SLAC Lab Physics Handout

Classical Mechanics

VECTORS

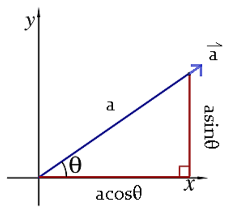
We deal with two types of quantities: Scalars and Vectors. Scalars, like temperature and speed, only have a magnitude (amount). Vectors, like displacement and velocity, have magnitude and direction and obey the rules of vector algebra.

Define: A *vector* is the product of magnitude a and direction $\hat{a}$, namely

$$\vec{a} = a\hat{a}$$

The components of a vector: The scalar components a_x, a_y of any 2-D vector $\vec{a}$ are

$$a_x = acos(\theta),\ a_y = asin(\theta)$$



The magnitude of $\vec{a}$ is

$$|\vec{a}| = a = \sqrt{(a_x)^2 + (a_y)^2}$$

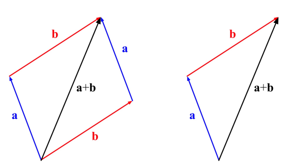
The orientation of $\vec{a}$ is

$$tan(\theta) = \frac{a_y}{a_x}$$

Unit vector Notation:

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

Vector Addition (Geometric/Algebraic):



$$\vec{a} + \vec{b} = [a_x\hat{i} + a_y\hat{j}] + [b_x\hat{i} + b_y\hat{j}] = (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j}$$

The Scalar Product:

$$\vec{a} \cdot \vec{b} = abcos(\theta) = a_x b_x + a_y b_y$$

The Vector Product:

$$\vec{a} \otimes \vec{b} = det\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{pmatrix}$$

$$|\vec{a} \otimes \vec{b}| = absin(\theta)$$

MOTION ALONG A STRAIGHT LINE

Displacement: $\Delta\vec{x} = \vec{x}_2 - \vec{x}_1$

Average Velocity: $\vec{v}_{avg} = \frac{\Delta\vec{x}}{\Delta t} = \frac{\vec{x}_2 - \vec{x}_1}{t_2 - t_1}$

Average Speed: $s_{avg} = \frac{\text{total distance}}{\Delta t}$

Instantaneous Velocity: $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$

Average Acceleration: $\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$

Instantaneous Accleration: $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$

Kinematic (Constant Acceleration) Equations:

Equation	Missing Quantity
$v = v_o + at$	Δx
$\Delta x = v_o t + \frac{1}{2}at^2$	v
$v^2 = v_o^2 + 2a(x - x_o)$	t
$\Delta x = \frac{1}{2}(v_o + v)t$	a
$\Delta x = vt - \frac{1}{2}at^2$	v_o

MOTION IN 2 AND 3 DIMENSIONS

Position vector: $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Displacement: $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

Average Velocity: $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$

Instantaneous Velocity: $\vec{v} = \frac{d\vec{r}}{dt} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

Average Acceleration: $\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$

Instantaneous Acceleration: $\vec{a} = \frac{d\vec{v}}{dt} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

PROJECTILE MOTION

$\rightarrow$ A projectile in motion with intial velocity v_o has $\vec{a}_x = 0$ and $\vec{a}_y = -g = -9.8\frac{m}{s^2}$ the constant acceleration due to gravity ($+g \neq -9.8\frac{m}{s^2}$). The Kinematic Equations become

$$\Delta x = (v_o cos(\theta))t$$

$$\Delta y = (v_o cos(\theta))t - \frac{1}{2}gt^2$$

$$v_y = v_o sin(\theta) - gt$$

$$v_y^2 = (v_o sin(\theta))^2 - 2g(y - y_o)$$

The Trajectory is given by:

$$y = (tan(\theta))x - \frac{gx^2}{2(v_o cos(\theta))^2}$$

The Horizontal Range (only when $y_i = y_f$) is given by:

$$R = \frac{v_o^2}{g}sin(2\theta)$$

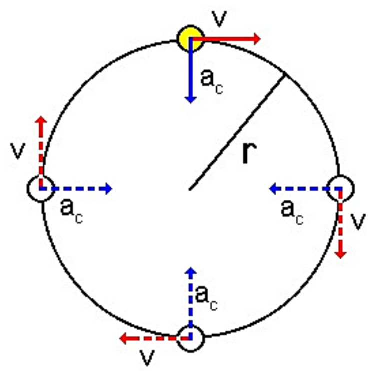
UNIFORM CIRCULAR MOTION

Circular Acceleration:

$$\vec{a}_c = \frac{v^2}{r}$$

Period:

$$T = \frac{2\pi r}{v}$$



FORCES

There are two *types* of force: Conservative, and Non-conservative. A more clear definition will be given below, but examples of Conservative forces are the gravitational force, spring force, and electromagnetic force, while examples of Non-conservative forces are friction force, applied force, drag force, normal force, and tension force.

Net force, $\vec{F}_{net}$, is the vector sum of all individual conservative and non-conservative forces acting on an object or in/on a system.

Newton's 3 Laws of Motion:

1) If there is no net force on a body, the body remains at rest if it is initally at rest or moves at constant speed if it is in motion.

2) The net force $\vec{F}_{net}$ on a body with mass m is related to the body's acceleration $\vec{a}$ by

$$\vec{F}_{net} = m\vec{a}$$

3) If a force $\vec{F}_{BC}$ acts on body B due to a body C, then there is a force $\vec{F}_{CB}$ on body C due to body B,

$$\vec{F}_{BC} = -\vec{F}_{CB}$$

Newton's Law of Universal Gravitation:

$$\vec{F}_G = G\frac{m_1 m_2}{r^2}$$

where G = 6.67 $\times 10^{-11}$

Free-Body Diagram: a FBD is a stripped-down diagram that considers the forces acting on only *one* body.

Some Particular Forces:

⋆ *Weight force* $\vec{W}$ is the gravitational force on a body due to the earth and is defined as proportional to the acceleration caused by Earth's gravity (g = 9.8 $\frac{m}{s^2}$).

⋆ *Normal force* $\vec{F}_N$ or $\vec{N}$ is always perpendicular to the surface that exerts a force on the object.

⋆ *Friction force* has two types: 1) Static and 2) Kinetic. When a body is acted on by $\vec{F}$ along a surface,

1) if the body does not move, the static frictional force $\vec{f}_s$ and the component of $\vec{F}$ are anti-parallel and equal in magnitude. The magnitude of $\vec{f}_s$ has a maximum of $f_{s,max}$ given by

$$f_{s,max} = \mu_s \vec{F}_N$$

2) if the body begins to slide on the surface, the magnitude of the frictional force decreases rapidly to a constant value of f_k given by

$$f_k = \mu_k \vec{F}_N$$

⋆ *Drag force* $\vec{F}_D$ or $\vec{D}$ occurs when there is relative motion between a body and air, or some other fluid, in the direction in which the fluid flows relative to the body, and is given by

$$\vec{D} = \frac{1}{2} C\rho A v^2$$

where C is the drag coefficient, ρ is the fluid density, and A is the effective cross-sectional area of the body.

⋆ *Spring force* $\vec{F}_s$ is the reactive force from a spring and is described by Hooke's Law,

$$\vec{F}_s = -k\Delta\vec{x}$$

⋆ *Tension force* $\vec{F}_T$ or $\vec{T}$ of a cord is the force directed along the cord away from the point of attachment to the body. For a massless cord, the pulls at both ends of the cord have equal magnitude T.

Terminal Velocity: When an object falls sufficiently far under free-fall, the magnitudes of $\vec{D}$ and $\vec{F}_g$ are equal in magnitude, meaning the body then falls at a constant terminal velocity given by

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}$$

Circular Motion:

$$\vec{F}_c = m\vec{a}_c = \frac{mv^2}{r}$$

ENERGY, WORK, POWER, AND CONSERVATION

Kinetic Energy: the energy associated with motion (well below the speed of light)

$$K = \frac{1}{2}mv^2$$

Work: the energy transferred to $(+W)$ or from $(-W)$ an object via a force acting on the object

$$W = \vec{F} \cdot \Delta\vec{x} = Fxcos(\theta)$$

where θ is the angle between $\vec{F}$ and $\Delta\vec{x}$.

*Note if $\vec{F}$ and $\Delta\vec{x}$ are parallel there is $+W$ done, if $\vec{F}$ and $\Delta\vec{x}$ are anti-parallel there is $-W$ done.

W_{net} of an object equals the sum of individual works done by individual forces acting on the object in a system.

Work-Kinetic Energy Theorem: $\Delta K = K_f - K_i = W_{net}$

Work done by Gravity: $W_g = \vec{F}_g \cdot \vec{d} = mgdcos(\theta)$

Work done by Spring: $W_s = -\frac{1}{2}kx^2$, where $x = |\Delta\vec{x}|$.

Work done by Variable Force: $W = \int_{x_i}^{x_f} F(x)dx$

When work done by a force in moving a particle between two points is path independent - the same value of work is calulated no matter the path chosen- the force is Conservative. If the work by the force depends on the path chosen the force is Non-Conservative. Also, a system consists of two or more objects.

Given a closed path ($x_i = x_f$), the W_{net} done by an internal force on a particle moving around that closed path is zero.

We assign *Potential Energy* to Conservative force.

Potential Energy: the energy associated with the arrangement of a system of objects that exert forces on one another

$$\Delta U = -W = -\int_{x_i}^{x_f} F(x)dx$$

Gravitational Potential Energy: Force due to Earth's gravity acts in the y-direction

$$\Delta U = mg\Delta y = mgh$$

where *h* is height measured from the ground.

Elastic Potential Energy: the energy associated with the compression or extension of an elastic body. For a spring

$$U(x) = \frac{1}{2}kx^2$$

Mechanical Energy: the energy associated with the motion and position of an object

$$E_{mec} = K + U$$

An *isolated system* only has *conservative forces* acting in it. Since only conservative forces are doing work, then the mechanical energy E_{mec} of the system cannot change, E_{mec} = constant! The **principle of conservation of mechanical energy**

$$\Delta E_{mec} = \Delta K + \Delta U = 0$$

Conservation of Energy: an experimental fact, it states that the total energy of a system E (the sum of its mechanical energy, thermal energy and internal energies) can change only by amounts of energy that are transferred to or from the system

$$W = \Delta E = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int}.$$

$\rightarrow$ For an isolated system, $W = 0$, so

$$\Delta E_{mec} + \Delta E_{th} + \Delta E_{int} = 0.$$

Power: due to a force is the *rate* at which that force does work on an object.

$$P_{avg} = \frac{U}{\Delta t} = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$$

$$P_{inst} = \frac{dW}{dt} = \frac{dE}{dt} = \vec{F} \cdot \vec{v}$$

LINEAR MOMENTUM

For a single particle, its linear momentum is

$$\vec{p} = m\vec{v}.$$

Newton's Second Law becomes

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

Conservation of Linear Momentum: The linear momentum of an isolated system remains constant

$$\vec{p} = constant, \text{ or } \vec{p}_i = \vec{p}_f$$

COLLISION AND IMPULSE IN 1-D

For an isolated system collision is either 1) **elastic** in which both conservation of kinetic energy and conservation of momentum are observed, or 2) **inelastic** in which only conservation of momentum is observed.

Elastic Collision: K & p conserved, so the respective velocities for two particles $(v_{2,i} = 0)$ after colliding are

$$v_{1,f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1,i} \text{ and } v_{2,f} = \frac{2m_1}{m_1 + m_2} v_{1,i}$$

Inelastic Collision: K is not conserved, p is conserved, so for two particles before and after collision

$$\vec{p}_{1,i} + \vec{p}_{2,i} = \vec{p}_{1,f} + \vec{p}_{2,f}$$

$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$$

If the bodies stick together, the collision is completely inelastic and the bodies have the same final velocity v_f.

Impulse: $\vec{J}$ of a force $\vec{F}$ for a given time interval

$$\vec{J} = \int_{t_1}^{t_2} \vec{F} dt$$

Impulse-Linear Momentum Theorem: in a collision of two bodies the change in one body's linear momentum $\Delta\vec{p}$ is the impulse due to the force exerted on it by the other body

$$\vec{J} = \vec{p}_f - \vec{p}_i = \Delta\vec{p}$$

CENTER OF MASS

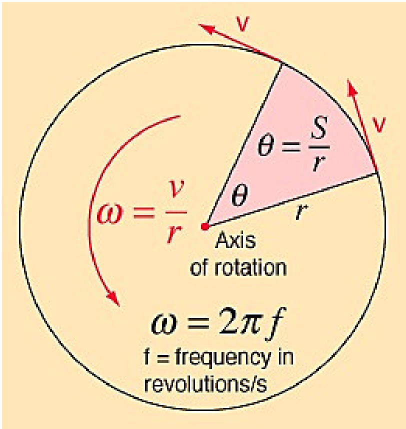
The center of mass of a system of n particles and total mass M is the point whose coordinate $(x_{com}, y_{com}, z_{com})$ is given by

$$x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i,\ y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i,$$

$$z_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i, \text{ -or- } \vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$$

The center of mass of an isolated system is not affected by a collision. In particular, the velocity $\vec{v}_{com}$ of the center of mass cannot be changed by collision.

ANGULAR (ROTATIONAL) MOTION



Angular Position: When θ is measured in radians,

$$\theta = \frac{s}{r},$$

where s is the arc length of a circular path of radius r and angle θ.

$\rightarrow$ Useful conversion: 1 revolution/rotation $= 360^o = 2\pi$

Angular Displacement: $\Delta\theta = \theta_2 - \theta_1$ where $\Delta\theta$ is positive for CCW rotation and negative for CW rotation.

Average Angular Velocity: $\vec{\omega}_{avg} = \frac{\Delta\theta}{\Delta t}$

Instantaneous Angular Velocity: $\vec{\omega} = \frac{d\theta}{dt}$

Average Angular Accleration: $\vec{\alpha}_{avg} = \frac{\Delta\vec{\omega}}{\Delta t}$

Instantaneous Angular Accleration: $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$

Kinematic (Constant Angular Acceleration) Equations:

Equation	Missing Quantity
$\omega = \omega_o + \alpha t$	$\Delta\theta$
$\Delta\theta = \omega_o t + \frac{1}{2}\alpha t^2$	ω
$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$	t
$\Delta\theta = \frac{1}{2}(\omega_o + \omega)t$	α
$\Delta\theta = \omega t - \frac{1}{2}\alpha t^2$	ω_o

Linear and Angular relationships: measured in radians

$\rightarrow$ Linear Velocity:

$$v = \omega r$$

$\rightarrow$ Linear Acceleration, $\vec{a}$ has tangential a_t and radial a_r components:

$$a_t = \alpha r,\ a_r = \frac{v^2}{r} = \omega^2 r$$

where $|\vec{a}| = \sqrt{(a_t)^2 + (a_r)^2}$

$\rightarrow$ Period of rotation is given by

$$T = \frac{2\pi}{\omega} = \frac{2\pi r}{v}$$

Angular Kinetic Energy and Inertia: The kinetic energy of a rigid body rotating about a fixed axis is given by

$$K = \frac{1}{2}I\omega^2,$$

where I is the rotational inertia of the body defined as

$$I = \sum m_i r_i^2,$$

for a system of discrete particles, or is defined experimentally based on the geometry of the rotating body.

*Note: Inertia is the tendency of a body to resist motion.

Torque: the tendency of a body to rotate when acted upon by force. If the force $\vec{F}$ is exerted at a point given by the position vector $\vec{r}$ relative to the axis,

$$\vec{\tau} = \vec{r} \otimes \vec{F} = rFsin(\theta),$$

where θ is the angle between $\vec{r}$ and $\vec{F}$.

Newton's Second Law in Angular Form:

$$\tau_{net} = I\alpha$$

Work, Power and Angular Kinetic Energy:

$$W = \int_{\theta_i}^{\theta_f} \tau d\theta$$

$$P = \frac{dW}{dt} = \tau\omega$$

$\rightarrow$ The Work-Kinetic Energy Theorem becomes

$$\Delta K = K_f - K_i = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = W$$

Angular Momentum

The angular momentum $\vec{L}$ of a particle (or point) with linear momentum $\vec{p}$, mass m, and linear velocity $\vec{v}$ is a vector defined relative to a fixed point as

$$\vec{L} = \vec{r} \otimes \vec{p}$$

The angular momentum $\vec{L}$ of a fixed mass that is rotating about a fixed axis is given by

$$\vec{L} = I\vec{\omega}$$

Conservation of Angular Momentum

The angular momentum $\vec{L}$ of an isolated system remains constant if the net external torque acting on the system is zero:

$$\vec{L} = constant, \text{ or}$$

$$\vec{L}_i = \vec{L}_f$$

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