Abstract

Let $G$ be a finite solvable group and $\text{cd}(G)$ the set of character degrees of $G$. The character degree graph $\Delta(G)$ is the graph whose vertices, $\rho(G)$, are the primes dividing the degrees in $\text{cd}(G)$ and there is an edge between two distinct primes $p$ and $q$ if their product $pq$ divides some degree in $\text{cd}(G)$. By Pálfy’s Condition, we know that the diameter of a character degree graph is at most three for a connected graph. Further, we can partition the vertices, $\rho(G)$ into four non-empty disjoint subsets $\rho_1 \cup \rho_2 \cup \rho_3 \cup \rho_4$ where the following is true: No prime in $\rho_1$ is adjacent to any prime in $\rho_3 \cup \rho_4$; no prime in $\rho_4$ is adjacent to any prime in $\rho_1 \cup \rho_2$; every prime in $\rho_2$ is adjacent to some prime in $\rho_3$; every prime in $\rho_3$ is adjacent to some prime in $\rho_2$; and $|\rho_1 \cup \rho_2| \leq |\rho_3 \cup \rho_4|$.

We will present the history on the character degree graphs of solvable groups with diameter three, and present some of the recent results.