Permutations and Combinations

Type	Formulas	Explanation of Variables	Example
Permutation with repetition (Use permutation formulas *when order matters* in the problem.)	n^r	Where *n* is the number of things to choose from, and you choose *r* of them.	A lock has a 5 digit code. Each digit is chosen from 0-9, and a digit can be repeated. How many different codes can you have? n = 10, r = 5 10^5 = 100,000 codes
Permutation without repetition (Use permutation formulas *when order matters* in the problem.)	$\frac{n!}{(n-r)!}$	Where *n* is the number of things to choose from, and you choose *r* of them. Sometimes you can see the following notation for the same concept: $P(n,r) = {}^nP_r = {}_nP_r = \frac{n!}{(n-r)!}$	How many ways can you order 3 out of 16 different pool balls? n = 16, r = 3 $\frac{16!}{(16-3)!} = 3{,}360\ ways$
Combination with repetition (Use combination formulas *when order doesn't matter* in the problem.)	$\frac{(n+r-1)!}{r!\,(n-1)!}$	Where *n* is the number of things to choose from, and you choose *r* of them.	If there are 5 flavors of ice cream and you can have 3 scoops of ice cream, how many combinations can you have? You can repeat flavors. n = 5, r = 3 $\frac{(5+3-1)!}{3!\,(5-1)!} = 35\ combinations$
Combination without repetition (Use combination formulas *when order doesn't matter* in the problem.)	$\frac{n!}{r!\,(n-r)!}$	Where *n* is the number of things to choose from, and you choose *r* of them. Sometimes you can see the following notation for the same concept: $C(n,r) = {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$	The state lottery chooses 6 different numbers between 1 and 50 to determine the winning numbers. How many combinations are possible? n = 50, r = 6 $\frac{50!}{6!\,(50-6)!} = 15{,}890{,}700\ combinations$

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Resource: http://www.mathsisfun.com/combinatorics/combinations-permutations.html
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Examples

1) **Mr. Smith is the chair of a committee. How many ways can a committee of 4 be chosen from 9 people given that Mr. Smith must be one of the people selected?**

Mr. Smith is already chosen, so we need to choose another 3 from 8 people. In choosing a committee, order doesn't matter, so we need the combination without repetition formula.

$\frac{n!}{r!(n-r)!} = \frac{8!}{3!(8-3)!}$ = 56 ways

2) **A certain password consists of 3 different letters of the alphabet where each letter is used only once. How many different possible passwords are there?**

Order does matter in a password, and the problem specifies that you cannot repeat letters. So, you need a permutations without repetitions formula. The number of permutations of 3 letters chosen from 26 is

$\frac{n!}{(n-r)!} = \frac{26!}{(26-3)!}$ = 15,600 passwords

3) **A password consists of 3 letters of the alphabet followed by 3 digits chosen from 0 to 9. Repeats are allowed. How many different possible passwords are there?**

Order does matter in a password, and the problem specifies that you can repeat letters. So, you need a permutations with repetitions formula.

The different ways you can arrange the letters = $n^r = 26^3 =$ 17,576
The different ways you can arrange the digits = $n^r = 10^3$ = 1,000
So the number of possible passwords = 17,576 × 1,000 = 17,576,000 passwords

4) **An encyclopedia has 6 volumes. In how many ways can the 6 volumes be placed on the shelf?**

This problem doesn't require a formula from the chart. Imagine that there are 6 spots on the shelf. Place the volumes one by one.

The first volume to be placed could go in any 1 of the 6 spots. The second volume to be placed could then go in any 1 of the 5 remaining spots, and so on. So the total number of ways the 6 volumes could be placed is
6! = 6 × 5 × 4 × 3 × 2 × 1 = 720 ways