RADIX AND REDUCED RADIX SUBTRACTION

Computers use Complementary Subtraction because it is faster and simpler than the addition of signed numbers (numbers with a "+" or "-" sign). This handout details radix and reduced radix subtraction problems which simulate the way computers perform subtraction operations.

Note: Radix and reduced radix subtraction utilizes radix and reduced radix complements. Therefore, it is assumed that the reader understands the concepts of complements and can perform radix and reduced radix complementation. If not, it is helpful to review the SLAC handout entitled Radix and Reduced Radix Complements or the appropriate sections in the text before continuing.

Radix Subtraction

Consider the following subtraction problem:

\[
\begin{array}{c}
5204_{10} \\
- 3986_{10} \\
\hline
1218_{10}
\end{array}
\]

Now compare this problem to the general format for a subtraction problem:

\[
\begin{array}{c}
X_{10} \\
- Y_{10} \\
\hline
Z_{10}
\end{array}
\]

Here, X equals 5204\textsubscript{10}, Y equals 3986\textsubscript{10}, and the goal is to find Z.

**Step 1**

For the first step in radix subtraction, find the radix complement of Y, the subtrahend.

The radix complement of 3986\textsubscript{10} is 6014\textsubscript{10}.

**Step 2**

Next, add this complement of Y to X.

\[
\begin{array}{c}
X_{10} \\
+ Y_{10} \\
\hline
Z_{10}
\end{array}
\]

\[
\begin{array}{c}
5204_{10} \\
+ 6014_{10} \\
\hline
11218_{10}
\end{array}
\]

**Step 3**

Now, if you discard the leading 1, you have 1218\textsubscript{10}, the same result you found using simple subtraction.

In the previous problem, the minuend was larger than the subtrahend, so the difference was positive. In the next example the subtrahend is larger than the minuend.
For example:

\[
\begin{array}{c}
1548_{10} \\
\hline
-3316_{10} \\
\hline
X \\
\hline
- \ Y \\
\hline
Z
\end{array}
\]

**Step 1**
As before, begin by finding the complement of Y.

The radix complement of \(3316_{10}\) is \(6684_{10}\).

**Step 2**
Again, you add the complement of Y to X.

\[
\begin{array}{c}
1548_{10} \\
\hline
+ 6684_{10} \\
\hline
8232_{10}
\end{array}
\]

**Step 3**
With this result, there is no extra, leading digit. This shows that the result will be a negative number since the subtrahend, \(3316_{10}\), is greater than the minuend, \(1548_{10}\). The last step is to take the radix complement of the result and attach a negative sign to it.

\[
8232_{10} \text{ becomes } -1768_{10}
\]

**Reduced Radix Subtraction**
The steps for using reduced radix subtraction are similar to those for using radix subtraction. However this time, use reduced radix complements.

Consider the following subtraction problem:
(Again, label these numbers according to the general subtraction format.)

\[
\begin{array}{c}
4352_{10} \\
\hline
-1678_{10} \\
\hline
X \\
\hline
- \ Y \\
\hline
Z
\end{array}
\]

**Step 1**
Find the reduced radix complement of Y.

\(Y = 1678_{10}\) so the reduced radix complement of Y is \(8321_{10}\)

**Step 2**
Add the complement of Y to X.

\[
\begin{array}{c}
4352_{10} \\
\hline
+ 8321_{10} \\
\hline
12673_{10}
\end{array}
\]
Step 3
Notice there is an extra leading 1 digit again. This time, drop the leading 1 digit and add it to the result, obtaining the correct answer.

\[
\begin{array}{c}
4352_{10} \\
+ 8321_{10} \\
\hline
12673_{10} \\
\end{array}
\]

Now consider an example in reduced radix subtraction where the subtrahend is greater than the minuend.

\[
\begin{array}{c}
2602_{10} \\
- 3781_{10} \\
\hline
Z \\
\end{array}
\]

Step 1
Once again, the first step is to find the reduced radix complement of Y. 
Y = 3781_{10}, therefore, the reduced radix complement of Y is 6218_{10}.

Step 2
Add the complement of Y to X.

\[
\begin{array}{c}
X \\
+ Y \\
\hline
2602_{10} \\
6218_{10} \\
8820_{10} \\
\end{array}
\]

Step 3
Expect a negative answer since the subtrahend is greater than the minuend. Again, notice the lack of a leading digit. This signifies that the result needs to be complemented. Therefore, take the reduced radix complement of the answer and place a negative sign before it to obtain the correct answer.

\[
8820_{10} \text{ becomes } -1179_{10}
\]

Conclusion:
While Step 1 and Step 2 are the same for each of the preceding problems, Step 3 yields different results for each case.

The following diagram summarizes the steps for using radix and reduced radix subtraction:
<table>
<thead>
<tr>
<th>Radix Subtraction</th>
<th>Reduced Radix Subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Find Radix Complement of Y</td>
<td>• Find Reduced Radix Complement of Y</td>
</tr>
<tr>
<td>• Add Y complement to X</td>
<td>• Add Y complement to X</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Extra Leading Digit</th>
<th>No Extra Digit</th>
<th>Extra Leading Digit</th>
<th>No Extra Digit</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Drop extra digit</td>
<td>• Take Radix Complement</td>
<td>• Drop extra digit</td>
<td>• Take Reduced Radix Complement</td>
</tr>
<tr>
<td></td>
<td>• Attach Negative</td>
<td>• Add extra digit to result</td>
<td>• Attach Negative</td>
</tr>
</tbody>
</table>