Po Leung Kuk 17th Primary Mathematics World Contest

Individual Contest

1. The LCM of a and b is 9800. The LCM of a and c is 2200. The LCM of b and c is 539. Find each of the numbers a, b and c.

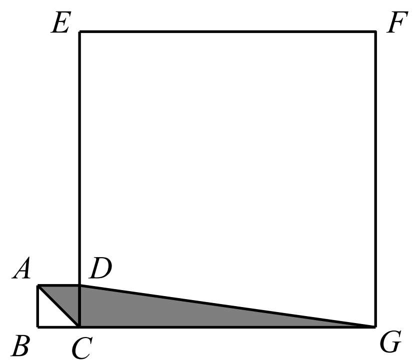
2. The numbers A, B, C, D, E, F, G and H form an arithmetic sequence. The sum of the two middle numbers (D and E) is 16. What is the sum of these eight numbers? (In an arithmetic sequence, each number, after the first one, is obtained by adding the same amount to the previous number.)

3. There are five consecutive positive integers, $a < b < c < d < e$. It is given that $b+c+d$ is a perfect square and $a+b+c+d+e$ is a perfect cube. What is the smallest possible value of c ?

4. Given $\overline{abcd}$ and $\overline{efgh}$ are both four-digit numbers (a, b, c, d, e, f, g and h are all different digits) such that $\overline{abcd} - \overline{efgh} = 2014$. Write down one possible solution on the answer sheet.

5. What is the largest six-digit number, $\overline{x2014y}$, that is divisible by 33?

6. Two squares $ABCD$ and $CGFE$ are joined as shown in the figure, the ratio of the shaded areas ACD to CGD is 1:7, what is the ratio of the unshaded areas ABC to $EDGF$?



7. On a particular day, a cyclist left town A at 9:00 AM and arrived in town B at 12:40 PM. On the same day, a motorcyclist left town A at 10:00 AM and arrived in town B at 12:00 PM. Given that they were both travelling at a constant speed, at what time of the day did the motorcyclist meet the cyclist?

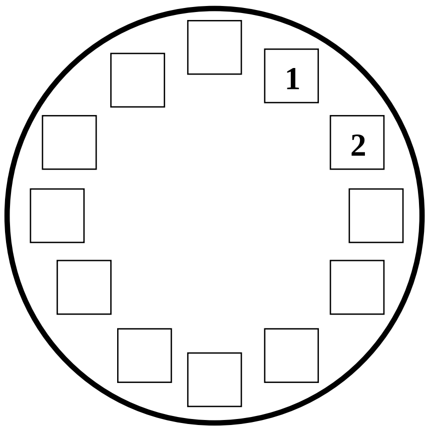
8. The sum of the reciprocals of four positive integers *a, b, c* and *d* (not necessarily different) is $\frac{7}{10}$, i.e. $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}=\frac{7}{10}$. What is the smallest possible sum of these four integers?

9. Alex must take the sum of 11 consecutive positive integers. But he is very careless and misses two consecutive numbers and gets a total of 9832. What total would he get if he counted correctly?

10. In a mathematics contest, 100 students had to solve four problems. No student solved all four problems. The first problem was solved by 90 students, the second problem was solved by 80 students, the third problem was solved by 70 students and the fourth problem was solved by 60 students. How many students solved both the third and fourth problems?

11. If the five numbers, $2^{147}\times 6^{49}$, $3^{98}\times 5^{49}$, $5^{98}\times 2^{49}$, 7^{98} and $128^{7}\times 23^{49}$, are arranged in ascending order, which number will be in the middle?

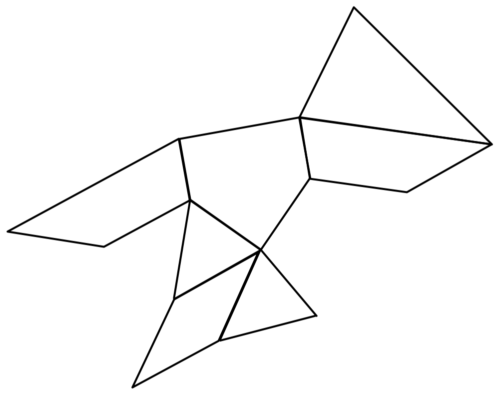
12.



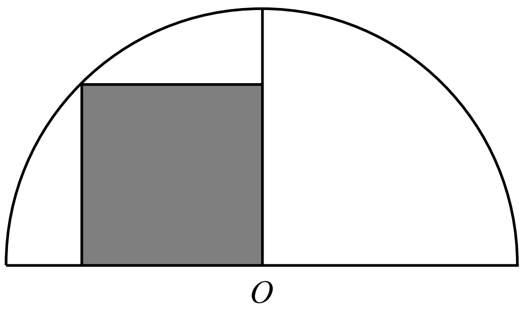
The positive integers 1 to 12 are placed in boxes around a circle. For any three consecutive boxes A, B and C, box B is between box A and box C on the circle. The numbers in the boxes A, B and C are a, b and c respectively. It is given that $b^2 - a \times c$ (not necessarily positive) is a multiple of 13. The numbers 1 and 2 have already been placed. Now place the numbers 3 to 12 in the rest of the boxes.

13. The positive integers m and n satisfy the identity $(m+6n)(n+6m)=2014$. Find the largest possible value of m.

14. The figure below represents the net of a solid, once the solid is formed how many vertices will it have?



15. Area of the shaded square region is 8 cm^2. Find the area of the semi-circle where O is the centre of the full circle. {Take $\pi = 3.14$}



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Team Contest

1. What is the maximum number of different positive integers that can be added to give a total of 2014?

2. Replace each star with one of the digits 1, 2, 3, 4, 5, 6, 7 and 8 in the equation below. Each digit must be used exactly once. Give one possible solution.

$$\frac{*}{*} + \frac{*}{*} = \frac{*}{*} + \frac{*}{*}$$

3. An integer *N* is formed by writing the integers from 1 to 50 in order. That is *N* = 123456789101112 … 484950. Some digits are removed from *N* to form a new integer such that the sum of the digits of the new number is 200. If *M* is the largest number that can be formed in this way, what are the first ten digits of *M*?

4. Tom is asked for the last digit of $1^1 + 2^2 + 3^3 + \cdots + 1007^{1007}$. Jack is asked for the last digit of $1008^{1008} + 1009^{1009} + 1010^{1010} + \cdots + 2014^{2014}$. What is the sum of the digits that Tom and Jack found?

5. It is known that in the numbers *A* and *B* each digit is larger than the digit to its left. $A^2 = B$ and *B* is a six-digit number. What is the number *A*?

6. Pooky writes down all the positive integers which have a sum of the digit(s) equal to 9 in increasing order: 9, 18, 27, 36,… What is the 2014^{th} number in this list?

7. Let point *K* lie on the side *AB* and point *M* lie on the side *BC* of triangle *ABC* such that $\frac{AK}{KB} = \frac{BM}{MC} = \frac{2}{1}$. *AM* intersects *CK* at point *T*, and *BT* intersects *AC* at point *N*. What is the area of triangle *ATN*, in cm^2, if the area of triangle *AKT* is 20 cm^2?

8. Choose five different digits from 0 to 9 to form a five-digit number such that the number formed is a multiple of 495. (The first digit **cannot** be zero.) How many such numbers are there?

9. Gary writes the digits ‘1’ to ‘9’ into the cells of a 3×3 square such that the sum of each row is 15 and the sum of each column is 15 (the diagonals do not necessarily add up to 15). The square is shown below and the digits are represented as letter ‘*A*’ to ‘*I*’.

A	*B*	*C*
D	*E*	*F*
G	*H*	*I*

Three students guess two digits in this square as follows:

- *Peter* guesses “A = 2” and “C = 9”.
- *Tom* guesses “E = 5” and “F = 8”.
- *Sam* guesses “G = 3” and “H = 7”.

Gary notes that one student guesses correctly for both digits, one student guesses only one digit correctly and one student guesses wrong for both digits. Find the product of $B \times D \times F \times H$.

10. The 125 unit-cubes are formed into a large 5×5×5 cube which is placed on a table as shown in the figure.
Step 1:
If at least 3 sides of any unit-cube can be seen, all those cubes must be removed. e.g. the four top shaded cubes are removed.
Step 2 to step 5:
This process is repeated on the new solid.
After step 5, how many unit-cubes still remain in the solid?