

Texas State Topology Seminar

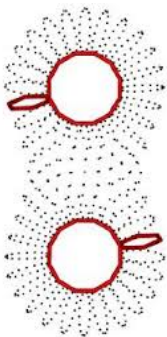
Thursday, 2018, September 27, 2:00-3:00 p.m., in DERR 227

Speaker: David Snyder

Topic: *Generalized Dunce Hats Are Not Splittable*

ABSTRACT

In 2008, David Gabai proved an astonishing result: the Whitehead manifold (a contractible non-compact 3-manifold that is not homeomorphic to \mathbb{R}^3) is *splittable*, a union of two open 3-manifolds whose intersection is a 3-manifold, all three of which are homeomorphic to \mathbb{R}^3 . A *generalized dunce hat* is a 2-dimensional polyhedron created by attaching the boundary of a disk Δ to a circle J via a map $f: \partial\Delta \rightarrow J$ with the property that there is a point $v \in J$ such that $f^{-1}(\{v\})$ is a finite set containing at least 3 points and f maps each component of $\partial\Delta - f^{-1}(\{v\})$ homeomorphically onto $J - \{v\}$. Theorem: No generalized dunce hat is the union of two proper subpolyhedra that each have finite first homology groups. This result undermines a strategy for proving that the interior of the Mazur compact contractible 4-manifold M is splittable in the sense of Gabai (i.e., $M = U \cup V$ where U, V and $U \cap V$ are each homeomorphic to Euclidean 4-space).



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