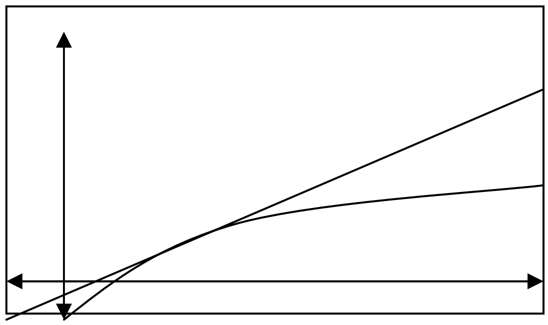
Finding the Equation of a Tangent Line

Using the First Derivative

Certain problems in Calculus I call for using the first derivative to find the equation of the tangent line to a curve at a specific point.

The following diagram illustrates these problems.



There are certain things you must remember from College Algebra (or similar classes) when solving for the equation of a tangent line.

Recall :

- A **Tangent Line** is a line which locally touches a curve at one and only one point.

- The slope-intercept formula for a line is $\mathbf{y = mx + b}$, where m is the slope of the line and b is the y-intercept.

- The point-slope formula for a line is $\mathbf{y - y_1 = m\,(x - x_1)}$. This formula uses a point on the line, denoted by (x_1, y_1), and the slope of the line, denoted by m, to calculate the slope-intercept formula for the line.

Also, there is some information from Calculus you must use:

Recall:

- The first derivative is an equation for the slope of a tangent line to a curve at an indicated point.
- The first derivative may be found using:

A) The definition of a derivative:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

B) Methods already known to you for derivation, such as:

- Power Rule
- Product Rule
- Quotient Rule
- Chain Rule

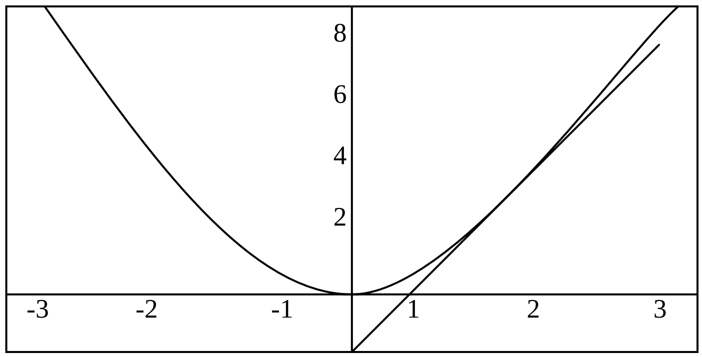
(For a complete list and description of these rules see your text)

With these formulas and definitions in mind you can find the equation of a tangent line.

Consider the following problem:

Find the equation of the line tangent to $f(x) = x^2 at\ x = 2.$

Having a graph is helpful when trying to visualize the tangent line. Therefore, consider the following graph of the problem:



The equation for the slope of the tangent line to $f(x) = x^2$ is $f'(x)$, the derivative of f(x). Using the power rule yields the following:

$$f(x) = x^2$$
$$f'(x) = 2x \qquad (1)$$

Therefore, at x = 2, the slope of the tangent line is $f'(2)$.

$$f'(2) = 2(2)$$
$$= 4 \qquad (2)$$

Now , you know the slope of the tangent line, which is 4. All that you need now is a point on the tangent line to be able to formulate the equation.

You know that the tangent line shares at least one point with the original equation, $f(x) = x^2$. Since the line you are looking for is tangent to $f(x) = x^2$ at x = 2, you know the x coordinate for one of the points on the tangent line. By plugging the x coordinate of the shared point into the original equation you have:

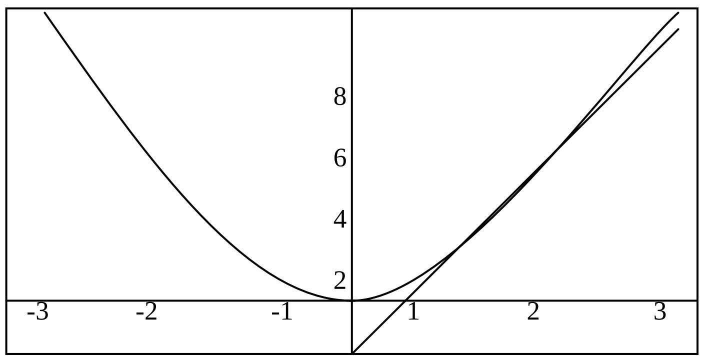
$$f(x) = (2)^2$$
$$= 4 \qquad \text{or} \qquad y = 4 \qquad (3)$$

Therefore, you have found the coordinates, (2, 4), for the point shared by f(x) and the line tangent to f(x) at x = 2. Now you have a point on the tangent line and the slope of the tangent line from step (1).

The only step left is to use the point (2, 4) and slope, 4, in the point-slope formula for a line. Therefore:

$$y - y_1 = m(x - x_1)$$
$$y - 4 = 4(x - 2)$$
$$y - 4 = 4x - 8 \qquad (4)$$
$$y = 4x - 4$$ This is the equation for the tangent line.

Finally, check with the graph to see if your answer is reasonable.



(5)

The tangent line appears to have a slope of 4 and a y-intercept at –4, therefore the answer is quite reasonable.

Therefore, the line y = 4x – 4 is tangent to $f(x) = x^2$ at x = 2.

Here is a summary of the steps you use to find the equation of a tangent line to a curve at an indicated point:

1) Find the first derivative of f(x).
2) Plug x value of the indicated point into f '(x) to find the slope at x.
3) Plug x value into f(x) to find the y coordinate of the tangent point.
4) Combine the slope from step 2 and point from step 3 using the point-slope formula to find the equation for the tangent line.
5) Graph your results to see if they are reasonable.

Bibliography

Larson, R.E. and Hostetler, R.P. (1994). Calculus: With Analytical Geometry (5^{th} ed.). Lexington, KY: D.C. Health and Co.

Revised Spring 2004
Created by Jay Whitehead, March 1999
Student Learning Assistance Center (SLAC)
Texas State University- San Marcos