classroom-ready activities

The Tortoise and the Hare: Investigating Rates

Nama Namakshi, Hiroko K. Warshauer, Sonalee Bhattacharyya, Christina Koehne, and Max Warshauer

Aesop's Fables are universally well-known short tales. Although each tale teaches a moral lesson, some of these stories can also be settings for teaching mathematics lessons.

Most students are familiar with the story of the tortoise and the hare, in which a tortoise challenges a hare to a race. In the race, we find that the tortoise plods along, slowly and surely, and beats the overconfident hare who decides to take a nap in the midst of the race. The hare awakens too late and realizes that it cannot catch up to the tortoise, which is way ahead and wins the race. The moral of the story is that the race is not always to the swift.

CENTRAL OBJECTIVE

Using this setting, we propose a mathematically rich activity that examines some of the "what ifs" that could have altered the outcome of the race between the quick but overconfident hare and the slow but persistent tortoise. For example, how long a nap can the hare take and still win the race? If the hare or the tortoise decides to slow down or speed up during the race, how does that affect the outcome of the race?

Students can explore the quantitative and algebraic relationships among distance, rate, and time by mathematically modeling the problems that can be posed about the race using pictures, tables, or equations. This activity involves algebraic reasoning and incorporates several of the Common Core's Standards for Mathematical Practices (CCSSI 2010), such as make sense of problems and persevere in solving them, reason abstractly and quantitatively, and model with mathematics. The activity also aligns with one of the NCTM Focal Points (NCTM 2008) that students be able to analyze and represent linear functions and solve linear equations and systems of linear equations.

This activity is appropriate for students in grades 6–8. A review of the relationships among distance, rate, and time may be helpful for students. Some prior knowledge of algebra can help students who use algebraic strategies and need to relate the quantities using expressions and equations. The sixth-grade and eighth-grade classes in which we implemented the activity were able to reason with equations as well as relate quantities of time, rate, and distance more formulaically without thinking formally of solving an equation.

TEACHER NOTES

In this section, we propose ideas that may help teachers implement the questions presented in the activity.

Students should work in small groups or pairs and be encouraged to try different strategies, such as drawing a picture, modeling the situation through linear equations, or keeping track of information through tables. The teacher can then lead a whole-class discussion in which each group can share their different strategies and reasoning.

In question 1, students investigate how much of a head start the hare could allow the tortoise and still tie in the mile race. Determining how much time elapses between when the hare completes the race and when the tortoise completes the race will help students answer this question. The teacher may wish to extend the students' thinking and pose such questions as these:

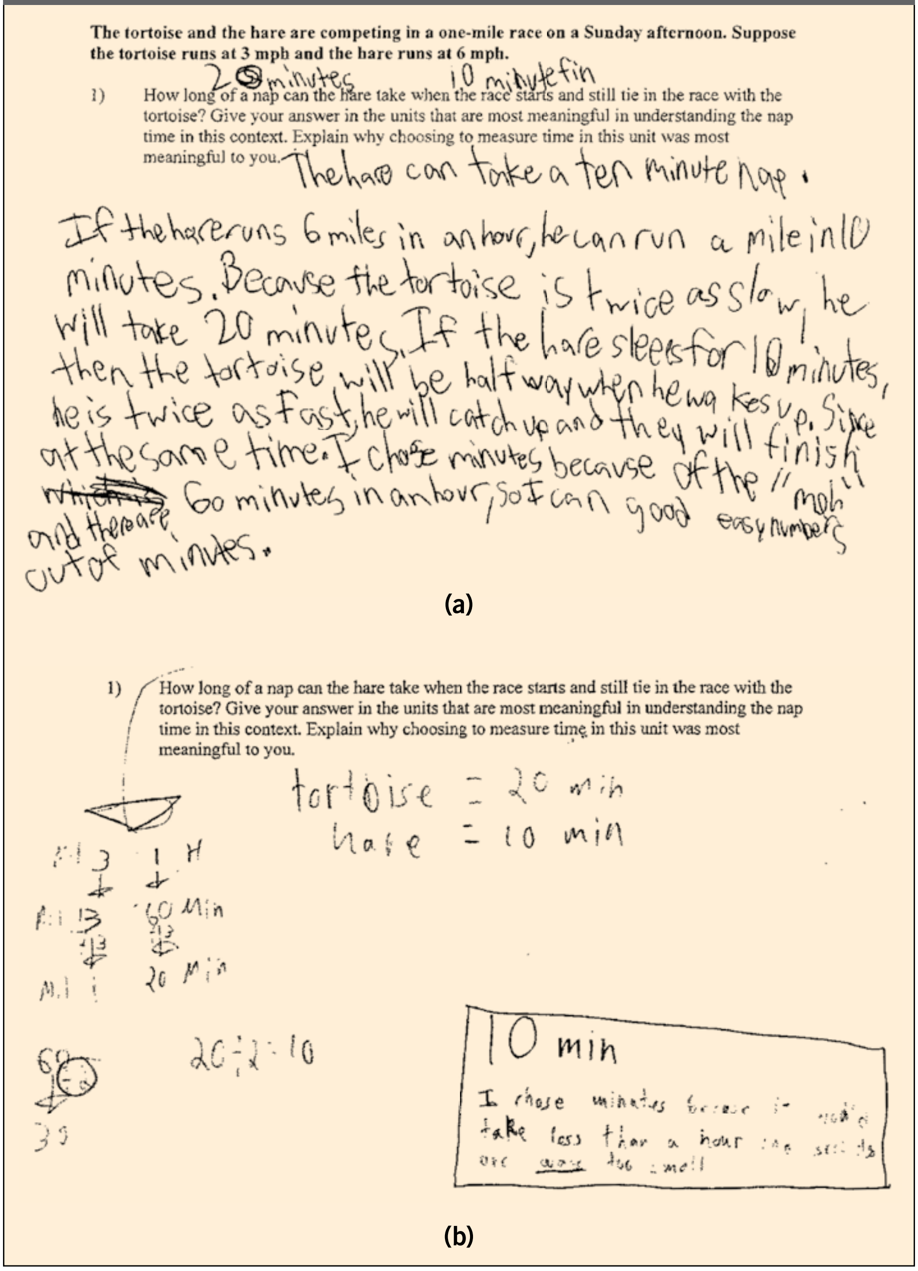
1. Is there a point beyond which the hare may never win or tie the race?
2. How does the concept of getting a head start help you beat someone who runs much faster than you do?
3. How would changing the distance of the race from 1 to 2 or 3 miles impact the nap time?

Edited by **Marilyn Howard**, marilyn-howard@utulsa.edu, University of Tulsa, Oklahoma; **Christine Kincaid Dewey**, dewey@wcskids.net, Macomb Mathematics Science Technology Center, Warren, Michigan; and **Hiroko K. Warshauer**, hiroko@txstate.edu, Texas State University, San Marcos. Readers are encouraged to submit classroom-tested activities through **http://mtms.msubmit.net**.

It may be helpful to have the students draw the scenarios and keep track of the tortoise's and hare's distances, given their rates and the elapsed time. Some students may also model the rates of the hare and tortoise as linear equations and find the point of intersection to see how formal algebra can also help answer this question. The question also allows students to think about choosing a unit of time that would be most meaningful in the context of this problem. Giving students the opportunity to think about what would be reasonable helps them think quantitatively and pay attention to the meaning behind the numbers with which they compute and the units that they use. **Figures 1** and **2** are examples of the different approaches that students took in a middle school classroom in which this activity was implemented.

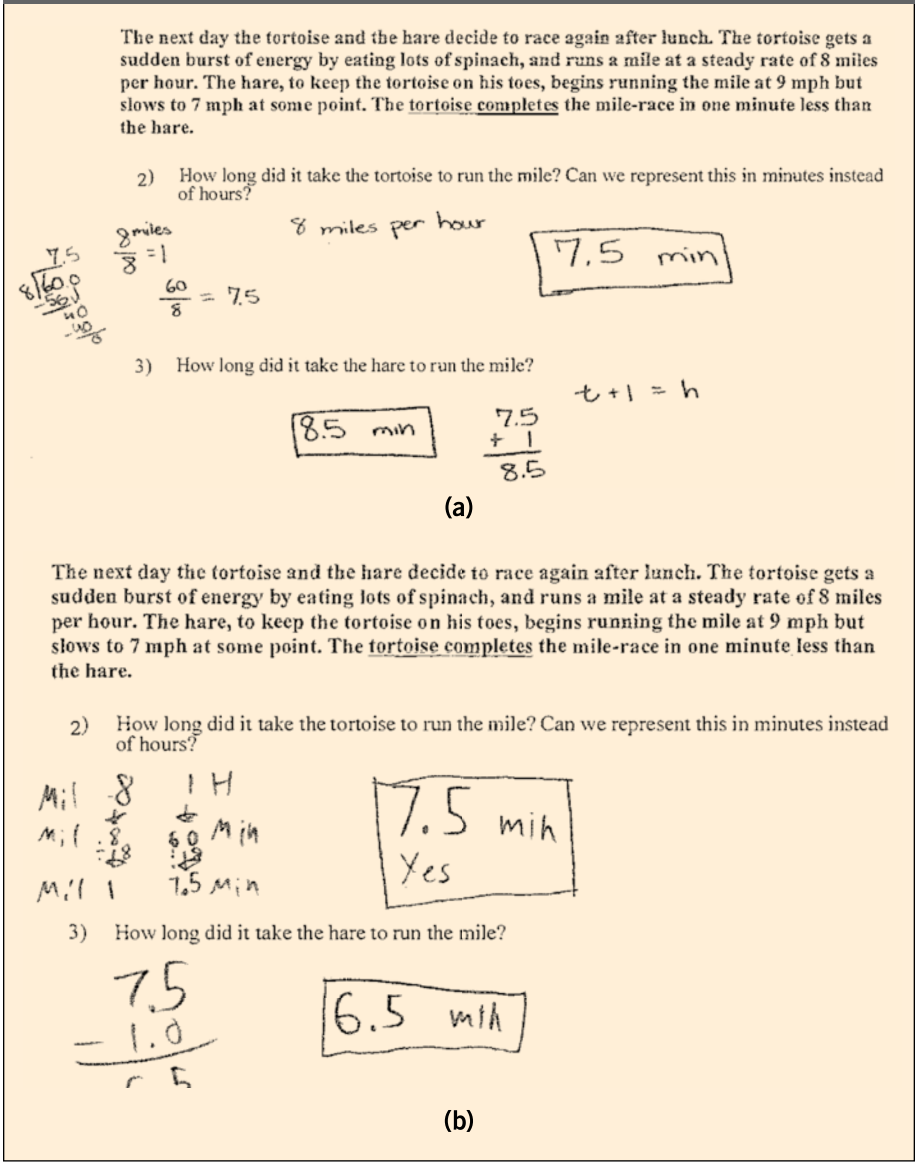
In **figure 1**, note that to find out how long a nap the hare can take and still tie in the mile race with the tortoise, the student first determined how long it will take the hare and the tortoise to finish the mile race. Explaining her reasoning, the student writes, "If the hare can run 6 miles in an hour, then it can run 1 mile in 10 minutes." Next, looking at the running rates given for the hare as 6 mph and the tortoise as 3 mph, the student reasoned that the tortoise is "twice as slow" and concluded that it would take 20 minutes for the tortoise to finish the mile race. Using this line of reasoning, the student was able to determine the nap time for the hare as 10 minutes. Furthermore, the student explains that choosing minutes as the unit of time in this problem was most meaningful to her because it helped her get "good easy numbers." The teacher may wish to discuss this explanation and elaborate on why looking at the nap time as 10 minutes is more meaningful to us in this context than 1/6 hour.

Fig. 1 In answering question 1, some students reasoned about rates (a) and others used a tabular format (b).



The student work illustrated in **figure 1b** contains a similar approach as that of the student work in **figure 1a**. However, instead of reasoning through writing about the process, this student chose to illustrate his thought process by looking at the relationship between miles and time in a tabular format. Using proportional thinking, the student determined that it would take the tortoise 20 minutes to finish the race. He then divided 20 minutes by 2 to determine that it would take 10 minutes for the hare to finish the race. Using this information, this student was also able to answer this problem. Furthermore, when explaining his choice for using minutes for his final answer, this student wrote,"I chose minutes because it would take less than an hour and seconds are way too small." This work

Fig. 2 In (a), this answer is straightforward, although the numbers have changed; and (b) shows a common mistake that students made.


The next day the tortoise and the hare decide to race again after lunch. The tortoise gets a sudden burst of energy by eating lots of spinach, and runs a mile at a steady rate of 8 miles per hour. The hare, to keep the tortoise on his toes, begins running the mile at 9 mph but slows to 7 mph at some point. The tortoise completes the mile-race in one minute less than the hare.

2) How long did it take the tortoise to run the mile? Can we represent this in minutes instead of hours?

3) How long did it take the hare to run the mile?

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shows how the student reasoned quantitatively to determine the appropriate and most meaningful unit for measure of time in the given context. The teacher may wish to have a class discussion about a similar explanation as the one in **figure 1b**. Teachers may ask the student to elaborate on his or her reasoning and explain what the student means by "seconds are way too small." This discussion would provide an excellent opportunity for students to communicate their mathematical thinking in meaningful terms using the language of mathematics.

Questions 2, 3, and 4 focus on comparing the running capabilities of the tortoise and the hare when the hare runs different parts of the race at different speeds while the tortoise maintains a constant speed.

Question 2 asks students to determine how long it takes the tortoise to run the mile. This question is straightforward although the rate of the tortoise is faster than it was in question 1. Question 3 can use the information from question 2 and note the relationship between the time it takes the tortoise to complete the race and the time it takes the hare. The figures are examples of student approaches. Note that **figure 2b** illustrates a common error that many students made when working on question 3.

Question 4 explores the question about the hare no longer taking a nap (going 0 mph) on his second day but merely slowing down from his 9 mph rate to a rate of 7 mph. The question is what fraction of the mile did the hare run at 9 mph? Notice that the students will need to refer to their previous work and use it to answer this question. You may wish to ask them what information would be useful to answer the question. For example, knowing the total time that it took the hare, as found in question 3, would be useful. The hare's time of 8.5 minutes would be the hare's total time, part of which was run at 9 mph and the other at 7 mph. You also may wish to discuss that mph stands for miles per hour and 8.5 is in minutes so a discussion of appropriate units might be in order: 9 mph could be viewed as 9/60 miles per minute and 7 mph = 7/60 miles per minute. We have the relationship:

Hare's total running time
= time hare runs at 9 mph
+ time hare runs at 7 mph

If we let the unknown fraction of the mile that the hare runs at 9 mph be called D_1, we have the following:

$$8.5 = \frac{D_1}{\frac{9}{60}} + \frac{1 - D_1}{\frac{7}{60}} \qquad (1)$$

In solving this equation for D_1, we have D_1 = 3/80 of a mile at 9 mph. Although rewriting equation (1) may be challenging for those students not yet accustomed to working with algebraic equations, you may wish to proceed using

$$8.5 = \frac{60D_1}{9} + \frac{60(1 - D_1)}{7},$$

or consider rewriting 8.5 minutes as 8.5/60 hr.

The extension problem should make for interesting conversation about what that could look like in a race. The hare chooses to run its speed race with no naps or slowing down. The tortoise runs its expected slow but persistent pace. As you might imagine, the distance between the two runners increases over time, with the hare moving farther and farther ahead of the tortoise. A visual representation or a physical model of the problem would help students make sense of what is being asked and would enrich the conversation without focusing on getting an answer. Understanding what the problem asks is critical, such as noting the relationships between the hare and tortoise distances. After 10 minutes, the distance remaining for the tortoise is twice that for the hare. After 15 minutes, the tortoise's distance to the finish line is five times that of the hare. Having students write these relationships in words and using equality where appropriate may help them work toward algebraic representations. Those students with extensive algebra background can still benefit from seeing where equations arise and what quantities are being equated.

CONCLUSION

In this activity, students explore algebraic reasoning by examining rates and ratios through a familiar fable from their childhood. By altering the rates of the tortoise and the hare, students engage in a fun process of discovering different outcomes of this famous age-old race. In the extension portion of the activity, students stretch their understanding of the relationship between distance, rate, and time by investigating clues given about the remaining distances in the race at different points in time to figure out how long it will take each character to finish the race. The Tortoise and the Hare Activity affords students the opportunity to translate mathematical scenarios and proportional thinking into algebraic equations as well as deepen their understanding of units of measure. In addition, the "what if?" questions enrich the task and encourage students to ask questions themselves as would mathematicians (Coomes and Lee 2017). By engaging with rich mathematical tasks in the context of an interesting and familiar work of children's literature, students can bring meaning to the algebraic concepts being explored.

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Nama Namakshi, namakshi@uark.edu, is a clinical assistant professor in the math department at the University of Arkansas. Her research interests include math camps and the effect they have on women's participation in STEM fields, development of teacher noticing skills among preservice teachers, and curriculum development at the elementary and middle school levels. **Hiroko K. Warshauer**, hw02@txstate.edu, is an assistant professor of mathematics at Texas State University. She is interested in teacher preparation and in-service professional development with a focus on teaching practices that support productive struggle and noticing of student thinking. **Sonalee Bhattacharyya**, sb1212@txstate.edu, is a doctoral student in mathematics education at Texas State University. Her research interests include sustaining teachers in the teaching profession, mathematics professional development, and interesting math problems. **Christina Koehne**, crz7@txstate.edu, is currently a doctoral student at Texas State University studying mathematics education and works as a researcher for a children's math camp program during the summers. **Max Warshauer**, max@txstate.edu, is a Regent Professor of mathematics at Texas State University. He founded and directs Texas Mathworks, a center for innovation in mathematics education at Texas State University, which has three pillars: summer math programs for K–12 students, curriculum development, and teacher training. His research interests include areas of teaching and learning related to these three pillars and developing model programs that can engage students and teachers in doing mathematics at a high level.

activity sheet

Name ______________________________

THE TORTOISE AND THE HARE

Are you familiar with Aesop's fable "The Tortoise and the Hare"? In this activity, you will explore a different take on the classic story and the mathematics involved.

The tortoise and the hare are competing in a 1 mile race on a Sunday afternoon. Suppose the tortoise runs at 3 mph and the hare runs at 6 mph.

1. How long a nap can the hare take when the race starts and still tie in the race with the tortoise? Give your answer in the units that are most meaningful in understanding the nap time in the context of the race. Explain why choosing to measure time in this unit was most meaningful to you.

The next day the tortoise and the hare decide to race again after lunch. The tortoise gets a sudden burst of energy by eating lots of spinach and runs a mile at a steady rate of 8 miles per hour. The hare, to keep the tortoise on his toes, begins running the mile at 9 mph but slows to 7 mph at some point. The tortoise completes the mile race in 1 minute less than the hare.

2. How long did it take the tortoise to run the mile? Can we represent this in minutes instead of hours?

3. How long did it take the hare to run the mile?

4. What fraction of the mile did the hare run at 9 mph?

EXTENSION

5. Suppose the tortoise and the hare began running at the same time. They both run the same distance but at different rates. Ten minutes after they start running, the remaining distance that the tortoise has in the race is twice as long as the remaining distance that the hare has in the race. Five minutes later the remaining distance that the tortoise has is five times the remaining distance that the hare has. After this, how long will it take each of them to reach the finish line?

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