DOMAIN AND RANGE

The **domain** of a function is the set of x values (along the x-axis) that gives a valid answer (y value) when the function is evaluated. Also, the set of all x values must be mapped to one and only one y value.

With linear equations such as:

$$y = x + 3$$
$$y = x^2 + 2x + 1$$
$$y = x^3 + 3x^2 + 2x + 6$$

The domain is all real numbers, written $\Re, (-\infty, \infty)\ or\ -\infty < x < \infty$. You can substitute any number
(x value) in for x and get a valid answer (y value).

For functions with a variable in the denominator:

$$y = 3x + \frac{1}{x^2} + 6$$
$$y = \frac{x+2}{x-4}$$
$$y = \frac{x^2 + 2x - 8}{3x^2 - 12x + 9}$$

the domain is all x values that do not make the denominator equal zero.

$$\left[\frac{1}{0}\ \textit{is undefined. If you place this in your calculator, you will get an error message.} \right]$$

To find the domain, set the denominator equal to zero. Solve. The answers you get are not part of your domain; all others are.

$$y = 3x + \frac{1}{x^2} + 6$$
$$x^2 = 0$$
$$x = 0$$
$$D = \{\Re \mid x \neq 0\} \quad \textit{read – the set of all reals such that x does not equal zero or } (-\infty, 0) \cup (0, \infty)$$

$$y = \frac{x+2}{x-4}$$

$$x - 4 = 0$$
$$x = 4$$
$$D = \{\Re \mid x \neq 4\} \textit{ read like above or } (-\infty,4) \cup (4,\infty)$$

$$y = \frac{x^2 + 2x - 8}{3x^2 - 12x + 9}$$

$$3x^2 - 12x + 9 = 0$$
$$(x-3)\,(3x-3) = 0$$
$$x - 3 = 0 \qquad 3x - 3 = 0$$
$$x = 3 \qquad 3x = 3$$
$$x = 1$$
$$D = \{\Re \mid x \neq 1, 3\} \textit{ or } (-\infty, 1) \cup (1, 3) \cup (3, \infty)$$

The domain of a square root function cannot have an *x* value where the expressions under the radical have a negative answer. (Negative numbers do not have square roots when working with real numbers.) Therefore, set what is under the square root greater than/equal to zero.

$$y = \sqrt{x-7}$$
$$x - 7 \geq 0$$
$$x \geq 7 \quad or \quad [7, \infty)$$

$$y = \sqrt{9-x}$$
$$9 - x \geq 0$$
$$x \leq 9 \quad or \quad (-\infty, 9]$$

$$y = \sqrt{4+x}$$
$$4 + x \geq 0$$
$$x \geq -4 \quad or \quad [-4, \infty)$$

$$y = \sqrt{3x+4}$$
$$3x + 4 \geq 0$$
$$3x \geq -4$$
$$x \geq \frac{-4}{3}$$

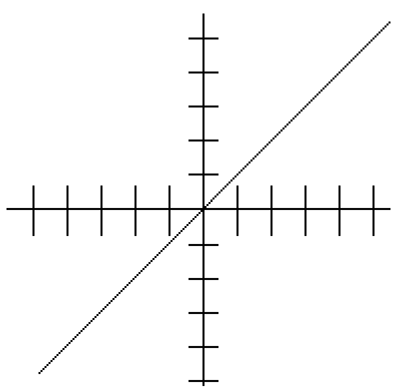
When a square root is in the denominator, set the expression under the square root greater than zero. (Remember, you cannot have zero in the denominator.)

$$y = \frac{x+5}{\sqrt{x-9}}$$

$$x - 9 > 0$$

$$x > 9 \quad or \quad (9, \infty]$$

The range of a function is all of the *y* values, or output, of the function. It is literally, the "range" of values that is achieved by carrying out the function in question. For example, if your equation is "y = x", your range would be all real numbers because the values you get from plugging all possible numbers for x gives back **all real numbers**. If your equation was "$y = x^2$", then your "range" of y values would only be from 0 to infinity!



y = x

Remember: **Range** is along the *y*-axis, and **domain** is along the *x*-axis!

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