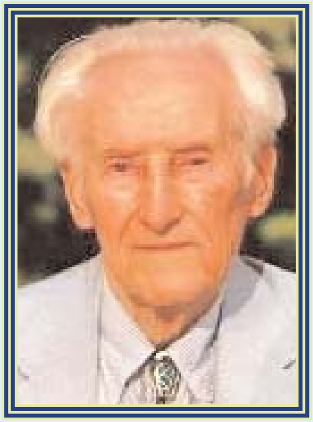


Arnold E. Ross

by Daniel Shapiro

Arnold Ephraim Ross was born in Chicago in 1906 and moved to Russia with his mother in 1909. As a teenager in Odessa; he was one of a small group of talented boys tutored by S.O. Shatunovsky, a distinguished University mathematician. This arrangement was unusual, but there were economic hardships after the Russian revolution and university salaries were small. Shatunovsky worked as a tutor, paid with hard candy rather than unstable paper money. Arnold Ross said often that he gained a deep love of mathematics from Shatunovsky.

Since Ross was an American citizen, he was able to leave the Soviet Union in 1923. He returned to Chicago, learned English, and enrolled at the University of Chicago. There he came under the influence of E.H. Moore, one of the founders of the University's Department of Mathematics. Ross learned from Moore that mathematics is best taught by getting students actively involved with hard problems. Ross continued on to graduate school at Chicago, worked as an assistant to the number theorist L.E. Dickson, and earned his doctorate in 1931.

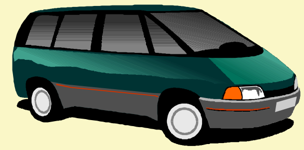
Ross was appointed head of the Math Department at the University of Notre Dame in 1946. In 1957 he developed a residential summer program for high school students talented in math. Following E.H. Moore's methods, Ross used problems in number theory to guide students from numerical explorations to deeper theoretical investigations. He chose number theory since the motivating questions can be understood easily, but the answers (if they are known) require a variety of methods, from elementary to advanced. Every summer he convinced students that mathematical abstraction is a powerful and accessible tool.

In 1964 that summer program moved to Ohio State University when Ross became the chairman there. Ross retired from OSU in 1976 (after nearly 30 years as a math department chairman), but he continued his summer program, stopping only in 2000 when incapacitated by a stroke. Arnold Ross died in September 2002. He is remembered by the hundreds of people who participated in his Program or were influenced by his ideas in math education. During his long career, Dr. Ross influenced generations of young mathematicians and scientists to "think deeply of simple things".

(continued on page 7)

1. Becky has 7 coins with a total value of 24 cents. How many nickels does Becky have?

2. A parking lot has 85 motor vehicles, mostly cars (4 wheels each) and some motorcycles (2 wheels each). If the total number of wheels is 320, how many motorcycles are there?



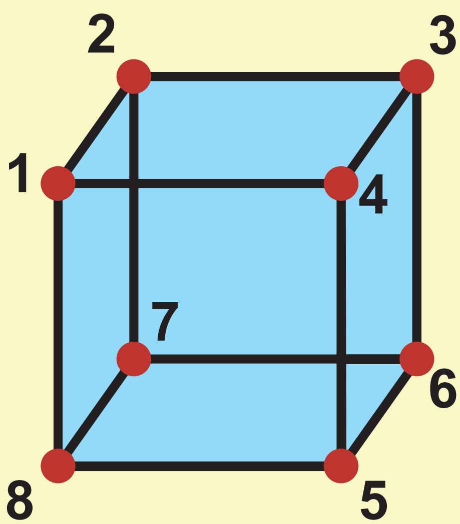
3. Jacob's phone number is 777-222x, and it is divisible by 11. What digit does the x represent?

4. What is the next term in the sequence: 1, 4, 27, 256,..?

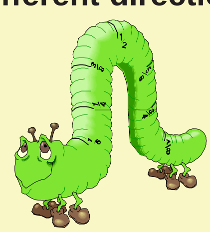
5. When Jack and Jill play chess, Jack wins 30% of the games, Jill wins 40%, and they draw the remaining 30% of the games. What is the probability that Jack will win the next decisive game?

6. The number 2003 has digit sum 2+0+0+3=5. How many 4 digit numbers have the same digit sum as 2003?

7. If Isaac spends 10% of his life watching television, on average how much time does he spend watching each day?

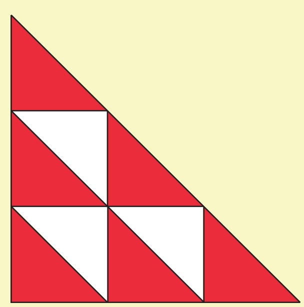


8. An inchworm starting at corner 1 of a room wants to visit each of the other corners exactly once and then return to its initial corner. How many routes can it take if it can only move along the edges? Count the routes 1-2-3-4-5-6-7-8-1 and 1-8-7-6-5-4-3-2-1 as different routes, since the inchworm is travelling in different directions.



9. The year number 2003 is prime. What is the next prime year number?

10. All of the red and white triangular tiles in the figure have the same size and shape. If each of the tiles has a perimeter of 1 foot, what is the perimeter of the tiled area?

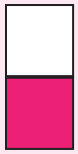


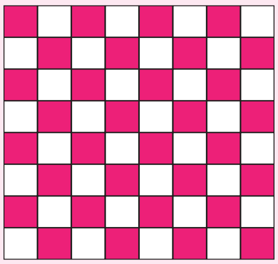
Answer: 1) 2 2) 10 3) 6 4) 3125 5) 3/7 6) 35 7) 2 hours 24 minutes 8) 12 9) 2011 10) 3 feet

The Tiling Problem

by Daniel Shapiro

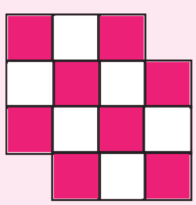
Can a checkerboard be tiled with dominoes? Here a "domino" is a 1 x 2 rectangle, fitting exactly into two adjacent squares of the board. A shape is "tiled" when it is exactly covered, with no missing areas, no overlaps, and no overhangs. A checkerboard is an 8 x 8 rectangle containing 64 squares, and it's easy to tile it with 32 dominoes. For instance they can all be aligned on the board vertically.





If one square of the checkerboard is removed, the resulting figure has no domino tiling. This is easy to see by thinking about areas. Each domino has area 2 (each square counts as one unit of area). If some number k of dominoes tiles a figure, then the total area covered is $2k$, an even number. However our figure (a checkerboard missing one square) has area $64 - 1 = 63$, an odd number. So it's impossible to tile it with dominoes.

If two opposite corners of a checkerboard are removed, the resulting figure has $64 - 2 = 62$ squares. The area is even and it seems that 31 dominoes might tile that figure. To begin, let's consider a smaller version of the problem, a 4 x 4 board with two corners removed. After experimentation with 7 dominoes, I suspected that this figure has no domino tiling. Is there a convincing way to prove this impossibility? Supposing a domino tiling does exist, let's try to discover some contradiction, some impossibility.



One domino must cover the upper left corner. Suppose that domino runs horizontally, and check that various other dominos are forced to lie in certain positions. After a sequence of these forced moves, an impossible situation arises. [Try it!] If that first domino runs vertically there is a different sequence of forced moves, again ending in an impossible situation. This shows that there can be no domino tiling.

That's a good method, but what about the original 8 x 8 case? The "forcing" arguments become harder to work through. Luckily there is a different proof that settles this question easily.

In the checkerboard coloring pictured, each square is white or red. Any domino placed on the board (whether vertical or horizontal) must cover one square of each color. Therefore 7 dominoes must cover 7 white and 7 red squares. There is no way that 7 dominoes can cover the given figure because it consists of 6 whites and 8 reds. Similarly the full 8 x 8 board has 32 white and 32 red squares. If we remove two squares of the same color (like two opposite corners) the remaining 62 squares form a figure with unequal numbers of white and red squares. But any figure tiled by dominoes must have an equal number of whites and reds. This "coloring proof" establishes the impossibility of domino tilings in a clever way.

What if we remove one white square and one red square from that 8 x 8 checkerboard? There are many different ways that this can be done. Must the resulting figure of 62 squares admit a domino tiling? I won't provide the answer here. Experiment by removing different squares and placing dominoes in that region. Proving that tilings exist requires explicit constructions, work that needs to be done for each different pair of missing squares. Here's an interesting point: Proving that tilings exist can be more difficult than proving they don't exist!

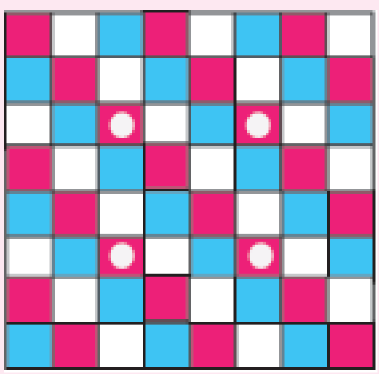
Since dominoes worked well, let's generalize to "trominoes."

We use the straight tromino made of three squares in a row. Which boards can be tiled with those trominoes?

For instance an 8 x 8 board has no tromino tiling. To see this, note that a tromino has area 3 so a figure tiled by k trominoes has area 3k. The checkerboard area is 64, and 64 = 3k is impossible for a whole number k. Therefore no tromino tiling can exist.

If one square is removed from the checkerboard, 63 squares remain. Here's our main question: Can that figure be tiled with 21 straight trominoes? The areas match, but I tried removing a square and couldn't find a valid tiling. Is a tiling always impossible no matter which square is missing? The red and white coloring trick doesn't work very well here. Instead let's use three colors as in the picture and try for a contradiction.

The key observation here is: Any straight tromino on the board (placed horizontally or vertically) must cover one square of each color.



Suppose a square is removed and 21 straight trominoes do tile the remaining figure. Using our key observation, we see that the whole figure must contain exactly 21 squares of each color. For the whole 8 x 8 board I count 22 reds, 21 blues, and 21 whites. So if a tiling exists, the square that was removed must be red.

For instance, if the upper right corner square is missing, then no tiling by straight trominoes is possible (because the square removed is not red in our color pattern). What if we removed the upper left corner, colored red in the picture? The impossibility of a tiling does not follow by counting those colors. However if there is a tiling for the board with a missing upper left corner, we can turn the picture by a 90 degrees to exhibit a tiling for a board with a missing upper right corner. Since that case was already proved impossible, the board with a missing upper left corner is also impossible to tile.

The color-counting and turning methods show that tiling by straight trominoes is impossible for most positions of the missing square. With a little thought we find that the only times there might possibly be tilings are on the boards where the missing square is one of the red squares marked with a dot in the picture to the left (they stay red when the picture is turned). If one of those squares is removed from the board, can the figure be tiled? Draw a board, erase one of those squares, and try to tile it with straight trominoes. It can be done. We have now completely answered our question of which checkerboard-with-a-missing-square figures can be tiled with straight trominoes!

Many tiling questions can be analyzed by similar methods. Here are some for you to try:

1. Which m x n boards can be tiled by the straight tetromino (four squares in a row)? For example, can a 10 x 10 board be tiled with 25 of those tetrominoes?

2. Which m x n boards can be tiled by copies of a 2 x 3 block? Areas show that if there is a tiling then mn is a multiple of 6. Can a 6 x 5 board be tiled?

3. Which m x n boards can be tiled with copies of the L-tromino? It's easy to tile a 2 x 3 board. Can a 5 x 9 board be tiled with L-trominoes?

Many puzzles, patterns, and problems of this nature appear in the wonderful book "Polyominoes", by Solomon W. Golomb. (Published by Princeton Univ. Press, Princeton NJ, 1994).

Daniel Shapiro is a professor of mathematics at Ohio State University. He is a frequent contributor to Math Explorer.

Puzzle Page

Math Explorers: We want to print your work! Send us original math games, puzzles, problems, and activities to swtMathworks, 601University Dr., San Marcos, TX 78666

If there are 295,123,015 people and each person has at most 999 hairs on his/her head, what is the smallest value for the maximum number of people with the same number of hairs?

Word Scramble

ITNGIL

YVLELRATCI

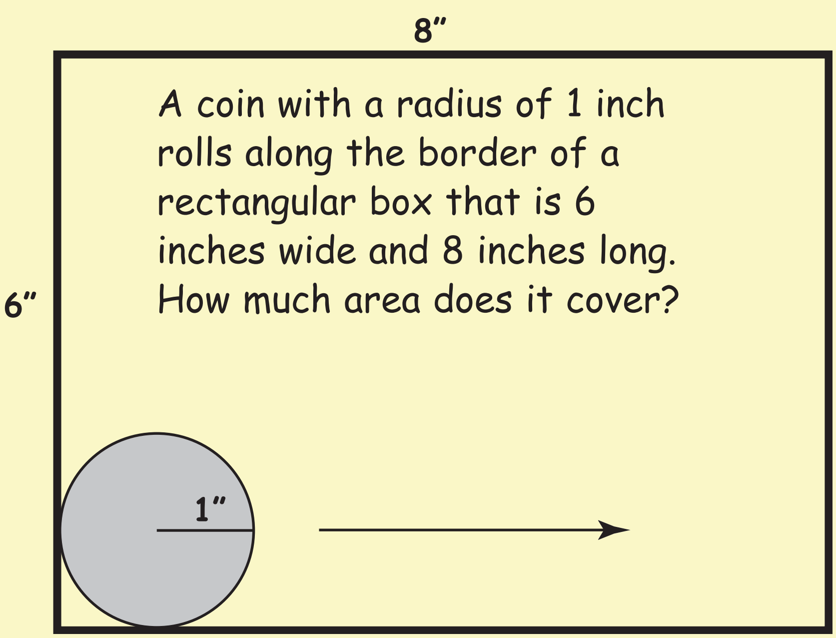
ONZTIARLOLHY

MDOISNEO

CBHOEACRKDRE

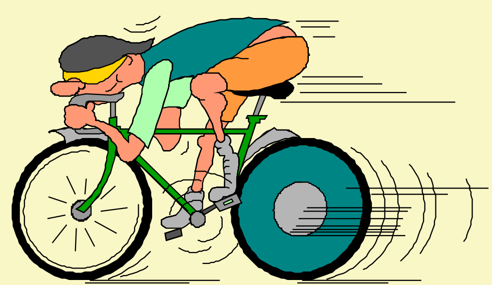
ESRQAU

SPNARTET



A coin with a radius of 1 inch rolls along the border of a rectangular box that is 6 inches wide and 8 inches long. How much area does it cover?

A bicyclist averaged 10 mph for 1/5 of his trip and 20 mph for the other 4/5 of his trip. What was his average speed?



Bulletin Board

April is Math Awareness Month

Celebrate mathematics all month long! Check the following website for some possible projects or events to have at your school. You can request a free poster! http://mathformu.org/mam/03

Ross Summer Math Program

The Ross Program at Ohio State University is an intensive course in mathematics for pre-college students. In this eight week summer program, students are immersed in a world of mathematical discovery. The program was founded by the late Dr. Arnold Ross and is now directed by Dr. Daniel Shapiro. For more information visit: http://www.math.ohio-state.edu/ross/introduction.html

Siemens Westinghouse Competition Regional Finalists

Congratulations to nine of the SWT Honors Summer Math Campers who competed in the prestigious Siemens Westinghouse Competition in Math, Science and Technology for 2002. The semifinalists were Felicia Alderete, Syed Ashrafulla, William Boney, Araceli Fernandez, Andrew Hsiau, and Diya Banerjee. The team of Hannah Chung, Alan Taylor and Jeremy Warshauer, mentored by SWT faculty member Daniela Ferrero, went on to be Regional Finalists.

Arnold E. Ross...continued from pg. 2

This educational legacy is being continued in three summer math programs run by Ross Program alumni:

* The Ross Mathematics Program at Ohio State University,
  http://www.math.ohio-state.edu/ross
* PROMYS at Boston University,
  http://math.bu.edu/INDIVIDUAL/promys
* Honors Summer Math Camp at Southwest Texas State University,
  http://www.swt.edu/mathworks

These programs continue the tradition of excellence passed from S. O. Shatunovsky and E. H. Moore, through Arnold Ross, and on to the present generation.

MATH ODYSSEY

by Nathan Warshauer

Repeating Decimals

When doing mathematics, division and multiplication are sometimes seen as opposites. When you multiply by the number 1/2 it is the same as dividing by the number 2. Dividing by 4 is the same as multiplying by the number 1/4. However, multiplying by 1/2 is actually multiplying by the number 0.5. This is called the decimal representation of a number. The decimal representation of 1/4 is 0.25. What is the decimal representation of the number 1/10? If you said 0.1, you're right! Finding the decimal representation of a number uses the technique of long division. Let's now explore finding the decimal representation of 1/3 . First, we set up the division of 3 into 1.

```
     .3
3 ) 1.00
   - 9
   ----
     10
```

How many times does 3 go into 1.0? 0.3 times is right, so we have a remainder of 1 and bring down a zero.

Disregarding the decimal point, how many times does 3 go into 10? 3 is right! Our remainder is 1 and we are again asked how many times 3 goes into 10. The problem here seems to be that the remainder is staying the same. We say that the number 1/3 has a repeating decimal representation 0.333333333… . Some mathematicians thought a good way to express this kind of repeating pattern was by writing a line over the repeated part, and therefore we write 1/3 as $0.\overline{3}$.

How about the number 1/6? Try using long division to figure out how to write 1/6 as a repeating decimal. Remember to write the line only over the repeating part. How about 5/6? For a challenge, try to figure out the decimal representation of the number 1/7.

Now what happens when we try to go in the other direction? How about the repeating decimal 0.11111…=$0.\overline{1}$? Can you figure out what fraction is equal to this number? A good place to start might be multiplication, so let's try multiplying the number by 10. This gives us 10 x 0.111…= 1.111… This has the same decimal part as our first number, so we can subtract the first number and get 1.111… - 0.111…= 1. Since we started with ten of 0.111… and subtracted one of 0.111… we know that nine of 0.111… must be 1. If we know that 9 multiplied by something equals 1, the something must be 1/9, because the opposite of multiplying by 9 is dividing by it!

Can you figure out what fraction is equal to the repeating decimal $0.\overline{4}$? If you get stuck, try multiplying by a number like 10 and subtracting the repeating decimal to get a number with no decimals. Did you get that 9 times this fraction should equal 4? Can you then find a number that when multiplied by 9 equals 4? (We sometimes call this the fraction 4/9.) So $0.\overline{4}$ = 4/9. Keep exploring to see if you can find any other symbols in mathematics that might be opposites. Happy hunting!

Dear Math Explorers

We dedicate this issue of Math Explorer to Dr. Arnold Ross, who influenced many young people through his summer mathematics program at Ohio State University. Daniel Shapiro, author of our main article and biography, is a former student in the Ross Program and is now its director. Our Math Odyssey writer, Nathan Warshauer, was also a student in the Ross Program for two years, as was his father, Max, a Senior Editor for *Math Explorer*. We are indebted to Dr. Ross for personally touching our lives and for the legacy he has left future participants. We include information on summer camps that you may wish to investigate for this summer or for future ones.

Our special thanks to Doug Dunham of the University of Minnesota, Duluth for the beautiful graphic that we have on the magazine cover. Have a safe and wonderful summer. Join us next fall as we continue our math exploration!

Sincerely,

Hiroko K. Warshauer

Hiroko K. Warshauer, editor