STRAIGHT LINES

Let's look at some equations: $y = -3x + 5$, $3x - 4y = 9$, $y - \frac{-2}{3}x$

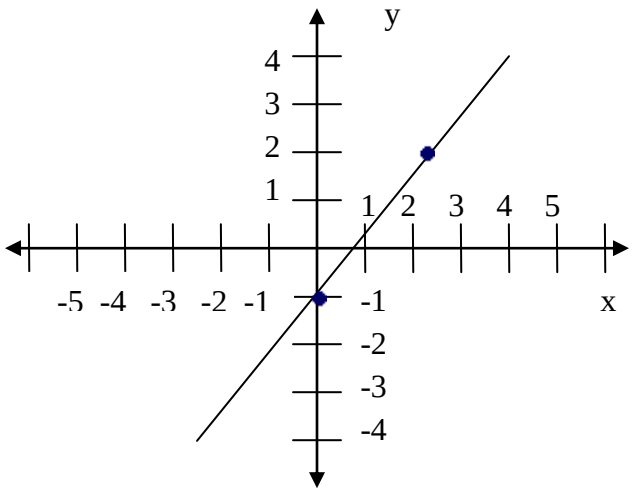
These are called first degree equations because the powers of the *x's* are (1). All of the above equations can be written as: $y=mx+b$, where *m* is the slope of the line and *b* is the y-intercept (where the line crosses the y-axis):

$y = -3x + 5$	slope = -3 and y – intercept = 5
$3x - 4y = 9$	slope = $\frac{3}{4}$ and y – intercept = $\frac{-9}{4}$
$y = \frac{-2}{3}x$	slope = $\frac{-2}{3}$ and y – intercept = 0

SLOPE:

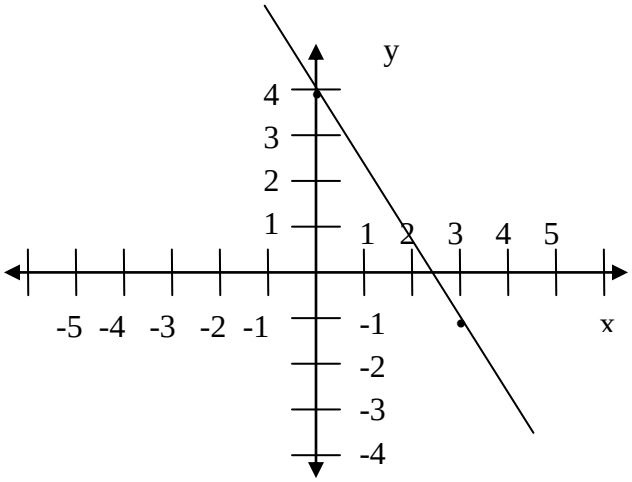
$$slope = \frac{rise}{run} = \frac{\text{vertical movement; upward (positive) or downward (negative)}}{\text{horizontal movement; forward (to the right ALWAYS)}}$$

Example 1: Graph the line whose y-intercept = -1 and slope = 3/2



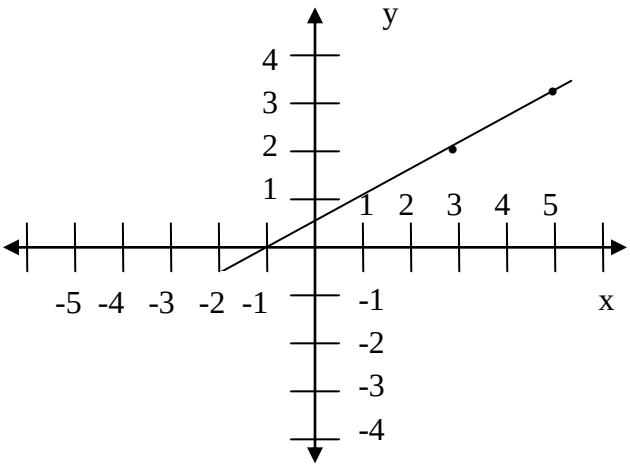
Notice that:
* y-intercept = -1
* go up 3 over 2
  (rise = 3, run = 2)

Example 2: Graph the line whose y-intercept = 4 and slope = -5/3



Notice that:
*y-intercept = 4
*go down 5 over 3

Example 3: Graph the line whose slope = ½ and passes through the (3, 2)



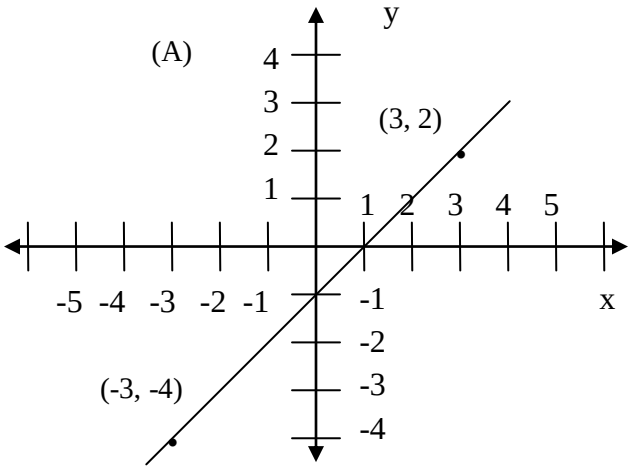
Notice that:
* point (3, 2)
* go up 1 over 2

To find the slope between two points (x_1, y_1) and (x_2, y_2) use the formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Example 4: Sketch a line through each pair of points and find the slope of each line:

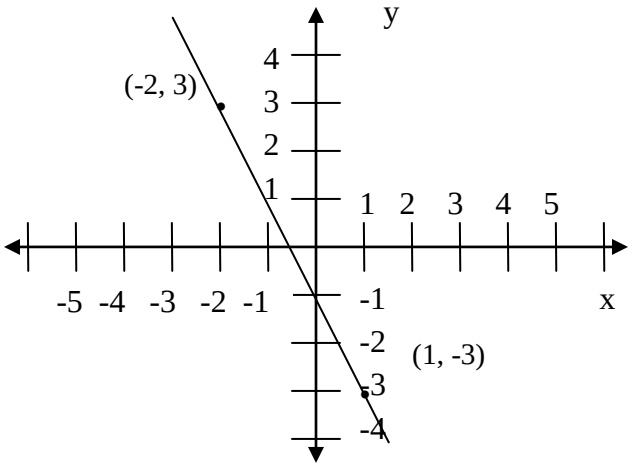
(A) (-3, -4), (3, 2)
(B) (-2, 3), (1, -3)
(C) (-4, 2), (3, 2)
(D) (2, 4), (2, -3)

(A)



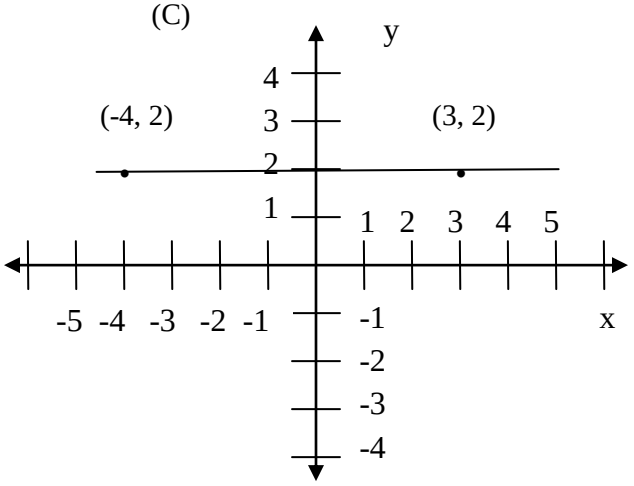
$$m = \frac{2-(-4)}{3-(-3)} = \frac{6}{6} = 1$$

(B)



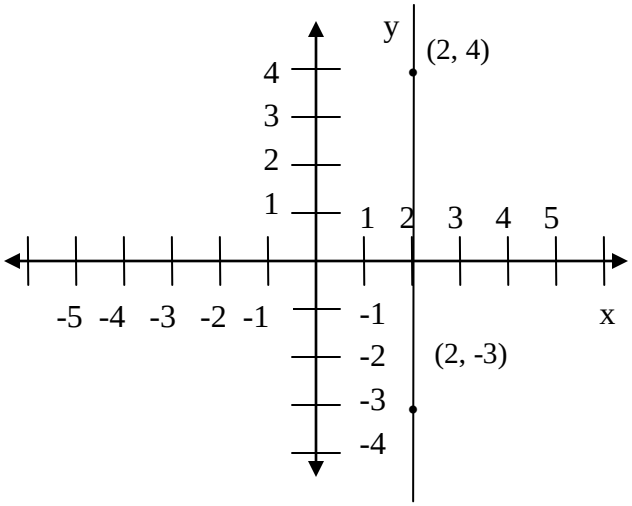
$$m = \frac{-3-3}{1-(-2)} = \frac{-6}{3} = -2$$

(C)



$$m = \frac{2-2}{3-(-4)} = \frac{0}{7} = 0$$

(D)



$$m = \frac{-3-4}{2-2} = \frac{-7}{0} = ND$$

Slope is not defined

Notice that:

- The slope (see above) can be one of four choices: (A) positive, (B) negative, (C) zero or (D) not defined.
- A horizontal line has a slope = 0.
- A vertical line has an undefined slope.

EQUATIONS OF THE LINE:

To find the equation of the line (if you have a point and slope) use the formula $y - y_1 = m(x - x_1)$.

Example 5: Find the equation of the line that has slope = 3 and passes through (2, -5). Using the above formula with m=3, $x_1 = 2$ and $y_1 = -5$ gives:

$y - (-5) = 3\,(x-2)$
$y + 5 = 3x - 6$
$y = 3x - 11$ is your answer.

Example 6: Find the equation of the line that passes through (-2, 1) and (6, -5).

To use the formula above we need a point and a slope just like example 5 above. To find the slope of the line we use the sloe formula and then pick any of the points for the equation:

$$m = \frac{-5-1}{6-(-2)} = \frac{-6}{8} = -3/4$$

if we pick (6, -5) as our point:

$y - (-5) = -3/4\,(x - 6)$
$y + 5 = -3/4\,x + 18/4$
$y = -3/4\,x + 9/2 - 5$
$y = -3/4\,x - 1/2$

if we pick (-2, 1) as our point:

$y - 1 = -3/4\,(x - (-2))$
$y - 1 = -3/4\,(x + 2)$
$y = -3/4x - 6/4 + 1$
$y = -3/4x - 1/2$

Both answers are the same (it does not matter which point you choose).

Example 7: Find the equation of the line that passes through (3, -2) and has an x-intercept 4.

x-intercept 4 means the line passes through (4, 0):

$$m = \frac{0-(-2)}{4-3} = \frac{2}{1} = 2$$

$y - (-2) = 2\,(x-3)$	*Point-slope form*	point (3, -2), slope = 2
$y + 2 = 2x - 6$		
$y = 2x - 8$	*Slope-intercept form*	slope = 2, y-intercept = -8
$y - 2x + 8 = 0$	*General form*	

PERPENDICULAR AND PARALLEL LINES:

Two lines are Parallel if and only if they have the same slope.
Two lines are Perpendicular if and only if they have negative reciprocal slopes.

Example 8: Find the equation of the line that passes through (5, 2) and:
(A) Parallel to the line passing through (4, -1) and (-8, 5).
(B) Perpendicular to the line passing through (4, -1) and (-8, 5).

$$m = \frac{5-(-1)}{-8-4} = \frac{6}{-12} = -1/2$$

1. The two lines are parallel; they have the same slope; m = -1/2.

$y + 2 = (-1/2)(x-5)$
$y = (-1/2)x + (5/2) + 2$
$y = (-1/2)x + (9/2)$

2. The two lines are perpendicular; they have negative reciprocal slopes; $m = +2/1 = 2$.

$y - 2 = 2(x - 5)$
$y = 2x - 10 + 2$
$y = 2x - 8$

Example 9: Find the equation of the line that passes through (-3, 5) and perpendicular to the line L: $3x - 2y = 5$.

First find the slope of L by writing $3x - 2y = 5$ in the equivalent slope-intercept form $y = m x + b$:

$3x - 2y = 5$
$-2y = -3x + 5$
$y = 3/2\ x - 5/2$ $\quad m = 3/2$

Our line is perpendicular to L and has a slope of -2/3:
$y - 5 = -2/3\ (x + 3)$
$y - 5 = -2/3\ x - 2$
$y = -2/3\ x + 3$

Example 10: Find the equation of the line that is the perpendicular bisector of (-3, 2) and (7, -4).

$$m = \frac{-4-2}{7-(-3)} = \frac{-6}{10} = \frac{-3}{5}$$

Our line has slope of 5/3 (perpendicular) and passes through the midpoint which is:

$$\left(\frac{-3+7}{2}, \frac{2-4}{2}\right) = \left(\frac{4}{2}, \frac{-2}{2}\right) = (2,-1)$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$ is the midpoint formula

$y + 1 = 5/3\ (x - 2)$
$y + 1 = 5/3\ x - 10/3$
$y = 5/3\ x - 13/3$