

A Collection of Problems

Max Warshauer, Hiroko Warshauer, Nama Namakshi
Department of Mathematics
Texas State University
San Marcos, TX 78666

1. Coffee and Cream

Suppose you have a cup of coffee and a cup of cream. Take a spoonful of the cream, pour it into the coffee, and stir it up. Then take a spoonful of the coffee and cream mixture, pour it back into the cream, and again mix it up. Determine if there is more coffee in the cream cup, more cream in the coffee cup, or the same amount of coffee in the cream cup as cream in the coffee cup.

2. Counting the Ways

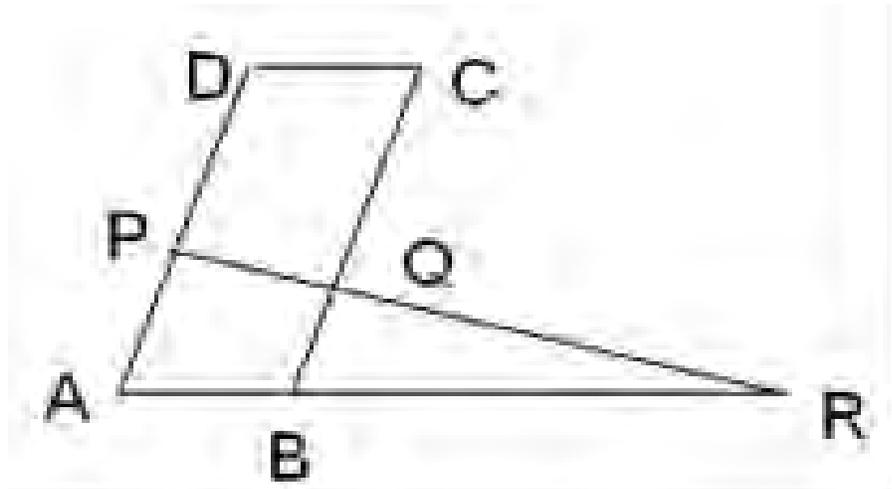
Archimedes, an ant, starts at the origin in the coordinate plane. Every minute he can crawl one unit up or one unit to the right, thus increasing one of his coordinates by 1. How many different paths can Archimedes take to the point (4,3)?

3. The Donut Problem

How many ways are there to select a dozen donuts from three different kinds --glazed, chocolate, and jelly-filled?

4. A Geometry Problem

Let $AB = 10$ units, $AP = PD = 8$ units. Parallelogram $ABCD$ has the same area as triangle APR . Compute the length of QB .



5. The Pan Balance Coin Problem

Among 13 coins of identical size and appearance there is one counterfeit which does not weigh the same as the others. It is not known whether the counterfeit is lighter or heavier than a genuine one (all genuine coins weigh the same). Using three weighings on a balance scale, how can the counterfeit be identified?

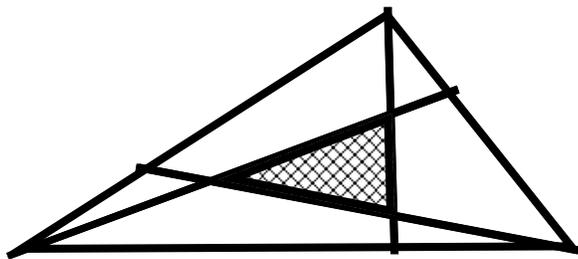
6. The Missionaries and the Cannibals

An island was inhabited by three cannibals and three missionaries. Because the typhoon season was approaching, it became necessary to evacuate the island. Thus, the cannibals and missionaries built a boat to carry themselves to the mainland. Unfortunately the boat would only hold two people, so it was necessary to take several trips.

However, the missionaries were aware that if there were ever more cannibals than missionaries in any location (either on the island or mainland), then the cannibals would revert to their old ways and eat the outnumbered missionaries. Hence it was necessary to plan the crossing in such a way that this never occurred. One last important consideration-- although each of the missionaries knew how to operate the boat alone, the missionaries were able to teach only one of the cannibals the necessary techniques. If either or both of the untrained cannibals attempted to operate the boat, the boat would immediately capsize and the passengers would be eaten by man-eating sharks. How could the cannibals and missionaries transport themselves to the mainland without loss of life.

7. Find that Area (Mass Point Geometry)

The three segments joining the vertices of a triangle to the alternate points of trisection on the opposite sides are drawn as shown below. What is the area of the shaded inner triangle in terms of the area of the original triangle?



8. The Shrewd Prisoner

A prisoner was scheduled to be executed. However, he was given one last reprieve to the effect that he would be released if he could draw a white ball from one of two similar urns. The rule was that he would be given 50 white and 50 black balls to distribute into the two urns. The story is that the prisoner drew a white ball and was released. How should the prisoner distribute these balls so that the chance of picking a white ball would be as great as possible? Would it help if there were more than two urns?

9. Football Teams

In how many different ways can 33 boys be divided into 3 football teams of 11 boys each?

10. Ice Cream Cones

A store sells 11 different flavors of ice cream. In how many ways can a customer choose 6 different ice creams cones, not necessarily of different flavors?

11. Making “Cents”

In how many ways can 200 cents be put together out of pennies and nickels?
Out of pennies, nickels, and dimes?

12. Weighing with unequal balance

A balance is constructed with arms of unequal length. In order to weigh 2kg of sugar, a cook proceeds as follows: first he puts a 1 kg weight on the left-hand scale and pours sugar on the right-hand one until the two scales are balanced; emptying the two scales, he then puts the 1 kg weight on the right-hand scale and pours sugar on the left-hand one until the two scales are balanced.

Are the two quantities of sugar taken together less than 2 kg, more than 2kg, or exactly 2 kg?
Hint: If one weighs two masses of weight m_1 and m_2 , and the masses are located at distances d_1 and d_2 respectively, then the scale will balance when $m_1d_1 = m_2d_2$.

13. Trick Dice

A gambler made three dice.

The red dice had the numbers 2,4,9 twice on its faces.

The blue dice had the numbers 3,5,7 twice on its faces.

The yellow dice had the numbers 1,6,8 twice on its faces.

The total on the faces of each dice was the same, but the gambler was confident that if he let his opponent choose a dice first and roll it he could select a dice which would give him a better chance of obtaining a higher score. Explain!

14. The Hundred Artisans

There was once a medieval Bavarian King named Gustav who loved to give lots of gold toys to the children of his kingdom every Christmas. To this end, he employed 100 skilled artisans to do nothing but make gold toys year-round. Every day each artisan was given 16 ounces of gold to make a one-pound toy.

A rumor suddenly reached Gustav one November that one of his artisans had been cheating him by using only 15 ounces of gold in his toys and keeping 1 ounce of gold for himself. The king was furious and decided that if the rumor were true and he could discover which of his artisans had been dishonest, that man would be executed immediately. In the castle there was a very large graduated scale (like a doctor's office scale) which (miraculously for Bavaria) registered in pounds and ounces. How could Gustav discover whether the rumor was correct and which artisan was guilty, by using the scale only once?

15. The Penny Triangle

Use 10 pennies to form a triangle, with 4 pennies on the bottom row, 3 pennies on the second row, two pennies on the third row, and 1 penny on the 4th row. Can you turn the triangle upside down moving only 3 pennies

16. Rows of Numbers

Begin writing numbers in rows, with 7 numbers in each row:

1 2 3 4 5 6 7
8 9 10 11 12 13 14
15...

Which column will the number 47 be in? The number 2012?

17. Medical Tests

A medical test for a rare (but fatal) disease has been developed that gives the correct result 99.9% of the time. The disease is so rare that only 1/10,000 people has it. If Mary is given the test, and the result is positive (i.e. according to the test, Mary has the disease), how worried should she be?

18. That Elusive Mailbox

A woman needs to walk to her mailbox, which is 30 steps away. However, each time she takes three steps forward, she must take two steps backwards. How many steps will it take to reach the mailbox?

19. The Shaded Circles (from PMWC 2012)

In the figure on the square grid below, the large circle has both shaded and unshaded areas. Find the ratio of the shaded areas to the unshaded areas.

