WORD PROBLEMS

The ONLY formula we need to solve this kind of problems is:

\[ \text{Base} \times \text{Rate} = \text{Percentage} \]

where:

- **Percent (Rate):** A fraction whose denominator is 100.
- **Percentage:** The product of a rate (percent) and another number called the base.

Percentages have the unit of the base and the description of the rate.

**Example 1:** ITT graduated 120 students from a math class after having "washed out" 40 of them. What percent of the class graduated?

120 = number of students graduated
40 = number of students not graduated
percent of students graduated = ???

The total number of students in the class is 120 + 40 = 160
If 120 graduated out of 160, then the percent is \( \frac{120}{160} = 0.75 = 75\% \)

**Example 2:** A company produced 800 good shafts and 30 defective shafts. What percent of their production was defective?

800 = number of good shafts
30 = number of defective shafts
percent of defective shafts = ???

The percent is \( \frac{30}{830} = 0.0361 = 3.61\% \) \( (830=800+30) \)

**Example 3:** Mary received $90 for her weekly allowance. Then she received two 15% increases. How much is her weekly allowance after the second increase?

**NOTE: you cannot add percents.**

90 = old allowance
15% = first increase
15% = second increase
allowance after the second increase = ???

After the first increase, her allowance was: \( 90 + 90(0.15) = 103.5 \)
After the second increase, her allowance is: \( 103.5 + 103.5(0.15) = 119.03 \)

**Example 4:** John just received a 10% raise on his salary. If he now receives $170 per week, what was his salary before the raise?

170 = current salary
10% = raise
salary before the raise = ???.

Looking at the previous example we know that: \( x + x(0.1) = 170 \) \( (Why?) \)
Solve for x: \( 1.1x = 170 \)
\[ x = \frac{170}{1.1} = 154.54 \]

Does the answer make sense?
Well, if he is getting $170 now, and we know that he got a raise; then, he used to get less than $170 or $154.54. So, yes, the answer makes sense.
Example 5: Tim got a 22.5% discount for a TV. He bought the TV for $345. What was the marked price of the TV?

22.5% = percent of discount
345 = selling price
marked price = ?? = x

Like the previous example: x -x(.225) = 345 (Why "-"?)
Solve for x: 755x = 345
x = $445.16

Does the answer make sense?

Example 6: A merchant paid $500 for a table. He then marked it $820. If he then allowed the buyer a 25% discount, how much was the selling price?

500 = cost
820 = marked price
25% = percent of discount
selling price = ???

For the complicated problems, we will use the following diagram:

\[ \text{percent of increase} \quad \text{marked price} \quad \text{percent of discount} \quad \text{selling price} \quad \text{profit} \quad \text{cost} \]

For our example the diagram looks like:

\[ \begin{align*}
\text{percent of increase} \quad & \quad \text{marked price} \\
& \quad \text{percent of discount} \\
& \quad \text{selling price} \\
& \quad \text{profit} \\
& \quad \text{cost} \\
\end{align*} \]

\[ \begin{align*}
\text{cost} & = 500 \\
\text{marked price} & = 820 \\
\text{percent of discount} & = 25\% \\
\text{selling price} & = ??? \\
\end{align*} \]

To find the selling price, we simply take 25% off the marked price:

\[ \begin{align*}
820 \times .25 & = 205 \\
820 - 205 & = $615
\end{align*} \]

Example 7: A merchant paid $500 for a table. He then marked the table 64% above the cost. If he then allowed the buyer a 25% discount, how much was the selling price?

500 = cost
64% = rate of mark up
25% = percent of discount
selling price = ???

To find the selling price, we first need to find the marked price; then, we take 25% off.

To find the marked price: 500 x .64 = 320
500 + 320 = $820 (marked price)
Example 8: A sofa cost a merchant $500. He priced the sofa so that he could allow a customer a 25% reduction from the marked price and still make a 23% profit. Find:  
   a. The marked price.  
   b. The rate of mark up.

\[
\begin{align*}
500 &= \text{cost} \\
25\% &= \text{rate of discount} \\
23\% &= \text{rate of profit} \\
\text{marked price} &= ??? \\
\text{rate of mark up} &= ???
\end{align*}
\]

First, find the selling price, and then the marked price. 
To find the selling price: 
\[500 \times .23 = 115 \text{ (profit)}\] 
\[500 + 115 = 615 \text{ (selling price)}\]

To find the marked price (let's call it "x"), we take 25% off "x" and get 615: 
\[x - .25x = 615\] 
\[.75x = 615\] 
\[x = 820\]

The rate of mark up: \((820 - 500) / 500 = 64\%\)

Example 9: An item cost a merchant $500. If it was marked 64% above the cost and later sold for a $115 profit, what was the rate of reproduction?

\[
\begin{align*}
500 &= \text{cost} \\
64\% &= \text{rate of mark up} \\
115 &= \text{profit} \\
\text{rate of discount} &= ???
\end{align*}
\]

To find the rate of discount, we need the marked price and the selling price. 
The marked price: \(500 \times .64 = 320\) 
\[500 + 320 = 820 \text{ (marked price)}\]

The selling price: \(500 + 115 = 615 \text{ (selling price)}\)

Rate of discount: \((820 - 615)/820 = 25\%\)