Caroline Herschel

by Jean Davis

Caroline Lucretia Herschel was born March 16, 1750 in Hannover, Germany. Her father was a musician and tried to give his six children, including the two girls, the best education he could. Caroline's mother, however, disapproved of learning and only wanted her daughters to do household chores.

All four of Caroline's brothers were trained as musicians. In 1772, Caroline went to live with her brother, William, who was an organist in England. Not only did he give her singing lessons, but he taught her mathematics and astronomy as well. She became quite an accomplished singer, giving many successful performances.

Caroline devoted the next few years to helping her brother with astronomy. She learned trigonometry so that she could do the tedious mathematical calculations for him. Caroline was always interested in mathematics as it applied to science.

In 1781, William discovered the planet Uranus and became a full-time astronomer. Caroline regretfully gave up music to devote herself to helping her brother. She also spent whatever time she could find on her own research. On August 1, 1786 Caroline discovered her first comet. She discovered a total of eight comets between 1786 and 1797. In 1788, William married, and Caroline turned her efforts to her nephew, John Herschel. With her help and guidance, he too became a famous mathematician and astronomer.

In 1828 Caroline completed her star catalogue of 2500 nebulae and was awarded a gold medal by the Royal Astronomical Society of London. She remained physically and mentally active all her life, entertaining the crown prince of Germany on her 97th birthday! She died January 9, 1848. In 1889, a minor planet was named Lucretia in her honor.

Reference: www.groups.dcs.st-and.ac.uk/~history

Jean Davis teaches math at Southwest Texas State University.
PROBLEMS OF THE MONTH

Read the main article before doing problems 1-3.

1. A composer decides to use the time signature $\frac{3}{4}$. How many different rhythmic ways can she write one measure using only quarter notes and eighth notes?

Here is an example:

\[
\frac{3}{4} \quad \boxed{\begin{array}{|c|c|c|c|}
\hline
\text{quarter notes} & \text{eighth notes} \\
\hline
\hline
\end{array}}
\]

2. Use a combination of two or more notes to equal the given notes below. Write at least two different possibilities for each and show how the fractions add up. Clap the rhythms to hear the difference.

\[
\begin{align*}
\frac{1}{\text{quarter note}} &= \\
\frac{1}{\text{eighth note}} &= \\
\frac{1}{\text{dotted eighth note}} &= 
\end{align*}
\]

3. Write the music notation for the measures that would have the following note values. What would be a reasonable time signature? Is there more than one possible answer?

\[
\begin{array}{|c|c|c|c|}
\hline
\frac{1}{8} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} & \frac{3}{8} + \frac{1}{8} + \frac{1}{4} & \frac{3}{8} + \frac{3}{8} & \frac{1}{4} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8} + \frac{1}{4} \\
\hline
\end{array}
\]

4. Draw a picture explaining why the sum of the fractions $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ is less than one. What is the sum? (You can use the picture or something else to help you.) What happens to the picture if you add the fraction $\frac{1}{16}$ to this sum?

5. The sum $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ is more than 1. How many more fractions in this pattern (the next two fractions would be $\frac{1}{5}$ and $\frac{1}{6}$) do you need to add to this sum to get a sum greater than 2? What about a sum greater than 3?

6. A 12 hour digital clock displays time in hours and minutes. In one 12 hour span, how many minutes will have a 1 as one of the digits displayed on the clock?

(continued on pg.7)
by Stephen Redfield and Hiroko Warshauer

Listening to music can affect our moods and feelings. Some songs give us energy and make us want to dance, while others soothe us and put us to sleep. Some songs can express happiness in one part and turn to sadness in the next. Though we might think of music as a purely emotional experience, the science of acoustics (a-KOO-stiks) describes the “physical basis of music,” or the way our ears actually hear music. The four categories of acoustics: pitch, timbre (TAM-ber), loudness and duration, can each be measured in mathematical terms. This can be done by counting the number of vibrations that travel through the air to our ear drums, as well as by graphing the shapes, widths and lengths of those vibrations. These pages will focus on duration, or in musical terms, rhythm (RITHeM).

Have you ever found yourself tapping your foot to music? If so, you were tapping to the beat! In music where the beat is regular, it acts like your body’s pulse, in an even, ongoing pattern. (The tick-tock of a watch or clock is another example of a regular beat.) Musicians combine and divide these beats into patterns to create rhythm. In order to communicate their ideas, they developed a system to write down rhythm called notation. Composers, the people who write music, add beats together into equal-sized groups, called measures. In example 1, we see four beats, each shown with an oval note-head and stem, divided into two measures along a horizontal line. The measures are divided by vertical measure lines.

(Ex. 1)

\[
\frac{4}{4}
\]

At the far left of Example 1 is a time signature, which looks a bit like a fraction: \( \frac{4}{4} \)

The upper number says that there are four beats in one measure. The lower number says that a quarter note gets one beat. As you can see in the example, the four quarter notes grouped together fill out a whole measure (\(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4}\)).

Composers can combine beats within the measures. A note that gets two beats, its sound lasting two beats without a break, is called a half note, (\(\frac{1}{4} + \frac{1}{4} = \frac{1}{2}\)). Notice that the note-head of a half note is hollow. Example 2 shows the first two beats of the first measure expressed with a half note, and the last two beats expressed with two quarter notes.

(Ex. 2)

\[
\frac{4}{4}
\]
COUNTS

Can you express a full measure using another combination of quarter and half notes in the first measure below? How about a full measure using only half notes in the next?

\[ \frac{4}{4} \]

You may have predicted that there is a whole note written with a hollow note head and no stem at all, which gets four beats. One whole note lasts a full measure, the same length of time it takes to sound two half notes or four quarter notes.

\[ 1 = \frac{1}{2} + \frac{1}{2}, \quad 1 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \]

Did you notice that a quarter note = 1 beat, a half note = 2 beats, a whole note = four beats? How do we show a note that gets three beats? The answer is a dotted half note. The small dot following the note head stands for half of that note head's value. This means:

\[ \dottedhalfnote = \frac{1}{2} + \cdot \]

or three beats = 2 beats + 1 beat. We should play a full two beats plus another single beat without a break.

Can you fill each of the empty three measures in Example 3 with exactly four beats, not fewer, not greater, using combinations of the notes discussed? All the note names in a measure should add up to 1 as in the example.

(Ex. 3)

\[ \begin{array}{c}
\dottedquarternote \\
\dottedquarternote \\
\hline
\frac{3}{4} + 1 = 4
\end{array} \]

So far, we have combined beats to make notes. Surely we can make even more notes by dividing beats! Guess the name of the note which gets half the length in sound of a quarter? Yes! An eighth note. Two eighth notes evenly divide a single beat (\( \frac{1}{8} + \frac{1}{8} = \frac{1}{4} \)). How many eighths does it take to fill out a full measure? Here's a sixteenth note:

It's value is half an eighth note. How many fit into a quarter note? What's half of a sixteenth note, and how do you suppose it looks?

<table>
<thead>
<tr>
<th>note name</th>
<th>symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Can you guess the name of this note, and how many of them fit into a whole note?

Who would have thought that we could use mathematics to define and explain something like rhythm in music? The math used in the other areas of acoustics is truly fascinating and ever so much more complicated! Whether listening to Bach or the Beatles, see if you can hear and understand the rhythmic structure in the songs and appreciate not only the music but the mathematics behind it!

Stephen Redfield is a violinist and teaches at the University of Southern Mississippi
Word Search

Forwards or backwards, up, slanted, or down.
Where can the words in this puzzle be found?

ACOUSTICS
QUAINSGATBUCOS
TERBMITCVHAJBX
EVDBUGFORYTQTZ

TIMBRE
BETYKCEKUPNHTC
EAMHRJBTFRTQS

RHYTHM
CMNIBPLSXMYACU
NEAHFUIBTGERMR

TEMPO
EAMHRJBTFRTQS

MEASURE
OSIMEJCCNNECI
BUCEAVGSNHRWN

BEATS
EAMHRJBTFRTQS

MUSIC
ZRSRBFTFYXKT

PITCH
KESTDJCISUMRKE
TETKVLXRXWEXNAO

DYNAMICS
KAJMHTYHRMLVMGQ
HONYMVOMZDWRG

QUARTER

Cut out the one-eared bunnies and position them so that every bunny has two ears.

Which frame below will this object exactly fit through? Can you draw an object that will exactly fit through the frames?
World Math Contest

The Po Leung Kuk 4th Primary Mathematics World Contest (PMWC) organized by Hong Kong Po Leung Kuk will be held July 15-20, 2000 in Hong Kong. For more info e-mail: eadmaths@poleungkuk.org.hk

Check it out!

Mathematician John Allen Paulos has a column posted the first day of every month on mathematics. See the ABC News website: http://abcnews.go.com/sections/science/whoscounting_index/whoscounting_index.html

Math in Flight

Looking for some math activities related to flight? Try the site from NASA Dryden Flight Research Center! Find them at: http://daniel.calpoly.edu/~dfrc/Robin/midd.html

Did you know?

In the year 2000 there will be a census of the U.S. population. This occurs every 10 years. Why? It's in the Constitution of the United States. Find a copy and look under Article I, Section 2. You can get online information from the US Census Bureau at http://www.census.gov/dmd/www/schindex.htm

Problems of the Month (Cont’d)

7. A double-dot is also used in music. For example, \( \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \) has note value \( 1/2 + 1/4 + 1/8 \). What is the note value of \( \frac{1}{4} \)? What would a triple dot \( \frac{1}{4} \) mean?

8. Is there some combination of notes (half, quarter, eighth, etc) that combine to produce a note value of \( 1/3 \)? For example, \( \frac{1}{4} \) has a note value \( 1/4 \), smaller than \( 1/3 \), and \( \frac{1}{4} + \frac{1}{8} \) has note value \( 1/4 + 1/8 = 3/8 \), greater than \( 1/3 \).
Dear Math Readers,

The fascinating relationship between mathematics and music is the theme of this last issue of Math Reader for 1999-2000. From counting beats to looking at intervals between notes as ratios, mathematics is an important part of music.

We at Math Reader have enjoyed exploring, learning, and growing with you over this past year. Next year we hope to continue bringing you exciting mathematics and relating it to the world around us.

Have a wonderful summer and we hope you'll join our Math Reader family again next fall.

Sincerely,

Hiroko K. Warshauer

Hiroko K. Warshauer

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Next year's magazines will be quarterly -- an issue each season -- bringing you biographies, puzzles, problems and articles that connect mathematics to the world around us. Be sure to subscribe now so you don’t miss the fall issue!